A QUEST FOR NEW PHYSICS

LOOP CALCULATIONS OF THE HIGGS DECAY TO TWO PHOTONS IN THE STANDARD MODEL EFFECTIVE FIELD THEORY

BY CHRISTINE HARTMANN
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A Quest for New Physics,
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Abstract

The discovery of a Higgs like boson in 2012 closed many chapters within particle physics, providing a completion of the standard model field content. As a result of this event, many new chapters have opened. These take the shape of unresolved issues within the standard model, as well as physics, which the standard model can not incorporate. Such open questions and insufficiencies of the standard model provide exciting hints of new physics, which can not be ignored. The quest for new physics has merely begun. The second run of the particle accelerator at CERN, LHC, is expected to provide interesting new data in the very near future. Such data requires careful treatment, also from a theoretical perspective.

New physics could manifest itself through the discovery of new particles. On the other hand, with new physics possibly being out of reach for present and future experiments, it could instead show up in the shape of deviations within existing interactions of the standard model. This thesis addresses the latter case through explicit calculations, incorporating new physics in the shape of higher dimensional operators. The standard model is taken to be an effective field theory and extended to include new interactions appearing in these additional terms of the Lagrangian.

The full one-loop contributions to the $h \rightarrow \gamma \gamma$ decay, within this standard model effective field theory, are presented. These contributions allow for deviations of the standard model expectations on the order of one percent. Such deviations will have an impact already with data from Run II, which will reach a comparable level of precision.

In conclusion, the results presented in this thesis are necessary to take into account to obtain a proper fit of the standard model effective field theory to experimental data. Following these main results, an outline of work in progress regarding the two-loop contributions will be given.

Den fulde udregning af et-loop bidragene til $h \rightarrow \gamma \gamma$ henfaldet, under denne udvidelse af standard modellen præsenteres. Disse bidrag tillader afvigelser af størrelsesordenen en procent i forhold til, hvad der forventes i standard modellen. De har derfor stor betydning for data, der vil blive tilgængelige under denne anden opstart af LHC, da disse data vil opnå samme præcisions niveau.

Resultaterne præsenteret i denne afhandling er dermed vigtige at tage med i betragtning, for at opnå en korrekt fortolkning af eksperimentelle data, set i lyset af standard modellen som værende en effektiv felt teori. Til sidst gives en oversigt over fremskridende arbejde inden for udregningen af tilsvarende to-loop bidrag.
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Beforehand, I obtained valuable skills within the area of implementing programming tools, under the guidance of Professor Gudrun Heinrich and PhD student Sophia Borowka, while staying abroad at the Max Planck Institute for astro- and particle physics in Munich. The gained experience within this more technical area of particle physics has had a big impact on the calculational part of this thesis. The obtained experience has helped increase the pace of the calculations, as well as provide independent checks on the results.

I appreciate having been given the opportunity to stay for nine months at CERN, while gaining knowledge within the field of research around this thesis. Interaction with frontline researchers and other PhD students allowed me to get a broader insight into the field of theoretical particle physics.

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Contents

Abstract i
Resumé i
Acknowledgements iii
Contents vii
Introduction 1

1 Motivation 3

2 The Standard Model of Electroweak Interactions 7
  2.1 Spontaneous Symmetry Breaking in Effective Field Theories 7
  2.2 The Higgs Mechanism 7
  2.3 Chiral Perturbation Theory 10
  2.4 The Standard Model Lagrangian 13
  2.5 Gauge Fixing 15
  2.6 Background Field Method 18
  2.7 The Choice of a Gauge 19
  2.8 The Unitarity Problem 20

3 Beyond the Standard Model 27
  3.1 Experimental Data 28
  3.2 The Standard Model Effective Field Theory 34
  3.3 Bases of Higher Dimensional Operators 35
  3.4 Power Counting in the Standard Model Effective Field Theory 37
7.3 Renormalization of the Two-Loop Calculation ............................................. 101
  7.3.1 Tree-Level Stage .................................................................................. 101
  7.3.2 One-Loop Level Stage ........................................................................... 103
7.4 SecDec ........................................................................................................ 104
7.5 The Two-Loop Program, FORM to SecDec .................................................. 104
7.6 Initiating the Two-Loop Calculation of the $h \rightarrow \gamma \gamma$ Decay - the $\lambda$ Dependence 105

8 Conclusion ....................................................................................................... 113

Bibliography ....................................................................................................... 117

Appendix ............................................................................................................. 129

A List of Abbreviations ........................................................................................ 129

B Calculational Methods .................................................................................... 131
  B.1 FORM ........................................................................................................ 131
  B.2 Passarino-Veltman Decomposition ............................................................ 131
  B.3 Passarino-Veltman Reduction .................................................................... 133
  B.4 Feynman Parametrization ......................................................................... 134
  B.5 Integrals in Dimensional Regularization .................................................. 134

C Feynman Rules in the Background Field Method ............................................ 137

D Identities for Gamma Matrices ....................................................................... 143

E The Standard Model One-Loop $h \rightarrow \gamma \gamma$ Decay Amplitude .................. 145

F Higgs Field Renormalization .......................................................................... 147

G Vacuum Expectation Value Renormalization ............................................... 153

H Feynman Rules for Effective Operators ....................................................... 155

I Direct Amplitudes ............................................................................................ 161

J Indirect Amplitudes .......................................................................................... 165

K Complete $R_\xi$ Result .................................................................................... 169
The standard model (SM) has been the solid periodic table of particle physics for many years and further established through the discovery of a boson in 2012, which behaves a lot like the Higgs particle. This discovery was the main accomplishment of the first run of the Large Hadron Collider LHC [1, 2]. After a two year shutdown, the beams of the LHC pipelines have started circulating once again, and collisions have already provided data from this second run.

There are still questions in particle physics, which the SM does not provide answers to. The SM does a good job in describing the particles, we know exist today. However, these only make up about four percent of our universe. The rest, such as dark energy and dark matter is unknown territory. Besides this insufficiency of the SM, a direct flaw has also been found within the SM, concerning neutrinos. The existence of neutrino oscillations, and therefore neutrino masses, requires certain revisions within the SM, perhaps even extensions.

Therefore, the goal of this Run II is undoubtedly to discover new physics. However, new physics might not show up, the way we expect, namely in the shape of newly discovered particles. Instead, it could manifest itself as deviations in what is expected to be found. This scenario becomes more likely, when experimental precision is increased. The search for deviations in measurements of the Higgs couplings is therefore an interesting area to investigate in the quest for new physics. The presence of such deviations would be the evidence that the SM is only part of the story. With no new particles in sight, or knowledge on where to look for such particles, the areas to search for new physics have therefore changed. Much attention is currently being addressed towards looking for deviations in known interactions.

Observation of deviations in experimental measurements could be interpreted as a hint of new physics. However, such new physics could be present at an energy scale, which is not yet possible to probe. The deviations will most likely only be weak traces, left behind for physicists to analyse and interpret. It is therefore necessary to have a theoretical tool, which can reveal what exactly such deviations mean in terms of underlying new physics.

A general approach is widely being used, which provides a possibility to eventually account for measured experimental deviations. In this approach, the SM is assumed to be an effective field theory (EFT) and extended in a general way, including operators of higher mass dimensions in...
the Lagrangian. This new extended general theory, the standard model effective field theory (SMEFT) will be the main focus of this thesis.

Experiments are at high speed showing increased precision in measurements, and this second run of the LHC will get data for $\Gamma(h \rightarrow \gamma \gamma)$ with a precision of $\leq 10\%$ at a luminosity of $\int \mathcal{L} \, dt = 300 \, \text{fb}^{-1}$ [3]. A higher luminosity run of the LHC, $\int \mathcal{L} \, dt = 3000 \, \text{fb}^{-1}$, is expected to push the precision of this measurement down to the few percent level. Future facilities, such as a Triple Large Electron Positron (TLEP) collider can further improve this precision by as much as an order of magnitude, as reported by the Snowmass Working Group [4].

It is important for theoretical calculations to match the level of experimental precision, in order to correctly interpret the measured data [5]. Loop level calculations within the SMEFT are already necessary with present available electroweak precision data (EWPD). The necessity for such calculations will furthermore increase in the near future, to account for the impressive increase of precision, expected from experiments. Loop contributions can modify interactions with percents, matching the level of precision obtained during the high luminosity run of the LHC.

In this thesis, contributions to the decay rate, $\Gamma(h \rightarrow \gamma \gamma)$, will be fully calculated at the one-loop level within the SMEFT using a linear parametrization of the scalar sector. These contributions will prove to be highly relevant when compared to measured data. Theoretical errors associated with such loop corrections are estimated to be of $O(1\%)$. This estimate will be confirmed, when carrying out the explicit calculations. Compared to the estimated uncertainty of the high luminosity Run II of the LHC, reaching $O(1\%)$ precision, the corrections from these loop calculations are of the same order of magnitude as the experimental error. To account for possible measured deviations in experiments and therefore to interpret the presence of new physics accurately, these loop calculations are necessary to include for a consistent fit. Either as estimated theoretical errors, or through explicit calculations. Especially in channels measured as precisely as the $h \rightarrow \gamma \gamma$ decay.

The renormalization group (RG) contributions, both previously known and unknown, will be revisited and included in the result [6–11]. However, calculations of the pure "finite terms" will make up the main accomplishment of the thesis. These turn out to be just as relevant as the RG contributions and therefore necessary to include in the full SMEFT contribution at one-loop, to account for a consistent analysis of the data. The complete result, including RG contributions and "finite" terms, will be presented for the specific decay, $h \rightarrow \gamma \gamma$. An outline will be given for the calculational methods behind this calculation.

Finally, an outlook will be presented initiating a two-loop calculation within the same process. Such two-loop contributions are highly suppressed compared to the one-loop contributions, as can be inferred from the theoretical uncertainty associated with such contributions, as will be shown. It is however not unlikely that such calculations become relevant in the future, to account for improved EWPD.
The Higgs mechanism makes up a mystery of the SM, which is yet to be solved. It provides a convenient and seemingly accurate solution to how particles of the SM obtain masses, as will be shown in section 2.2 [12–15]. However, the Higgs mechanism itself demands answers. For the Higgs mechanism to take place, electroweak symmetry breaking (EWSB) is required. The question is, which underlying mechanism breaks this symmetry? Is it strongly or weakly interacting?

Finding answers to such questions could help shed light on possible underlying new physics. It could also help solve another problem associated with the Higgs mechanism, or more explicitly, the Higgs boson. A problem of naturalness shows up with the Higgs mechanism, since there is no support for light scalar fields, such as a light Higgs, in quantum field theory (QFT). Some solutions to this hierarchy problem, as well as answers to the above questions, hint towards the Higgs mechanism, and therefore the SM, being itself an EFT.

In particle physics, symmetries play an important role as guidelines, when looking for new physics. The approach to account for experimental discoveries is often to match these with underlying symmetries. This has so far proven to be an adequate guideline for describing existing physics, as well as for discovering new physics. The building blocks of the SM are symmetries, which dictate what is allowed and what is not. Gell-Mann’s *Totalitarian Principle* says it well: "Everything that is not forbidden is compulsory" [16]. The quest for new physics presently follows the same approach. New physics manifesting itself through new particles is generally associated with an underlying symmetry, dictating how these particles behave.

The problem associated with the Higgs mass, the hierarchy problem, is one of the main concerns that have led to speculations in Higgs physics. Following Gell-Mann’s *Totalitarian Principle*, large contributions to scalar masses are expected from ultraviolet (UV) corrections, since these are not forbidden by the existing symmetries within the SM. As an example, the Higgs mass gets contributions from one-loop interactions with the top quark, the SU(2) gauge bosons and the Higgs itself, as seen in figure 1.1. These diagrams only give contributions to the Higgs mass at the electroweak (EW) scale. Therefore, no significant finetuning is needed to account for such contributions. However, if particles were to exist at a higher energy scale, Λ, such particles could induce large contributions to the Higgs mass, proportional to this scale, Λ. Following dimensional analysis, this scale must enter with a term like \((\Lambda^2 + v^2)H^\dagger H\), unless
some symmetry or mechanism forbids it. The Higgs mass could therefore be as large as the Planck scale.

To contain the Higgs mass, a cutoff scale $\Lambda$ is assumed to exist much below the Planck scale. Since no knowledge currently exists regarding the nature of the new physics around this scale, writing up the full theory is impossible, without making assumptions about the underlying theory. The most general approach is therefore to assume the SM to be an EFT, where the heavy fields do not enter the Lagrangian explicitly, but are assumed to have been integrated out. This leaves traces behind in the SM Lagrangian manifesting themselves as higher dimensional operators. The cutoff scale $\Lambda$ corresponds to the mass scale of the associated new particles, which appears as a suppression scale in these operators in the effective Lagrangian.

Within the SM, everything which is not forbidden by its symmetries is compulsory. The large contributions to the Higgs mass on account of this, is generally seen as unnatural. However, the unnaturalness of nature could be a fact to accept, accounted for through fine-tuning. Such a point of view finds the naturalness problem to be of purely aesthetic nature. The overwhelming successes of the SM could be convincing enough to overrule the importance of such a, perhaps minor, flaw.

On the other hand, the quest to solve this naturalness problem is widespread within the community of particle physics. A reason for this could be that similar hierarchy problems have arguably been solved before. The solutions turned out to include the existence of particles and symmetries. In one case, it led to discoveries.

This happened in the chiral sector of the SM and regarded the electron [17, 18]. Before the era of quantum mechanics (QM), classical electromagnetic theory only consisted of electrons, the electric field and the magnetic field. Around the electron, a Coulomb electric field was known to prevail, an electro-static energy making one end of the electron repulsive to the other. The size of this energy is classically proportional to the inverse radius of the electron mass $\Delta E = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r_e}$. This energy must therefore contribute to the electron mass,

$$m = m_0 + \Delta E.$$  \hspace{1cm} (1.1)

The mass and the radius of the electron are measured to be very small. The Coulomb energy, $\Delta E$ is therefore very large both according to, and in contradiction to, the measured data. To account for the contradiction, another contribution to the electron mass had to enter, to cancel this large energy.

The solution to the electron hierarchy problem involved QM and the discovery of the anti-
Motivation

![Figure 1.2: Electron and positron creation with a photon. The electron self-energy really involves the emission of a photon with the creation of a positron.](image)

electron, the positron. In QM, quantum fluctuations are possible, where a pair of an electron and a positron is generated with a photon out of nothing, see figure 1.2. The self-energy of the electron, described classically as a Coulomb energy, is instead in QM seen as an emission and absorption of a photon, see figure 1.2. This generates a large contribution to the electron self-energy, proportional to the cutoff, introduced to contain the amplitude of the diagram. This cutoff is accounted for by the existence of the positron and quantum fluctuations.

Intuitively, this corresponds to the existence of $e^+ e^-$ pairs smearing out the electric charge of the electron over a much larger area, thus making the radius bigger and the self-energy, being inversely proportional to the radius, smaller.

The appearance of the positron furthermore introduced a new symmetry, the chiral symmetry. If this symmetry is exact, the electron would be massless. The explicit breaking of this symmetry, however, introduces a small mass for the electron, in accordance with reality. This symmetry therefore also prevents the electron mass from getting large self-energy corrections. Without quantum mechanics entering to solve the electron hierarchy problem, as described above, this could have provided an alternative solution. In the case of the electron, this symmetry protection is merely a consequence of the solution. However, this type of solution is being exploited in other areas of physics, also in the area concerning the Higgs hierarchy problem.

In the area of QCD, the naturalness problem has also been solved, although the solution existed before the problem was formulated. It concerns the strongly interacting pions, two charged and one neutral [18, 19]. The charged pions, like the electron, receive contributions to their masses due to electromagnetic interactions with the photon, as seen in figure 1.3. Their masses are accordingly bigger than the mass of the neutral pion, which does not undergo such electromagnetic interactions, namely

\[
m^2_{\pi^\pm} - m^2_{\pi^0} \sim (35.5 \text{ MeV})^2
\]

This mass difference is therefore expected to be a result of the extra electromagnetic interaction, undergone by the charged pions. However, such an interaction would induce a contribution to the charged pion masses of the size,

\[
\delta m^2 \sim \frac{3 e^2}{16 \pi^2} \Lambda^2
\]

depending on the cutoff of the theory. The cutoff is therefore expected to be of the size $\Lambda \sim 850 \text{ MeV}$, to account for the measured mass difference of the pions. The solution to this
Figure 1.3: The pion receives electromagnetic contributions to its mass from interacting with the photon.

hierarchy problem already existed in the shape of the $\rho$ meson, which has a mass of 770 MeV, naturally cutting off the contribution to the mass difference.

From these success stories, the same things are believed to happen for the Higgs field and its undefined mass. The large self-energy corrections should be contained by some structure around the TeV scale. This could be in the form of new particles, whose interactions would generate amplitudes canceling off the self-energy diagrams of the Higgs. Some new symmetry could also come in to protect the Higgs mass, requiring it to be massless at leading order. After explicit breaking of the symmetry, the Higgs acquires mass only through small radiative corrections.

Such broken symmetries involving new particles have been widely considered, to protect the Higgs potential in this way and stabilize its mass to the measured value. The solution to the electron hierarchy problem has today mainly inspired the development of supersymmetry (SUSY) [20], where space time symmetry is extended to include a symmetry between fermions and bosons. The underlying theory is in this case weakly coupled. The solution to the pion hierarchy problem has inspired the development of composite Higgs models, where a broken shift symmetry occurs, assuming an underlying strongly coupled theory. In both cases, contributions from radiative corrections are accounted for through new particles entering at the cutoff scale.

In this thesis, an approach will be implemented, where the SM is considered to be an EFT. The EFT approach is an efficient technique for going beyond the SM, without having met signs of new physics. The concept of EFTs was already introduced in the 70’s [21, 22]. This extension of the SM, the SMEFT, is a general approach, since it does not assume any underlying particular model, weakly or strongly interacting. The idea behind this approach is that some mechanism exists to solve the hierarchy problem, perhaps also accounting for further flaws known to exist in the SM. No knowledge around this mechanism is necessary. Calculations and fits to experimental data carry no assumptions and act to observe and interpret rather than describe. Therefore, this approach provides a solid and unprejudiced attempt to learn about new physics beyond the SM.
The SM, or the Glashow-Weinberg-Salam model, was first proposed for leptons in [23–25] and extended to include quarks in [26]. It is a renormalizable anomaly free theory [27], which describes the world of particles known to exist today. The latest accomplishment was the discovery of a Higgs like boson on the 4th of July, 2012, presumably completing the last part of the SM puzzle [1, 2].

2.1 Spontaneous Symmetry Breaking in Effective Field Theories

Spontaneous symmetry breaking (SSB) is an important technique within the SM. Even though it makes up the most uncertain area of the SM, as described in section 1, many models beyond the SM have implemented this technique as a fundamental building block. These models include SUSY, technicolor and composite Higgs models. In the following sections, two representative cases within the SM will be addressed, where SSB occurs on very different grounds, namely the Higgs mechanism and chiral perturbation theory (χPT). In the Higgs mechanism, the dynamics responsible for the SSB is unknown and weakly coupled. In χPT, describing the low energy limit of QCD, the underlying theory is strongly interacting.

2.2 The Higgs Mechanism

The Higgs mechanism, embodied in the SM, represents a weakly interacting sector breaking the EW symmetry [12–15]. With the discovery of a boson with the same properties as those expected for the Higgs boson, experiments highly favour such a mechanism, since this boson, very likely to be the Higgs boson, is light and therefore undergoes weakly coupled self-interactions.

The SM Higgs potential is invariant under the global symmetry group $SU(2)_L \times SU(2)_R$. This global symmetry is spontaneously broken by the vacuum expectation value (VEV) of the
2.2. The Higgs Mechanism

The Standard Model of Electroweak Interactions

Higgs into the diagonal subgroup $SU(2)_{L+R}$, the custodial symmetry group, from which the masses of the gauge fields $W^\pm$ and $Z$ are found to be related,

$$m_w^2 = \frac{1}{4} g_2^2 v^2, \quad m_z^2 = \frac{m_w^2}{c_w^2}.$$  \hspace{1cm} (2.1)

The covariant derivative of the Higgs is given by

$$D_\mu H = \partial_\mu H + ig_2 W_\mu^a T^a H + ig_1 H B_\mu T_3,$$  \hspace{1cm} (2.2)

such that the $SU(2)_L$ subgroup is fully gauged, whereas the $SU(2)_R$ subgroup is gauged by $T_3$. This induces an explicit breaking of the custodial symmetry [28]. The generator of $U(1)_Y$ is the hypercharge, $Y = T_3 + \frac{1}{2} (B - L)$ with $B$ and $L$ being the baryon and lepton numbers. This gauging as well as Yukawa couplings being different for up and down type quarks, cause the global custodial symmetry group to break.

The symmetry group $SU(2)_L \times U(1)_Y$ is thereafter spontaneously broken by the scalar field, the Higgs. The Higgs field is in a complex doublet representation within $SU(2)_L$,

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} i \phi^+ \\ h + v + i \phi_0 \end{pmatrix},$$  \hspace{1cm} (2.3)

which transforms under the symmetry as

$$H \rightarrow e^{i \theta^I \tau^I} H,$$  \hspace{1cm} (2.4)

where $\tau^I$ are the Pauli matrices,

$$\tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$  \hspace{1cm} (2.5)

The Higgs kinetic and potential part of the Lagrangian,

$$\mathcal{L}_{\text{Higgs}} = (\partial_\mu H)^\dagger \partial^\mu H = V(H^\dagger H),$$  \hspace{1cm} (2.6)

is invariant under the $SU(2)_L \times U(1)_Y$ symmetry where the potential is

$$V(H^\dagger H) = \mu^2 H^\dagger H - \lambda (H^\dagger H)^2.$$  \hspace{1cm} (2.7)

When the Higgs acquires its VEV, $\langle H^\dagger H \rangle = v^2$, the symmetry breaks down, $SU(2)_L \times U(1)_Y \rightarrow U(1)_Q$. The VEV is determined minimizing the potential,

$$\frac{\partial V}{\partial (H^\dagger H)} = \mu^2 - \frac{1}{2} \lambda \langle H^\dagger H \rangle = 0, \quad \Rightarrow \quad \langle H^\dagger H \rangle = \frac{\mu^2}{2 \lambda} \equiv \frac{1}{2} v^2, \quad \Rightarrow \quad \mu^2 = \lambda v^2.$$  \hspace{1cm} (2.8)
Plugging this VEV into the Lagrangian,

\[ L_{\text{Higgs}} = \frac{1}{2} (\partial_{\mu} \phi_{+})^2 + \frac{1}{2} (\partial_{\mu} h)^2 + \frac{1}{2} (\partial_{\mu} \phi_{0})^2 + \lambda v^2 h^2 - \frac{1}{4} \lambda v^4. \]  

(2.9)

Under the breaking of the symmetry, a single linear combination of the \( SU(2)_{L} \times U(1)_{Y} \) generators is unbroken, namely \( Q = T^3 + Y \) and three fields in the doublet have become massless, the Goldstone bosons, \( \phi_{\pm} \) and \( \phi_{0} \). To remove these unphysical massless fields, gauge fields, the \( W^{\pm} \), \( \gamma \) and \( Z \) fields are introduced through the covariant derivative,

\[ D_{\mu} = \partial_{\mu} + i g_{2} W_{\mu}^{a} T^{a} + i g_{1} B_{\mu} Y, \]  

(2.10)

as well as a pure kinetic term, such that the Higgs Lagrangian, including these Yang-Mills terms becomes [29],

\[ L_{\text{Higgs}+\text{YM}} = (D_{\mu} H)^{\dagger} D^{\mu} H - \frac{1}{4} W_{\mu \nu}^{a} W^{a \mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} - \lambda \left( H^{\dagger} H - \frac{1}{2} v^2 \right)^2. \]  

(2.11)

It is appropriate to introduce the following rotation of the gauge fields,

\begin{align*}
W_{\mu}^{3} &= c_{w} Z_{\mu} + s_{w} A_{\mu}, \\
B_{\mu} &= -s_{w} Z_{\mu} + c_{w} A_{\mu}, \\
W_{\mu}^{1} &= \frac{1}{\sqrt{2}} (W_{\mu}^{+} + W_{\mu}^{-}), \\
W_{\mu}^{2} &= \frac{i}{\sqrt{2}} (W_{\mu}^{+} - W_{\mu}^{-}),
\end{align*}  

(2.12)

where

\[ s_{w} = \frac{g_{1}}{\sqrt{g_{2}^2 + g_{1}^2}}, \quad c_{w} = \frac{g_{2}}{\sqrt{g_{1}^2 + g_{2}^2}}. \]  

(2.13)

This results in three massive gauge fields, which can absorb the three unphysical Goldstone bosons, as well as one massless field, corresponding to the photon. The terms of interest, expanding the Higgs kinetic term are given by

\[ (D_{\mu} H)^{\dagger} D^{\mu} H = \left( \partial_{\mu} \phi_{+} + \frac{1}{2} g_{2} v W_{\mu}^{+} \right) \left( \partial^{\mu} \phi_{-} + \frac{1}{2} g_{2} v W_{\mu}^{-} \right) + \frac{1}{2} (\partial_{\mu} \phi_{0} - \frac{1}{2} \frac{g_{2}}{c} v Z_{\mu})^2 + ... \]  

(2.14)

The kinetic Higgs term involves mixing amongst the gauge fields and the unphysical Goldstone boson. The gauge fields are therefore redefined to absorb the unphysical massless fields,

\begin{align*}
W_{\mu}^{\pm} &\rightarrow W_{\mu}^{\pm} - \frac{2}{g_{2} v} \partial_{\mu} \phi_{\pm}, \\
Z_{\mu} &\rightarrow Z_{\mu} + \frac{2 c}{g_{2} v} \partial_{\mu} \phi_{0},
\end{align*}  

(2.15)
such that only the mass terms for these gauge fields are recovered,

\[(D_\mu H)^\dagger D^\mu H = \frac{1}{4} g_2^2 v^2 W_\mu^+ W_\mu^+ + \frac{1}{4} \frac{g^2}{c^2} v^2 Z_\mu Z^\mu + \ldots\] (2.16)

The Higgs mechanism therefore proves to be an effective way of accounting for the SSB and the gauge field masses.

### 2.3 Chiral Perturbation Theory

When addressing an EFT, assuming an underlying strongly coupled theory, the approach within chiral perturbation theory (χPT) can be followed [19, 22, 30, 31]. This theory involves an underlying global symmetry group, a chiral symmetry, such as \(SU(3)_L \times SU(3)_R\), describing the light quarks, \(u, d\) and \(s\). Under this symmetry group, the quarks are in left-handed and right-handed triplet representations, transforming under this symmetry as

\[
\psi_L \rightarrow L \psi_L \\
\psi_R \rightarrow R \psi_R
\] (2.17)

The global symmetry group is spontaneously broken at the chiral symmetry breaking (χSB) scale, \(\Lambda_\chi\) to the diagonal subgroup \(SU(3)_V\). This leads to eight Goldstone bosons, the \(\pi\)'s, K's and the \(\eta\). They undergo a shift under infinitesimal chiral transformations, which prevents them from obtaining masses, unless this symmetry is broken. Assuming the confinement scale, \(\Lambda_{QCD}\), where quarks and gluons are bound in hadrons, to be less than the scale \(\Lambda_\chi\), an EFT can be studied in this intermediate area, where quarks, gluons and the above mentioned Goldstone bosons are considered to be fundamental particles [32].

Besides the spontaneous symmetry breaking, an explicit symmetry breaking of the global symmetry group occurs, when the three massless quarks are allowed to acquire masses, \(m_u\), \(m_d\) and \(m_s\). The Goldstone bosons become approximately massless, pseudo Goldstone bosons, with their masses vanishing as the symmetry breaking scale, \(f_\pi\) goes to infinity.

χPT is a bottom up approach, where the specific underlying UV model, responsible for the SSB is unknown. Only the pattern of the symmetry breaking is known. This however, leads to some predictivity through a power counting tool. The power counting depends on what kind of parametrization is being used for the symmetry breaking. In the case of having only two light quarks, \(u\) and \(d\), transforming under the global symmetry \(SU(2)_L \times SU(2)_R\), the scalar sector, consisting of a sigma field \(\sigma\) and three pions \(\pi = (\pi_1, \pi_2, \pi_3)\), can be parametrized linearly in the linear sigma model [33], in the following way,

\[\pi = \sigma + i \tau \cdot \sigma\] (2.18)

where \(\tau = (\tau^1, \tau^2, \tau^3)\) are the Pauli matrices. The chiral transformation of these scalar fields therefore occurs linearly under the global symmetry group, \(\pi \rightarrow L \pi R^\dagger\). Under SSB, the \(\sigma\) field is shifted by the VEV and remains massive, whereas the three pions, \(\pi_i\), become massless, as was also seen for the Higgs field \(h\) and the three Goldstones \(\phi^\pm\) and \(\phi_0\) respectively, in
2.3. Chiral Perturbation Theory

the Higgs mechanism. Under the unbroken symmetry group $SU(2)_V$, the sigma field remains invariant and the massless pions transform as $\pi \to V \pi V^\dagger$.

To acquire an exponential form, the $\sigma$ and $\pi$ fields can be shifted appropriately to form the following representation,

$$\sigma + i \mathbf{\tau} \cdot \mathbf{\pi} \to (f_\pi + S)\Sigma$$

(2.19)

where $\Sigma = e^{i \mathbf{\tau} \cdot \mathbf{\pi}/f_\pi}$ and $f_\pi \simeq 93$ MeV [34] is the Goldstone boson decay constant. In this exponential case, the fields transform as $S \to S$ and $\Sigma \to L\Sigma R^\dagger$. The $\pi$ field itself therefore transforms non-linearly. The two representations parametrize completely equivalent underlying theories and one can pick any parametrization, which is most convenient.

When studying the pions in an EFT, the $\sigma$ field, or the $S$ field is considered to be heavy compared to the pseudo Goldstone pions. Therefore, this field can be integrated out. In the exponential parametrization, this can simply be done setting the $S$ field to zero. This leads to the so-called non-linear EFT parametrization [35–50].

In this non-linear parametrization, a more sophisticated power counting tool exists. This can be seen from comparing the two Lagrangians, the linear parametrization giving

$$\mathcal{L}_\chi^{\text{lin}} = \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - 2 \mu^2 \sigma^2 \right) + \frac{1}{2} \partial_\mu \mathbf{\pi} \partial^\mu \mathbf{\pi} - \lambda f_\pi \sigma \left( \sigma^2 + \mathbf{\pi}^2 \right) - \frac{\lambda}{4} \left( \sigma^2 + \mathbf{\pi}^2 \right)^2,$$

(2.20)

and the leading and next-to-leading order non-linear Lagrangian, with the $S$ field integrated out giving

$$\mathcal{L}_\chi^{\text{non-lin}} = \mathcal{L}_2 + \mathcal{L}_4 + ... = \frac{f_\pi^2}{4} \text{Tr} \left( \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right) + \frac{f_\pi^2}{\Lambda^2} \text{Tr} \left( \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right)^2 + ...$$

(2.21)

In the low energy limit, the expansion parameter is $\frac{p^2}{f_\pi^2}$. The power counting structure can be analyzed, calculating the pion $(\pi \pi \to \pi \pi)$ scattering, as seen in figure 2.1. In the non-linear parametrization the power counting is already captured in the four-point vertex, diagram a) of figure 2.1. In the linear case, diagram b) also contains some momentum structure to be included in the power counting. This is evident from the fact that in the linear case, the heavy field has not been integrated out, and therefore interactions with this field need to be considered. The non-linear parametrization therefore allows for a power counting method, which is not allowed in the linear parametrization. In the linear parametrization, it is not possible to say anything about the momentum structure just from calculating diagram (a) of figure 2.1.

Since $\Sigma$ can be expanded in terms of the pion fields, the Lagrangian, already at leading order in this parametrization, is non-renormalizable,

$$\mathcal{L}_{\text{LO}} = \mathcal{L}_{\text{ren}} + \mathcal{L}_\Sigma$$

(2.22)

comprised of a renormalizable Lagrangian with SM fields undergoing renormalizable interactions, and a non-renormalizable Lagrangian of Goldstones described in terms of the non-linear $SU(2)$ matrix field. The renormalization of this Lagrangian therefore happens order by order
2.3. Chiral Perturbation Theory

Figure 2.1: $\pi \pi \rightarrow \pi \pi$ scattering.

in the loop expansion. At NLO, counterterms are introduced to cancel the UV divergences entering at leading order. This can be demonstrated looking at the pion $\pi \pi \rightarrow \pi \pi$ scattering at loop level in figure 2.2 [32]. The Feynman rule for the vertex involving 4 pions, $\sim \frac{\mu^2}{f_\pi^2}$, can be deduced from expanding the above kinetic term in the effective Lagrangian. Therefore, the amplitude will go like

Figure 2.2: $\pi - \pi$ loop scattering.

$$A^{\text{loop}}_\pi \approx \frac{p^4}{f_\pi^4} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2)^2} \approx \frac{p^4}{f_\pi^4} \frac{1}{(4\pi)^2} \log \left( \frac{\Lambda^2}{\mu^2} \right).$$ (2.23)

The same structure of the amplitude would occur, if it was produced by the effective vertex, the higher dimensional operator, suppressed by $\Lambda^2$ in the above Lagrangian, as seen in figure 2.3, leading to the effective interaction,

Figure 2.3: $\pi \pi \rightarrow \pi \pi$ scattering with effective vertex.

$$\frac{f_\pi^2}{\Lambda^2} \text{Tr}(\partial_\mu \Sigma \partial_\nu \Sigma^\dagger \partial^\mu \Sigma \partial^\nu \Sigma^\dagger),$$ (2.24)

resulting in the Feynman rule, $\sim \frac{p^4}{\Lambda^2 f_\pi^2}$ for the effective four-point vertex. The coefficients of the two amplitudes induced from the two methods are assumed to be of the same order for the theory to be renormalizable order by order. The changes in the renormalization point $\mu$ can be absorbed into the coefficient in front of the effective higher order term. This coefficient is therefore expected to be equal to or larger than the coefficient in front of the loop amplitude,

$$\frac{1}{\Lambda^2 f_\pi^2} \geq \frac{1}{f_\pi^4 (4\pi)^2}.$$ (2.25)
2.4. The Standard Model Lagrangian

In ∊PT, as well as in strongly coupled EFTs inspired from this model, an appropriate assumption is therefore, that the ∊SB scale is taken to be

$$\Lambda_\chi \simeq 4\pi f_\pi .$$

(2.26)

A power counting scheme can therefore be implemented in the non-linear case, to determine how the non-renormalizable terms get suppressed. This power counting tool, or naïve dimensional analysis (NDA) provides a measure of the size of the coefficients of the various operators at each order in the perturbative expansion. From the non-linear chiral Lagrangian,

$$\sum_n f_\pi^2 \Lambda_\chi^n \frac{L_n}{\Lambda_\chi^n} = f_\pi^2 \Lambda_\chi^2 \left[ \frac{L_2}{\Lambda_\chi^2} + \frac{L_4}{\Lambda_\chi^4} + \ldots \right] ,$$

(2.27)

setting \( \Lambda_\chi \simeq 4\pi f_\pi \), the power counting goes as

$$\sum_n f_\pi^2 \Lambda_\chi^n \frac{L_n}{\Lambda_\chi^n} = \frac{\Lambda_\chi^4}{16\pi^2} \left[ \frac{L_2}{\Lambda_\chi^2} + \frac{L_4}{\Lambda_\chi^4} + \ldots \right] .$$

(2.28)

The power counting can be implemented in EFT models, where the underlying UV physics is assumed to be strongly interacting, as will be addressed in section 4.2.

The approach of using a non-linear parametrization to describe the scalar sector of a theory, such as the sector involving the Higgs in the SM can therefore prove very useful. A non-linear parametrization could capture more of the underlying UV physics, when studying the SMEFT, than a linear parametrization, due to the stronger constraints imposed by the power counting tools.

In this thesis, the simplest approach of using a linear parametrization will however be implemented and addressed, unless stated otherwise. No assumption on the underlying theory being weakly or strongly interacting will be made. Therefore, the power counting only depends on the canonical dimension of the higher dimensional operators. The expansion is carried out in terms of \( 1/\Lambda \), the inverse cutoff scale and the Higgs is assumed to be in a doublet representation.

2.4 The Standard Model Lagrangian

Based on a \( SU(3)_C \times SU(2)_L \times U(1)_Y \) gauge symmetry, the SM makes up a solid theory of all known particles to date. The \( SU(3)_C \) gauge group describes the strong interactions, whereas the \( SU(2)_L \times U(1)_Y \) gauge group describes the EW interactions. Only the EW sector is relevant to address for the calculations of this thesis. It will be assumed throughout this thesis that the EW scalar sector is linearly realized and gets spontaneously broken by the VEV of the Higgs field, as described in section 2.2. This is a choice based on the fact that it gives the simplest interpretation of current experimental data, and at this point, no attempt will be made to fit to an underlying UV model.

Table 2.1 lists the fields of the EW SM Lagrangian, where left-handed and right-handed fermions are defined as
2.4. The Standard Model Lagrangian

<table>
<thead>
<tr>
<th>Field</th>
<th>$SU(3)_C$</th>
<th>$SU(2)_L$</th>
<th>$U(1)_Y$</th>
<th>Lorentz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^i_L$</td>
<td>3</td>
<td>2</td>
<td>$\frac{1}{6}$</td>
<td>$(\frac{1}{2},0)$</td>
</tr>
<tr>
<td>$u^i_R$</td>
<td>3</td>
<td>1</td>
<td>$\frac{2}{3}$</td>
<td>$(0,\frac{1}{2})$</td>
</tr>
<tr>
<td>$d^i_R$</td>
<td>3</td>
<td>1</td>
<td>$-\frac{1}{3}$</td>
<td>$(0,\frac{1}{2})$</td>
</tr>
<tr>
<td>$l^i_L$</td>
<td>1</td>
<td>2</td>
<td>$-\frac{1}{2}$</td>
<td>$(\frac{1}{2},0)$</td>
</tr>
<tr>
<td>$e^i_R$</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>$(0,\frac{1}{2})$</td>
</tr>
<tr>
<td>$H$</td>
<td>1</td>
<td>2</td>
<td>$\frac{1}{2}$</td>
<td>(0,0)</td>
</tr>
</tbody>
</table>

Table 2.1: Particles of the SM EW sector.

\[ \psi_L = P_L \psi, \quad \psi_R = P_R \psi, \] (2.29)

where

\[ P_L = \frac{1}{2} (1 - \gamma_5), \quad P_R = \frac{1}{2} (1 + \gamma_5). \] (2.30)

The SM Lagrangian involving interactions amongst these particles, invariant under the EW gauge symmetry of the theory is given by

\[ \mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Y}}. \] (2.31)

The Lagrangian involving the Higgs has already been encountered,

\[ \mathcal{L}_{\text{Higgs}} = (D_\mu H)\dagger D^\mu H - \lambda \left( H\dagger H - \frac{1}{2} v^2 \right)^2, \] (2.32)

where

\[ H = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sqrt{2} i \phi^+ \\ h + v + i \phi_0 \end{array} \right), \] (2.33)

is the $SU(2)_L$ scalar doublet with hypercharge $Y_H = \frac{1}{2}$. The mass of the physical Higgs field $h$ is given by $m_h^2 = 2 \lambda v^2$, with $v \sim 246$ GeV.

The covariant derivative has already been defined as $D_\mu = \partial_\mu + i g_2 W_\mu^I T^I + i g_1 B_\mu Y$, with $Y$ the $U(1)$ hypercharge generator, $T^I = \frac{T^I}{2}$, the $SU(2)_L$ generator, with $\tau^I$, the Pauli matrices. The Yang-Mills part of the Lagrangian is [11, 29, 32],

\[ \mathcal{L}_{\text{YM}} = -\frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} = -\frac{1}{2} W_+^{\mu\nu} W_-^{\mu\nu} - \frac{1}{4} W_3^{\mu\nu} W_3^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}, \] (2.34)
with
\[
W_{\mu \nu}^a = \partial_\mu W_{\nu}^a - \partial_\nu W_{\mu}^a - g_2 \epsilon_a \epsilon^b \epsilon^c W_{\mu}^b W_{\nu}^c,
\]
\[
D_{\mu} W_{\nu}^a = \partial_\mu W_{\nu}^a - g_2 \epsilon_a \epsilon^b \epsilon^c W_{\mu}^b W_{\nu}^c.
\] (2.35)

These equations are rewritten in terms of mass eigenstates, the $W$’s, $Z$ and $A$, through equation (2.12). The Yukawa part of the Lagrangian is given by
\[
L_Y = \sum_{\psi=q,u,d,l,e} \bar{\psi} i \not{D} \psi - \left[ H^{ij} \bar{d} Y_d q_j + \tilde{H}^{ij} \bar{u} Y_u q_j + H^{ij} \bar{e} Y_e l_j + H.c. \right],
\] (2.36)
where $\tilde{H}_j = \epsilon_{jk} H^{jk}$, $\epsilon_{12} = 1$, $\epsilon_{jk} = -\epsilon_{kj}$, and $j, k = 1, 2$. After SSB, the fermion mass matrices become
\[
M_{u,d,e} = \frac{1}{\sqrt{2}} Y_{u,d,e} v,
\] (2.37)
where the Yukawa couplings $Y_{u,d,e}$ are matrices in flavor space, such that the fermions and these matrices carry extra indices, $p, r, s, t$, eg.
\[
H^{ij} \bar{d} Y_d q_j = H^{ij} \bar{d}_p [Y_d]_{pr} q_{rj}.
\] (2.38)

### 2.5 Gauge Fixing

The functional integral for a free non-Abelian gauge field theory is given by
\[
Z[J] \sim \int \mathcal{D} A^a_\mu e^{i S[A, J]} = \int \mathcal{D} A^a_\mu e^{i \int d^4x \left[ -\frac{1}{4} F^{a \nu}_{\mu \nu} + J^\mu A^a_{\mu} \right]},
\] (2.39)
where $A^a_\mu$ are the gauge fields and $F^{a \nu}_{\mu \nu}$ their field strengths.

Due to the action $S[A, J]$ and the integration measure $\mathcal{D} A^a_\mu$ being gauge invariant, a gauge fixing condition is needed in order to contain the functional integral to one field configuration, making it possible to define the propagators in the gauge sector. The Faddeev-Popov method is implemented for this purpose [27, 51–53].

The necessity of introducing gauge fixing is accounted for in the following derivations. Integrating by parts, the action contains, in Fourier space,
\[
S'[A] = \frac{1}{2} \int \frac{d^4 k}{(2 \pi)^4} \tilde{A}^a_\mu(k) (-k^2 g^{\mu \nu} + k^\mu k^\nu) \tilde{A}^a_\nu(-k),
\] (2.40)
which vanishes when $\tilde{A}^a_\mu(k) \sim k_\mu$, a consequence of gauge invariance. In this case, the propagator, defined through
\[
(-k^2 g_{\mu \nu} + k_\mu k_\nu) \tilde{D}^\nu_\rho = i \delta^\rho_\mu
\] (2.41)
2.5. Gauge Fixing

has no solution, since the matrix to be inverted has at least one zero eigenvalue, the one corresponding to the eigenvector component of $A^\mu_a$ being $k_\mu$. The functional integral occurs in this case over a continuous infinity of field configurations, which are physically equivalent. To avoid this divergence, the integration has to be constrained to cover only one of these field configurations. Since the $k_\mu$ component of $\hat{A}_\mu^a$ causes the action to vanish, the integral should only happen over the remaining basis vectors of the matrix. This is obtained, if the condition $k^\mu \hat{A}_\mu^a = 0$ is satisfied, which implies the Lorentz gauge fixing condition, $G(A^\mu_a) \equiv \partial^\mu A^a_\mu(x) \equiv 0$. It is implemented through including a functional delta function, $\delta(G(A^\mu_a))$, in the integral, using the relation,

$$1 = \int D\alpha(x) \delta(G(A^\mu_a)) \det \left( \frac{\delta G((A^\alpha)^\mu_a)}{\delta \alpha_b} \right), \quad (2.42)$$

where

$$A^\alpha = (A^\alpha)^\mu_a \epsilon^a,$$  

and for infinitesimal changes,

$$(A^\alpha)^\mu_a = A^\mu_a + \frac{1}{g} \partial^a \alpha^a + f^{abc} A^b \alpha^c = A^\mu_a + \frac{1}{g} \partial^a D_{a}^\mu,$$  

where the covariant derivative $D_{a}^\mu = \delta_{ab} \partial^a - g f^{abc} A^c$ acts on the adjoint representations, and the $f^{abc}$ are the structure constants of the non-Abelian group. Therefore,

$$\delta G((A^\alpha)^\mu_a) = \frac{1}{g} \partial^a D_{a}^\mu \cdot \quad (2.45)$$

The integral with the new insertion becomes,

$$\int D A^\mu_a e^{iS[A,J]} = \int D\alpha(x) \int D A^\mu_a e^{iS[A,J]} \delta(G((A^\alpha)^\mu_a)) \det \left( \frac{\delta G((A^\alpha)^\mu_a)}{\delta \alpha_b} \right). \quad (2.46)$$

Due to gauge invariance, the shift $A^\alpha$ to $\hat{A}$ can be made in the action and integration variable,

$$\int D A^\mu_a e^{iS[A,J]} = \int D\alpha \int D A^\mu_a e^{iS[A,J]} \delta(G(A^\mu_a)) \det \left( \frac{1}{g} \partial^a D_{a}^\mu \right). \quad (2.47)$$

To interpret this functional, the Lorentz gauge condition can be generalized as,

$$G(A^\mu_a) = \partial^\mu A^a_\mu(x) - \omega^a(x) = 0. \quad (2.48)$$

The functional integral to begin with is itself independent of $\omega(x)$. Therefore, an integration over $\omega(x)$, including a Gaussian weighting function centered around $\omega(x) = 0$, can be carried out to take care of the delta function, changing only the overall normalization of the integral,

$$Z[J] \sim N(\xi) \int D\omega \exp \left[ -i \int d^4 x \frac{\omega^2}{2 \xi} \right] \left( \int D\alpha \right) \int D A^\mu_a e^{iS[A,J]} \delta((\partial^\mu A^a_\mu - \omega^a)) \det \left( \frac{1}{g} \partial^a D_{a}^\mu \right) \quad (2.49)$$

$$= N(\xi) \int D\alpha \int D A^\mu_a e^{iS[A,J]} \exp \left[ -i \int d^4 x \frac{(\partial^\mu A^a_\mu)^2}{2 \xi} \right] \det \left( \frac{1}{g} \partial^a D_{a}^\mu \right). \quad (2.49)$$
2.5. Gauge Fixing

The determinant within the functional integral is not independent of the gauge field. It can be rewritten as a functional integral over a new set of anticommuting fields belonging to the adjoint representation,

$$\det \left( \frac{1}{g} \partial_{\mu} D^\mu \right) = \int D\bar{u} D\bar{u} \exp \left( i \int d^4 x \bar{u} (-\partial_{\mu} D^\mu) u \right),$$

which further introduces new terms into the Lagrangian. These fields are the Faddeev-Popov unphysical ghosts, ensuring unitarity and renormalizability.

With the new gauge fixing and ghost terms, the Lagrangian has been extended to

$$\mathcal{L} = -\frac{1}{4} F_{\mu}^a F_{\mu}^{a\nu} - \frac{1}{2} \left( \partial_{\mu} A_{\nu}^a \right)^2 + \bar{\psi} \left( i \not{D} - m \right) \psi + u^a \left( -\partial_{\mu} D_{\mu}^a u \right) u^c.$$  \hspace{1cm} (2.51)

The correlation function for interactions amongst fields is given in terms of the above functional integral in the denominator, and the functional integral times a gauge independent operator specifying the interaction fields in the numerator,

$$\langle \Omega | T \mathcal{O}(\mathcal{A}) | \Omega \rangle = \lim_{T \to \infty (1 - i \epsilon)} \frac{\int D\mathcal{A} \mathcal{O}(\mathcal{A}) e^{i \int_{-T}^{T} d^4 \mathcal{L}}}{\int D\mathcal{A} e^{i \int_{-T}^{T} d^4 \mathcal{L}}}.$$ \hspace{1cm} (2.52)

Therefore, in this ratio, all terms independent of $\mathcal{A}$ in the functional integral cancel. The gauge fixing has therefore only altered this correlation function by introducing the above mentioned extra terms into the Lagrangian. From this new Lagrangian, the gauge propagator is found to be

$$\langle A^a_{\mu}(x) A^b_{\nu}(y) \rangle = \int \frac{d^4 k}{(2\pi)^4} \frac{-i}{k^2 + i \epsilon} \left( g_{\mu\nu} - (1 - \xi) \frac{k_\mu k_\nu}{k^2} \right) \delta^{ab} e^{-i k \cdot (x-y)}.$$ \hspace{1cm} (2.53)

Including the Higgs sector in the Lagrangian, the gauge fixing can be extended to account for SSB,

$$\mathcal{L} = -\frac{1}{4} W_{\mu}^a W_{\mu}^{a\nu} - \frac{1}{4} B_{\mu}^a B^{a\nu} + (D_{\mu} H)^\dagger (D^\mu H) - \lambda (H^\dagger H - \frac{1}{2} v^2)^2,$$ \hspace{1cm} (2.54)

where the Higgs doublet after SSB is defined as

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} i \phi^+ \\ h + v + \delta v + i \phi^0 \end{pmatrix}, \quad H^\dagger = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{2} i \phi^- \\ h + v + \delta v - i \phi^0 \end{pmatrix}.$$ \hspace{1cm} (2.55)

This leads to off-diagonal elements of unphysical couplings between the gauge fields and the Goldstone bosons,

$$\mathcal{L} = \frac{g_2 v}{2} \left( W_{\mu}^+ \partial^\mu \phi_- + W_{\mu}^- \partial^\mu \phi_+ - \frac{g_2 v}{2} c_w Z_{\mu} \partial^\mu \phi_0 \right),$$ \hspace{1cm} (2.56)

which are accounted for by extending the gauge fixing condition to get rid of these off-diagonal elements, in an appropriate way. The way this is done in the background field method (BFM) will be described in the next few sections.
2.6 Background Field Method

When imposing gauge fixing conditions, gauge invariance is lost within these quantum corrections of the gauge field theories, in the intermediate steps towards the calculation of the once again gauge independent S matrix elements. In the background field method (BFM), this loss of gauge invariance can be partly avoided. Fields are split into classical background and quantum components [54–62]. The background fields are not integrated over in the functional integral, and therefore remain gauge invariant, when gauge fixing terms are added. Quantum effects are computed for the quantum fields independently, which are the fields to be integrated over, and these remain gauge dependent [54, 57, 60]. External sources are taken to couple only to quantum fields. This allows one to perform the quantum calculations without breaking gauge invariance of the classical fields.

Giving the background fields a hatted and the quantum fields an unhatted superscript notation, the background covariant derivative is given by,

$$\hat{D}_\mu = \partial_\mu - igT^a A^a_\mu. \quad (2.57)$$

The Lorentz gauge fixing condition therefore changes accordingly, having

$$G(A^a_\mu) = (\hat{D}_\mu)^{ab} A^b_\mu, \quad (2.58)$$

with the covariant derivative acting on the quantum field. In the spontaneously broken theory, a coupling occurs between the gauge fields and the unphysical Goldstones,

$$\mathcal{L} = \frac{g_2 v}{2} \left( W^{+}_\mu \partial^{\mu}(\phi_+ + \phi_-) + W^{-}_\mu \partial^{\mu}(\phi_+ + \phi_-) - \frac{g_2 v}{2c_w} Z_\mu \partial^{\mu}(\phi_0 + \phi_0) \right), \quad (2.59)$$

which is taken into account, when defining the gauge fixing. In the BFM, the gauge fixing Lagrangian accounts for the spontaneously broken theory in the following way,

$$\mathcal{L}_{GF} = -\frac{1}{2\xi_W} \sum_a \left[ \partial_\mu W^{a,\mu} - g_2 e^{abc} \bar{W}_{b,\mu} W^{c}_\mu + ig_2 \frac{\xi}{2} \left( \hat{H}_i^{\uparrow a}_{ij} H_j - H_i^{\downarrow a}_{ij} H_j \right) \right]^2, \quad (2.60)$$

Choosing $\xi_B = \xi_W$, as done in the SM to avoid $A_\mu - Z_\mu$ mixing, the above can be rewritten as

$$\mathcal{L}_{GF} = -\frac{1}{2\xi} \left[ (G^A)^2 + (G^Z)^2 + 2 G^+ G^- \right], \quad (2.61)$$

where
2.7. The Choice of a Gauge

\[
G^A = \partial_\mu A^\mu + i e \left( \bar{W}_\mu^+ W_\mu^- - W_\mu^+ \bar{W}_\mu^- \right) + i e \xi \left( \hat{\phi}^- \phi^+ - \hat{\phi}^+ \phi^- \right),
\]
\[
G^Z = \partial_\mu Z^\mu + i e \frac{c_w}{s_w} \left( \bar{W}_\mu^+ W_\mu^- - W_\mu^+ \bar{W}_\mu^- \right) + i e \xi \frac{1}{2 c_w s_w} (c_w^2 - s_w^2) \left( \hat{\phi}^- \phi^+ - \hat{\phi}^+ \phi^- \right),
\]
\[
- e \xi \frac{1}{2 c_w s_w} \left( \hat{\phi}_0 h - \hat{h} \phi_0 - v \phi_0 \right),
\]
\[
G^\pm = \partial_\mu W_\mu^\pm \pm i e \left[ \hat{A}_\mu + \frac{c_w}{s_w} \hat{Z}_\mu \right] W_\mu^\pm \pm i e \left( A_\mu + \frac{c_w}{s_w} Z_\mu \right) \bar{W}_\mu^\pm,
\]
\[
- e \xi \frac{1}{2 s_w} \left( (v + \hat{h} \pm i \hat{\phi}_0) \phi^\pm - (h \mp i \phi_0) \hat{\phi}^\pm \right).
\]

(2.62)

which is consistent, taking different conventions into account, with [62]. The ghost kinetic part of the Lagrangian becomes in the BFM,

\[
\mathcal{L}_{gh} = \bar{u}^a \hat{D}_\mu a b \bar{D}_\mu b c u^c = - (\hat{D}_\mu u)^a (D_\mu u)^a.
\]

(2.63)

Using equation (2.45) and integrating by parts,

\[
\mathcal{L}_{gh} = - u^a \frac{\delta G^a}{\delta \alpha^b} \bar{u}^b,
\]

(2.64)

where \(a\) and \(b\) are summed over the physical mass eigenstate fields, and \(\delta G^a / \delta \alpha^b\) is the variation of the gauge fixing terms under the infinitesimal quantum gauge fixing transformations. The gauge transformations, following [62] are given by

\[
\delta W_\mu^\pm = \partial_\mu \delta \theta^\pm \mp i e (W_\mu^\pm + \bar{W}_\mu^\pm) (\delta \theta^A + \frac{c_w}{s_w} \delta \theta^0) \mp i e \left( A_\mu + \hat{A}_\mu \right) \left( Z_\mu + \hat{Z}_\mu \right) \delta \theta^\pm,
\]
\[
\delta Z_\mu = \partial_\mu \delta \theta^0 + i g_2 c_w \left( (W_\mu^+ + \bar{W}_\mu^+) \delta \theta^- - (W_\mu^- + \bar{W}_\mu^-) \delta \theta^+ \right),
\]
\[
\delta A_\mu = \partial_\mu \delta \theta^A + i e \left( (W_\mu^+ + \bar{W}_\mu^+) \delta \theta^- - (W_\mu^- + \bar{W}_\mu^-) \delta \theta^+ \right),
\]
\[
\delta \phi^\pm = - \frac{g_2}{2} \left( \hat{h} \mp \hat{v} \pm i (\phi_0 \mp \hat{\phi}_0) \right) \delta \theta^\pm \mp i e (\phi^\pm + \hat{\phi}^\pm) (\delta \theta^A + \frac{c_w^2 - s_w^2}{2 c_w s_w} \delta \theta^0),
\]
\[
\delta h = \frac{g_2}{2} \left( (\phi^+ \mp \hat{\phi}^+) \delta \theta^- + (\phi^- \mp \hat{\phi}^-) \delta \theta^+ \right) - \frac{g_2}{2 c_w} (\phi_0 \mp \hat{\phi}_0) \delta \theta^0,
\]
\[
\delta \phi_0 = - \frac{i g_2}{2} \left( (\phi^+ \mp \hat{\phi}^+) \delta \theta^- - (\phi^- \mp \hat{\phi}^-) \delta \theta^+ \right) + \frac{g_2}{2 c_w} (\hat{h} + \hat{v}) \delta \theta^0.
\]

(2.65)

These transformations are used following equation (2.64) to derive the ghost Lagrangian.

2.7 The Choice of a Gauge

Under EWSB, Goldstone bosons are generated, which in the Higgs mechanism become the longitudinal components of the weak gauge bosons as described in section 2.2. In a general gauge, the Goldstone modes appear in the gauge boson propagators in the following way,
2.8 The Unitarity Problem

$$\mathcal{P}_{\nu}^{\mu}(q^2) = \frac{-i}{q^2 - m_V^2 + i\epsilon} \left[ g^{\mu\nu} + (\xi - 1) \frac{q^\mu q^\nu}{q^2 - \xi m_V^2} \right]$$

$$= \begin{cases} 
-\frac{i}{q^2 - m_V^2 + i\epsilon} g^{\mu\nu}, & \xi \to \infty, \text{ Unitary} \\
\frac{-i}{q^2 - m_V^2 + i\epsilon} g^{\mu\nu}, & \xi = 1, \text{ Feynman} \\
\frac{-i}{q^2 - m_V^2 + i\epsilon} g^{\mu\nu}, & \xi = 0, \text{ Landau} 
\end{cases}$$

The three different choices of gauge must be physically equivalent. The inequivalence seen above is therefore compensated in the Goldstone propagator,

$$\mathcal{P}_{GB}(q^2) = \frac{-i}{q^2 - \xi m_V^2 + i\epsilon}.$$ (2.67)

In the unitary gauge, the Goldstones are promoted to become the massive part of the gauge bosons, such that the Goldstone modes are hidden. The Goldstones do not propagate and the propagator vanishes. This gauge works well to establish unitarity, however, it fails in providing renormalizability. The physical propagators diverge.

In the Feynman gauge, the Goldstones are included as particles with masses corresponding to the masses of the gauge fields. At lowest order, the poles of the ghosts, Goldstone bosons and gauge fields therefore coincide. The propagator structure is mostly simplified in this gauge.

In the Landau gauge, the Goldstones are massless particles. Unitarity is lost, but the propagators fall off as $\sim \frac{1}{k^2}$, making this gauge renormalizable.

Both unitarity and renormalizability are therefore satisfied, when gauge invariance is ensured. To maintain the most general possible approach, $R_\xi$ gauge is implemented in this thesis, mainly letting $\xi$ be undefined. This allows for choosing an arbitrary gauge, which serves as an efficient check of the result.

2.8 The Unitarity Problem

Unitarity is equivalent to having conserved probability, since unitarity of the S matrix implies $S^\dagger S = SS^\dagger = 1$. Before the discovery of the Higgs boson, its existence was hinted towards in the unitarity problem occurring in $WW$-scattering \cite{63–66}.

The vertex entering $WW$-scattering as seen in figure 2.4 has the following Feynman rule,

$$g_2^2 (2 g^{\mu\alpha} g^{\nu\beta} - g^{\mu\nu} g^{\alpha\beta} - g^{\alpha\beta} g^{\mu\nu}).$$ (2.68)
At rest, the vector bosons have momenta \( p_\mu = (m_w, 0, 0, 0) \). They are polarized, which is represented by their polarization vectors, \( \epsilon^\mu(p) \), which are linear combinations of the three unit vectors, \((0, 1, 0, 0), (0, 0, 1, 0)\) and \((0, 0, 0, 1)\). If the field is boosted along the \( \hat{3} \) direction, the momentum becomes \( p_\mu = (E, 0, 0, p) \). The polarization vectors are defined through the following identities,

\[
\epsilon^\mu p_\mu = 0, \quad \epsilon^\mu \epsilon_\mu = -1. \quad \text{(2.69)}
\]

A convenient solution is given by

\[
\epsilon_\mu = (0, 1, 0, 0), \quad (0, 0, 1, 0), \quad \left( \frac{p}{m_w}, 0, 0, \frac{E}{m_w} \right), \quad \text{(2.70)}
\]

where the first two vectors are the transverse polarization vectors and the third the longitudinal polarization vector, \( \epsilon^\mu_L \), boosted with the momentum. The third component of the last polarization vector grows with energy. In the limit \( E \gg m_w \),

\[
p = \sqrt{E^2 - m_w^2} \simeq E - \frac{m_w^2}{2E} + \ldots, \ \Rightarrow
\]

\[
\epsilon^\mu_L = \left( \frac{E}{m_w} + \mathcal{O}(m^2_w/E), 0, 0, \frac{E}{m_w} \right) = \frac{p^\mu}{m_w} + \mathcal{O}(m^2_w/E). \quad \text{(2.71)}
\]

The longitudinal polarization vector is now parallel to the momentum, by construction, which can be exploited in the calculation of the amplitude for the \( WW \) scattering. With \( E \gg m_w \), the transverse external polarization vectors can be replaced by their respective external momenta,

\[
A \simeq i \frac{g^2}{m_w} \frac{p_\mu ^+}{m_w} \frac{p_- ^\mu}{m_w} \frac{q_+ ^\alpha}{m_w} \frac{q_- ^\beta}{m_w} \left[ 2g^{\mu \alpha} g^{\nu \beta} - g^{\mu \nu} g^{\alpha \beta} - g^{\mu \beta} g^{\nu \alpha} \right] = i \frac{g^2}{m_w^2} \left[ 2 (p_+ q_+) (p_- q_-) - (p_+ p_-) (q_+ q_-) - (p_+ q_-) (p_- q_+) \right]. \quad \text{(2.72)}
\]

Using this limit that

\[
s = (p_+ + p_-)^2 \simeq 2 p_+ p_- = 2 q_+ q_- \]
\[
t = (p_+ - q_+)^2 \simeq -2 p_+ q_+ = -2 p_- q_- \]
\[
u = (p_+ - q_-)^2 \simeq -2 p_+ q_- = -2 p_- q_+ , \quad \text{(2.73)}
\]

the amplitude can be found,

\[
A \simeq i \frac{g^2}{m_w^2} \left[ 2 \left(-\frac{t}{2}\right) \left(-\frac{t}{2}\right) \left(\frac{s}{2}\right) \left(\frac{s}{2}\right) - \left(-\frac{u}{2}\right) \left(-\frac{u}{2}\right) \right] = i \frac{g^2}{4 m_w^4} \left[ 2 t^2 - s^2 - u^2 \right]. \quad \text{(2.74)}
\]

As can be seen, for increasing energy, the diagram diverges. In a similar way, the diagrams with an exchange of a \( \gamma \) or \( Z \) boson as seen in figure 2.5 can be calculated. The amplitudes are,
2.8. The Unitarity Problem

W^+ + W^- ≠ W^+ + W^-
Z, \gamma
p_\mu + p_\nu ≠ q_\alpha + q_\beta
\mu ≠ \nu
p_\mu + \nu ≠ q_\alpha + q_\beta
– \gamma
p_\mu + \nu ≠ q_\alpha + q_\beta

Figure 2.5: W W-scattering with interchanges of a \gamma or a Z field.

\[ A_a = -i e^2 \left( 1 + \frac{c_w^2}{s_w^2} \right) \frac{1}{s m_w^4} \frac{1}{u m_w^4} \frac{p_\mu^+ p_\nu^- q_\alpha^+ q_\beta^-}{g_{\beta \sigma}} \]
\[ \times \left( -2 p_\mu^+ g_\mu^\nu - p_\nu^+ g_\nu^\mu + 2 p_\mu^+ g_\mu^\nu + p_\nu^+ g_\nu^\mu + (p_- - p_+) \rho g_\rho^\nu \right) \]
\[ \times \left( -2 q_\alpha^- g_\sigma^\beta - q_\alpha^- g_\gamma^\beta + 2 q_\alpha^- g_\sigma^\alpha + q_\beta^- g_\gamma^\alpha + (q_- - q_+) \sigma g_\gamma^\nu \right) \]
\[ \simeq -i e^2 \frac{1}{s m_w^4} \frac{1}{u m_w^4} s^2 (u - t), \]
\[ A_b = -i e^2 \left( 1 + \frac{c_w^2}{s_w^2} \right) \frac{1}{u m_w^4} \frac{1}{s m_w^4} \frac{p_\mu^- p_\nu^+ q_\alpha^+ q_\beta^-}{g_{\beta \sigma}} \]
\[ \times \left( 2 q_\mu^- g_\gamma^\beta - p_\mu^- g_\gamma^\mu + 2 p_\mu^- g_\gamma^\beta - q_\beta^- g_\gamma^\mu - (p_+ + q_-) \rho g_\rho^\mu \right) \]
\[ \times \left( -2 q_\nu^+ g_\sigma^\alpha + q_\beta^- g_\gamma^\alpha - 2 p_\nu^+ g_\sigma^\nu + q_\nu^+ g_\gamma^\nu + (p_+ + q_-) \sigma g_\gamma^\nu \right) \]
\[ \simeq -i e^2 \frac{1}{u m_w^4} \frac{1}{s m_w^4} u^2 (s - t). \] (2.75)

Lorentz invariance implies the relation \( s + t + u = 4 m_w^2 \) at high energies, so that taking \( e^2 \simeq e^2 \frac{c_w^2}{s_w^2} \simeq g_2^2 \equiv g^2 \), it can be found that

\[ A \simeq i \frac{g^2}{4 m_w^4} \left( 2 t^2 - s^2 - u^2 - s (u - t) - u (s - t) \right) = i \frac{g^2}{4 m_w^4} \left( 2 t^2 - (s + u)^2 + t (s + u) \right) \]
\[ \simeq i \frac{g^2}{4 m_w^4} \left( 2 t^2 - (t + 4 m_w^2)^2 + t (-t + 4 m_w^2) \right) \]
\[ \simeq i \frac{g^2}{4 m_w^4} t m_w^2 \simeq -i \frac{g^2}{m_w^4} (s + u). \] (2.76)

To order \( E^4 \), the diagrams cancel amongst themselves. However, the amplitude blows up as \( \sim E^2 \) at large energy.

The energy, at which unitarity no longer holds can be found from a partial wave analysis. The total cross section for this scattering is given by

\[ \sigma(W W \rightarrow W W) = \frac{1}{32 \pi E_{CM}^2} \int d \cos \theta |A(\theta)|^2, \] (2.77)
2.8. The Unitarity Problem

where the amplitude is decomposed into partial waves,

\[ A(\theta) = 16 \pi \sum_{j=0}^{\infty} a_j (2j + 1) P_j(\cos \theta), \quad (2.78) \]

and \( P_j \) are the Legendre polynomials, with \( P_j(1) = 1 \). In terms of these, the cross section is given by

\[ \sigma(WW \rightarrow WW) = \frac{16 \pi}{2 E_{CM}^2} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} (2j + 1)(2k + 1) |a_j|^2 \int_{-1}^{1} P_j(\cos \theta) P_k(\cos \theta) \, d\cos \theta. \quad (2.79) \]

Using that

\[ \int_{-1}^{1} P_j(\cos \theta) P_k(\cos \theta) \, d\cos \theta = \frac{2}{2j+1} \delta_{jk}, \quad (2.80) \]

it becomes,

\[ \sigma(WW \rightarrow WW) = \frac{16 \pi}{E_{CM}^2} \sum_{j=0}^{\infty} (2j + 1) |a_j|^2. \quad (2.81) \]

The optical theorem relates the imaginary part of the amplitude with the total cross section. At \( \theta = 0 \), \( \text{Im}(A) \leq |A|^2 \). This leads to the constraints \( |a_j| \leq 1 \), \( 0 \leq \text{Im}(a_j) \leq 1 \) and \( |\text{Re}(a_j)| \leq \frac{1}{2} \). The upper bound, namely \( \text{Im}(a_j) = |a_j|^2 \) gives a circle in the complex plane, as seen in figure 2.6.

![Figure 2.6: Partial wave analysis. The condition \( \text{Im}(a_j) = |a_j|^2 \) gives this circle in the complex plane.](image)

In the case of \( WW \)-scattering, there is no angular dependence, so \( j = 0 \) and equation (2.78) leads to \( |a_0| \simeq \frac{E^2}{16 \pi v^2} \). The saturation of the unitarity bound, is therefore \( E \simeq 4 \sqrt{\pi} v \simeq 2 \) TeV.

The above unitarity problem therefore requires some new symmetry breaking dynamics below this energy scale, acting as a UV completion of the EW Lagrangian to cancel the calculated divergent amplitude. There are two ways to solve this. Either, new particles associated with new dynamics enter to restore unitarity, or the interaction becomes strong and non-perturbative. The latter possibility has been explored in Higgsless models before the Higgs discovery. After the discovery, the unitarity problem was resolved in the former way, where the Higgs enters to restore unitarity, contributing to the \( WW \)-scattering with new interactions,
as seen in figure 2.7. The new interactions enter from the kinetic Higgs term \((D_\mu H)^\dagger D^\mu H\), generating the Feynman rule for the \(WW h\) coupling,

\[ W^+_\mu W^- h : \ g_2 m_w g_{\mu \nu} \]  
\[ (2.82) \]

The amplitudes for these diagrams are

\begin{align*}
A_a &= i g_2^2 m_w^2 g_{\mu \nu} g_{\alpha \beta} \frac{1}{s} \frac{1}{m_w} p^\mu_\alpha p^\nu_\beta q^+ q^- \simeq i g_2^2 \frac{s}{m_w^2}. \\
A_b &= i g_2^2 m_w^2 g_{\mu \beta} g^{\alpha \nu} \frac{1}{u} \frac{1}{m_w^2} p^\mu_\beta p^\nu_\alpha q^+ q^- \simeq i g_2^2 \frac{u}{m_w^2}. 
\end{align*}
\[ (2.83) \]

which can be seen to have the right form to cancel off the pure gauge contribution.

When introducing new physics in terms of higher dimensional operators, the unitarity problem has to be addressed as well. The new couplings amongst SM fields entering due to higher dimensional operators will alter the above solution to the unitarity problem. This will occur differently in the case of having a linear parametrization than in the case of having a non-linear parametrization in the scalar sector, due to the extra scale of underlying strong interactions \(f_\pi\) being present in the non-linear realization.

In a linear realization of the SMEFT, the unitarity problem reemerges at the cutoff scale of the theory. The divergent part of the amplitude will receive a correction to the \(WW\) scattering from dimension six operators, with coefficients \(C_i\), of the form

\[ A \simeq \frac{1}{v^2} E^2 \frac{C_i v^2}{\Lambda^2} = E^2 \frac{C_i}{\Lambda^2}. \]
\[ (2.84) \]

The unitarity bound including this extra term is in this case corrected to be \(\sqrt{s} \simeq 4 \sqrt{\frac{E}{C_i}}\). Unitarity is therefore only lost at a scale proportional to the cutoff, which does not result in an immediate problem in the EFT.

In a non-linear realization, implementing the power counting tools of NDA, the effect from higher dimensional operators could give a contribution suppressed by the smaller strong scale encountered, as in \(\chi PT\), \(f_\pi\),

\[ A \simeq \frac{1}{v^2} E^2 \frac{C_i v^2}{f_\pi^2} = E^2 \frac{C_i}{f_\pi^2}. \]
\[ (2.85) \]
2.8. The Unitarity Problem

Unitarity gets violated at a smaller scale than the cutoff, namely at \( \sqrt{s} \simeq 4 \sqrt{\pi} \frac{f}{\sqrt{\eta}} \). This requires new physics to show up at a lower scale than in the linear realization. However, there is still no way of saying what this lower lying energy scale \( f_x \) should be. Unitarity is again only lost at a scale proportional to this strong scale, which does not cause an immediate problem.
2.8. The Unitarity Problem
Chapter 3

Beyond the Standard Model

The successes of the SM are evident. It is however interesting that much is hinting towards there being more to our universe than what is covered by the SM. Problems still exist within the SM, demanding attention, as well as the 96 percent of the universe, which the SM can not yet describe. New physics could linger around the corner, and especially at loop level, experimental bounds are compatible with deviations from SM predictions.

The search for undiscovered particles is one way to probe new physics. However, our only knowledge of such particles is, that these have not been found to date by modern experiments. The cutoff scale has seemingly been pushed up considerably and might not be reachable even by future experiments. The search for new physics through interactions already within the SM is another way to probe new physics. The discovery of a Higgs like boson, however, satisfies the expectations within the SM. The question is, to what precision? Can this precision still accommodate new physics?

In this thesis, the latter method is implemented in the quest for new physics. The scenario is addressed, where the scale of new physics is not pushed up further than to a scale, which still leaves behind evidence for it at the low energy scale, found through an EFT approach, the SMEFT. This way of searching for new physics does not rely on discovering new particles, but on deviations in interactions amongst particles already known to exist. The approach is to expand the SM Lagrangian to include new terms, generated through interactions with new particles, where these have been integrated out. This leaves behind higher dimensional operators in an EFT, suppressed by the mass scale of the new particles. To have new physics manifest itself only through such higher dimensional operators, such new physics is required to occur only at a level much above $O(100 \text{ GeV})$ to allow for such an EFT approach. This coincides with experiments, which have so far not showed evidence of new particles. This method of implementing a SMEFT provides a way of probing new physics without being able to reach the energy scale at which it shows up, or for some reason, without being able to detect existing new physics at a reachable energy level.
3.1 Experimental Data

Future experiments will most certainly probe more knowledge on the nature of the Higgs. Especially the Higgs production and decay channels will become important. The LHC is highly suitable for exploring new resonances and will also provide very precise measurements on interactions, such as the one addressed in this thesis. A clean process such as the $h \rightarrow \gamma \gamma$ decay, has already been measured at high precision by the LHC. Current and future runs of the LHC will improve this measurement considerably. A percent level of precision for such interactions will most likely be obtained with upcoming data. Furthermore, with a cleaner environment, electron-positron colliders, such as a TLEP collider could provide a sensitivity below one percent for $h \rightarrow \gamma \gamma$ and $e^+ e^- \rightarrow Z \gamma$.

The two main detectors of the LHC, ATLAS and CMS have both provided precise results on this decay channel with Run I data. For a center of mass energy of $\sqrt{s} = 7$ TeV and $\sqrt{s} = 8$ TeV, ATLAS reports their results in [67] with luminosities of $4.5 \text{fb}^{-1}$ and $20.3 \text{fb}^{-1}$ respectively for the two energies, whereas CMS in [68] provides data with luminosities of $5.1 \text{fb}^{-1}$ and $19.7 \text{fb}^{-1}$. These experiments measure the decay channel $h \rightarrow \gamma \gamma$ from Higgs production channels as illustrated in figure 3.1.

![Diagram](image)

**Figure 3.1:** Figure from [69] showing the various production channels of the Higgs, before its decay into two photons. Diagram a) shows the gluon fusion, $gg \rightarrow h$, diagram b) shows the vector boson fusion (VBF), diagram c) the associated production of the Higgs with vector bosons, and diagram d) the associated production of the Higgs with top quarks.

Within each production category, a background curve and a signal plus background curve are generated from the amount of events within the category. This leads to the individual signal strengths for the production channels. The signal strengths for each class can be seen in figures 3.2 and 3.3. The curves from the different classes are summed over, providing the combined signal plus background curve and background curve, as seen in figures 3.4 and 3.5 for ATLAS and CMS respectively.

The signal strength, $\mu = \frac{\sigma \cdot \text{BR}}{\sigma \cdot \text{BR}_{\text{SM}}}$, is the measured cross section of the specific production channel and subsequent decay, $\sigma$, times the various branching fractions, BR, divided by the ones expected in the SM. It can be extracted from maximum likelihood fits to the invariant mass distribution of the two photons, $m_{\gamma \gamma} = \sqrt{2 E_1 E_2 (1 - \cos \alpha)}$, where $E_1$ and $E_2$ are the energies of the photons, and $\alpha$ measures the opening angle between the two photons from the
decay vertex. The individual signal strengths for the various classes of production channels probe both the Higgs production and decay rates. They are eventually combined to form the best fit signal strength for the $h \to \gamma \gamma$ decay.

The measured signal strength from ATLAS as seen in figure, is 3.6,

$$\mu_{\gamma \gamma} = 1.17 \pm 0.27,$$  

(3.1)

for $m_h = 125.4 \pm 0.4$ GeV, and from CMS, it is given by

$$\mu_{\gamma \gamma} = 1.14^{+0.26}_{-0.23},$$  

(3.2)
3.1. Experimental Data

Figure 3.4: Figure from the ATLAS experiment showing the combined signal plus background and background signals, obtained from summing over all 12 production classes, [67].

Figure 3.5: Figure from the CMS experiment showing the combined signal plus background and background signals, obtained from summing over all 11 production classes for $\sqrt{s} = 7$ TeV and 14 classes for $\sqrt{s} = 8$ TeV, [68].

for $m_h = 124.70 \pm 0.34$ GeV. So far, these measurements show consistency in the Higgs mass results, [70]. Therefore, there is seemingly no evidence for a deviation of the Higgs decay to photons, from the expectation within the SM. The precision of these measurements
will however improve greatly during this second run of the LHC, and especially, when the luminosity is enhanced. For the $h \rightarrow \gamma \gamma$ decay, the precision increases to the percent level, as can be seen in figure 3.7. A deviation seen at this level could still provide evidence for new physics, when implementing the approach of the SMEFT. This precision at which the $h \rightarrow \gamma \gamma$ decay will be measured in the near future, highly motivates calculating contributions to this interaction in the SMEFT at loop level. Therefore, loop calculations within the SMEFT will in this thesis concern this precisely measured interaction.

![Figure 3.6](image-url)

**Figure 3.6:** Figure from the ATLAS experiment showing the signal strengths of various decay channels, including the $h \rightarrow \gamma \gamma$ channel, [71].

When considering new physics through the inclusion of higher dimensional operators, the effects of these operators enter the $h \rightarrow \gamma \gamma$ decay. The contribution at this stage is the main focus of this thesis.

The signal strength gives a good measure of how close to the SM expectations the actual measurements are. However, it is found for a particular Higgs mass, which in turn specifies the SM couplings of the Higgs to other fields. Furthermore, as can be seen from the method of extraction of the signal strength, the pure $h \rightarrow \gamma \gamma$ decay can not be probed by the overall $h \rightarrow \gamma \gamma$ signal strength, which also depends on the various production channels. The signal strength is found by summing up the signal strengths from all of the production channels. Therefore, the specific vertices within an interaction are not addressed, but all the processes involved become entangled in the signal strength. To correctly fit with the signal strength, it is necessary to also account for the possible inclusion of new physics entering the various production channels. Each production channel has to be considered individually, accounting for the new physics entering already at the production level. Therefore, a $\mu$ framework provides a good compatibility test for the SM, but needs to be broken up to extract information on deviations of these couplings from the expected SM values.
### 3.1. Experimental Data Beyond the Standard Model

**Figure 3.7:** The uncertainty on the signal strength \( \mu \) expected to be performed by the ATLAS experiment during Run II of LHC with 14 TeV and an intensity of respectively 300 fb\(^{-1}\) and 3000 fb\(^{-1}\). "comb." gives the combination of the measurements of all sub-categories of the same final state. "incl." gives the measurement from the inclusive analysis Figure from ATL-PHYS-PUB-2014-016.

In order to use measurements to find deviations within specific decay or production channels, which can eventually be used to make a global fit of the new parameters within the SMEFT, a different framework has been developed, disentangling the information contained in the \( \mu \).

The various channels are separated in the so-called kappa framework [5, 72–74], which is suited for analysing deviations from the SM. SM predictions, including higher order QCD and EW corrections are taken into account. On top of this, the deviations of the SM couplings from possible new physics is included. This is done dressing the individual cross sections and partial decay widths with the scale factors, \( \kappa \) [75].

For the production channel involving gluon fusion, the various cross sections and branching ratios are dressed with different \( \kappa \)'s,

\[
(\sigma \cdot \text{BR})(gg \rightarrow h \rightarrow \gamma \gamma) = \sigma_{\text{SM}} \cdot \text{BR}_{\text{SM}} \kappa_{g}^2 \kappa_{\gamma}^2 \frac{\Gamma_{\gamma\gamma}}{\Gamma_{\gamma\gamma}^{\text{SM}}},
\]

where

\[
\kappa_{\gamma}^2 = \frac{\Gamma_{\gamma\gamma}}{\Gamma_{\gamma\gamma}^{\text{SM}}}, \quad \kappa_{g}^2 = \frac{\sigma_{gg\rightarrow h}}{\sigma_{gg\rightarrow h}^{\text{SM}}}, \quad \kappa_{h}^2 = \frac{\Gamma_{h}}{\Gamma_{h}^{\text{SM}}}. \tag{3.4}
\]

With the presence of additional particles or underlying new physics, the SMEFT can be treated in the context of these effective coupling parameters, making up a model independent
Beyond the Standard Model

3.1. Experimental Data

approach. With the inclusion of higher dimensional operators, the kappa framework takes into account the combination of Wilson coefficients contributing to a particular interaction, the pseudo-observables [76–78]. Limits on these can be found disentangling the various $(\sigma \cdot BR)$. Measurements of these parameters within an EFT from the experiments CMS and ATLAS can be seen in figures 3.8 and 3.9. For the $h \rightarrow \gamma \gamma$ decay, ATLAS has measured $\kappa_\gamma$ in the one sigma range to be [71]

$$\kappa_\gamma = 0.93^{+0.36}_{-0.17}. \quad (3.5)$$

CMS has likewise measured this value to be [79]

$$\kappa_\gamma = 0.98^{+0.17}_{-0.16}. \quad (3.6)$$

These values will be used in a numerical analysis in section 6. In this section, the effects from one-loop contributions to the $h \rightarrow \gamma \gamma$ decay in the SMEFT will be extracted and compared to experimental data.

![Figure 3.8](image-url)

**Figure 3.8:** Figure from the ATLAS experiment showing the effective couplings of various interactions, including the $h \rightarrow \gamma \gamma$ interaction, [71].
The exploration and opportunities of EFTs within various areas of physics are vast and the \( \chi PT \), as described in section 2.3, illustrates the concept well. A plausible method for introducing new physics is therefore to have a general low energy Lagrangian as an EFT and measure its parameters accurately at the LHC. It is assumed that new physics appears at \( \Lambda \gg \Lambda_h \) allowing for this effective description \cite{80}. An EFT means however that at scales higher than \( \Lambda \), unitarity and predictivity is lost, as discussed in section 2.8.

In the SMEFT, heavy particles of new physics interacting with SM particles at a high energy scale are integrated out and the details specifying these particles are ignored. This integrating out of the heavy fields is done at the path integral level. The generating functional can be written up in terms of the light (\( \phi_l \)) and heavy particles (\( \phi_h \)) \cite{81},

\[
Z[J_l, J_h] = \int \mathcal{D} \phi_l \int \mathcal{D} \phi_h \exp \left[ i \int dx \left( \mathcal{L}'(\phi_l) + \Delta \mathcal{L}'(\phi_l, \phi_h) - J_l \phi_l - J_h \phi_h \right) \right],
\]  

where \( \Delta \mathcal{L}'(\phi_l, \phi_h) \) contains the high energy interactions between heavy and light fields. The heavy fields are integrated out, obtaining the effective action,

\[
Z[J_l, 0] = \int \mathcal{D} \phi_l \exp \left[ i \int dx \left( \mathcal{L}_{\text{eff}}(\phi_l) - J_l \phi_l \right) \right],
\]  

where

\[
\exp \left[ i \int dx \mathcal{L}_{\text{eff}}(\phi_l) \right] = \exp \left[ i \int dx \mathcal{L}'(\phi_l) \right] \int \mathcal{D} \phi_h \exp \left[ i \int dx \Delta \mathcal{L}'(\phi_l, \phi_h) \right].
\]  

This can also be understood as expanding the massive heavy propagators in a momentum expansion,

\[
\frac{1}{p^2 - M^2} \approx -\frac{1}{M^2} + \frac{p^2}{M^4}.
\]  

Figure 3.9: Figure from the CMS experiment showing the effective couplings of various interactions, including the \( h \to \gamma \gamma \) interaction, \cite{79}.
According to the above expansion, a scattering where a heavy particle $Z'$ with mass $M$ couples to two SM particles with a coupling $g$, as seen in figure 3.10, becomes a four fermion interaction, with a coupling that goes like $\frac{g^2}{\Lambda^2}$. Heavy new physics can therefore be parametrized through effective couplings of SM fields, which are suppressed by the heavy scale, following the Appelquist-Carazzone theorem [82]. The effective Lagrangian is therefore expressed in terms of an expansion in inverse powers of the heavy mass scale, or the cutoff scale, $\Lambda$.

![Figure 3.10: Heavy particle $Z'$ intermediates two SM particles and gets integrated out.](image)

$$L_{\text{eff}} = L_0(\phi_i) + \sum_i \frac{1}{\Lambda^2} C_i O_i(\phi_i),$$

(3.11)

where the lowest order new physics, assuming baryon and lepton number conservation, is given by dimension six operators,

$$L_{\text{eff}}^{(6)} = \sum_i \frac{1}{\Lambda^2} C_i O_i(\phi_i),$$

(3.12)

and $C_i$ are anomalous couplings, eventually modifying the SM couplings. These operators are generated at the scale of new physics, $\Lambda$. The SM can be extended in such a way, using gauge invariance as a guideline. The gauge symmetries $SU(3)_C \times SU(2)_L \times U(1)_Y$ of the SM are assumed to be preserved within these higher dimensional operators, suppressed by the energy scale $\Lambda$ of new physics. The new interactions introduced through higher dimensional operators therefore decouple from the SM in the limit where the energy scale goes to infinity, $\Lambda \to \infty$, recovering the SM.

### 3.3 Bases of Higher Dimensional Operators

When extending the SM to include higher dimensional operators, Gell-Mann’s *Totalitarian Principle* also captures the used approach well; “*Everything which is not forbidden is compulsory*” [16]. All possible effective couplings of the SM fields are required, following only the guidelines of having gauge invariance and a linearly realized scalar sector. The order of higher dimensional operators in this expansion goes as $(1/\Lambda)^{d-4}$. Therefore, the largest effect comes from operators of smaller mass dimension. The smallest mass dimension, above dimension four, which respects lepton and baryon number conservation is of dimension six. More than 100 dimension six operators have been written up in [83], when considering only one flavour family. However, this number was narrowed down to 59 plus Hermitian conjugates, in the Warsaw basis, when implementing equations of motions (EOM), see table 3.1 [84]. This amounts to 2499 parameters including all families and Hermitian conjugates [11].

EOMs correspond to field redefinitions, shifting the variables in the path integral and therefore do not affect the S-matrix element [9, 35, 36, 85, 86]. The implementation of EOMs is allowed
3.3. Bases of Higher Dimensional Operators

![Figure 3.11: One loop diagram contributing to the Z decay through a four-fermion dimension six effective interaction.](image)

from the equivalence theorem, which states that the choice of operators is arbitrary, since it does not affect the S-matrix elements. Therefore, redundant operators do not contribute to the on-shell matrix elements and can safely be removed in favour of other operators. This has been proven to be true also at the quantum level [85]. The implementation of EOMs to reduce the number of operators is a method that has not been applied in the SM, but becomes possible only for higher dimensional operators.

The EOMs are established through the leading order Lagrangian in the EFT. The fields are treated as classical to make use of these equations, found through varying the Lagrangian with respect to the various fields [84].

As can be seen by a couple of EOMs derived from the SM Lagrangian,

\[
\begin{align*}
\bar{l} : & \quad i \not\! D l + Y_e e H = 0, \\
H^I : & \quad D^2 H - Y^I e l - Y_q q u - Y^I d q = 0,
\end{align*}
\]  

(3.13)

derivatives acting on the SM fields can be replaced through the EOMs by non derivative terms. In general, all operators with covariant derivatives acting on fermions can be removed. This has been implemented in the Warsaw basis, in which a minimal amount of derivatives is obtained. This makes it a very useful and well-defined basis, since redundant operators can be removed simply by removing derivatives when possible through the EOMs.

There are subtleties involved with removing abundant operators. Starting out with an independent basis with non-abundant operators, abundant operators can reappear later in a calculation, which needs to be taken care of using EOMs. When calculating loop diagrams with insertions of effective vertices from dimension six operators, operators outside of the initially chosen basis can appear in the counterterms. After the renormalization counterterms have been computed, these have to be converted back to the standard basis. For example, in the case of having a Z decaying into two fermions, a four-fermion dimension six operator contributes to this decay as seen in figure 3.11. The dimension six operator entering the loop is part of the Warsaw basis. However, the counterterm involves an operator like \( D^\nu Z_{\nu \mu} \bar{\psi} T^A \gamma^\mu P_L \psi \), which is not part of the Warsaw basis. It therefore needs to be redefined as four-fermion operators of the basis in question.

The Warsaw basis is chosen for the purpose of calculations in this thesis, to easily deal with these subtleties associated with removing redundant operators. In some bases, the redundancy is not as obvious [11]. Especially in bases, where a subset of operators is assumed to characterize the full basis structure of a particular type of interactions, eg interactions involving only gauge and Higgs fields. Only when the complete S-matrix element is computed is it clear whether redundancies have appeared. Otherwise, they can be hidden away in redundant
forces.

3.4 Power Counting in the Standard Model Effective Field Theory

Various operators within the SMEFT can be assigned a certain importance, depending on the power counting implemented. Naive dimensional analysis (NDA) is a power counting tool based on assuming an underlying strongly coupled UV theory, as described in section 2.3. The UV theory is in this case itself a non-renormalizable EFT, possibly consisting of composite fields, where a non-linear parametrization is implemented. NDA was developed to understand the non-relativistic quark model in terms of the effective chiral quark theory [19].

As was shown, for such a non-linear parametrization, at leading order, the Lagrangian is already non-renormalizable,

\[ \mathcal{L}_{\text{LO}} = \mathcal{L}_{\text{ren}} + \mathcal{L}_\Sigma, \tag{3.14} \]

where \( \mathcal{L}_{\text{ren}} \) is renormalizable and \( \mathcal{L}_\Sigma \) parametrizes the scalar sector non-linearly, as described in section 2.3. Implementing NDA, an operator at a scale \( \Lambda \) can be written with the following power counting structure,

\[ f^2 \Lambda^3 \left( \frac{H}{f} \right)^A \left( \frac{\psi}{f \sqrt{\Lambda}} \right)^B \left( \frac{g X}{\Lambda^2} \right)^C \left( \frac{D}{\Lambda} \right)^D \left( \frac{y H}{\Lambda} \right)^E, \tag{3.15} \]

where fundamental and composite fields are suppressed by different scales, the cutoff scale, \( f \) and the scale of the breaking of the underlying symmetry in the strong sector, \( \Lambda \) respectively.

The NDA can be applied more generally to EFTs, as well as to the SMEFT [48, 72]. For operators in the Warsaw basis, the power counting goes as follows,

\[ \frac{f^2}{\Lambda^4} g^3 X^3, \quad \frac{\Lambda^2}{f^3} H^6, \quad \frac{1}{f^2} H^4 D^2, \quad \frac{1}{\Lambda^2} g^2 X^2 H^2, \]

\[ \frac{1}{f^2} y \psi^2 H^3, \quad \frac{1}{\Lambda^2} y \psi^2 g X H, \quad \frac{1}{f^2} \psi^2 H^2 D, \quad \frac{1}{f^2} \psi^4. \tag{3.16} \]

where \( X \) is a gauge field. As explained in section 2.3, the renormalization of the chiral Lagrangian happens order by order in the loop expansion. At next-to-leading order (NLO), counterterms are canceled by UV divergences at leading order. This cancellation was shown in section 2.3 to put an estimate on the size of the cutoff \( \Lambda \), namely \( \Lambda \sim 4 \pi v \).

In the opposite case of assuming the underlying theory to be weakly coupled and renormalizable, following a linear parametrization, operators can be classified as tree or loop level suppressed, [87]. In this case, the Higgs comprises the Goldstones in a doublet field. The leading order Lagrangian is renormalizable and the cut-off scale, \( \Lambda \) is allowed to decouple completely, not being determined through cancellation between leading order operators entering in loop interactions and NLO operators entering at tree level as in the non-linear case. NDA power counting therefore no longer applies and the scale of the strong sector, \( f \) has been removed. The ordering is governed only by dimensional counting.
3.4. Power Counting in the Standard Model Effective Field Theory

\[ \begin{align*}
1 & : X^3 \\
2 & : H^6 \\
3 & : H^4D^2 \\
5 & : \psi^2H^3 + \text{H.c.} \\
4 & : X^2H^2 \\
6 & : \psi^2XH + \text{H.c.} \\
7 & : \psi^2H^2D
\end{align*} \]

<table>
<thead>
<tr>
<th>Operator</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_G )</td>
<td>( g_3^3 f^{ABC} G^{\mu_1}_A G^{\mu_2}_B G^{\mu_3}_C )</td>
</tr>
<tr>
<td>( O_{\bar{G}} )</td>
<td>( g_3^3 f^{ABC} \bar{G}^{\mu_1}_A G^{\mu_2}_B G^{\mu_3}_C )</td>
</tr>
<tr>
<td>( O_W )</td>
<td>( g_3^2 \epsilon^{JK} W_{\mu_1}^I W_{\mu_2}^J W_{\mu_3}^K )</td>
</tr>
<tr>
<td>( O_{\bar{W}} )</td>
<td>( g_3^2 \epsilon^{JK} \bar{W}<em>{\mu_1}^I W</em>{\mu_2}^J W_{\mu_3}^K )</td>
</tr>
<tr>
<td>( O_{\phi} )</td>
<td>( \lambda (H^I \overline{H})^3 )</td>
</tr>
<tr>
<td>( O_{H\square} )</td>
<td>( (H^I \overline{H})(H^I \overline{H}) )</td>
</tr>
<tr>
<td>( O_{H_D} )</td>
<td>( (H^I \overline{D}<em>{\mu \nu} H)(H^I \overline{D}</em>{\mu \nu} H) )</td>
</tr>
<tr>
<td>( \psi^2XH + \text{H.c.} )</td>
<td>( O_{\psi^2XH} )</td>
</tr>
<tr>
<td>( \psi^2H^2D )</td>
<td>( O_{\psi^2H^2D} )</td>
</tr>
</tbody>
</table>

8 : \((\overline{LL})(LL)\) + H.c.

<table>
<thead>
<tr>
<th>Operator</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_{\psi} )</td>
<td>( \overline{q}<em>p \gamma</em>\mu \gamma_\nu )</td>
</tr>
<tr>
<td>( O_{\psi}^{(1)} )</td>
<td>( \overline{q}<em>p \gamma</em>\mu \gamma_\nu )</td>
</tr>
<tr>
<td>( O_{\psi}^{(2)} )</td>
<td>( \overline{q}<em>p \gamma</em>\mu \gamma_\nu )</td>
</tr>
<tr>
<td>( O_{\psi}^{(3)} )</td>
<td>( \overline{q}<em>p \gamma</em>\mu \gamma_\nu )</td>
</tr>
<tr>
<td>( \psi^2XH + \text{H.c.} )</td>
<td>( O_{\psi^2XH} )</td>
</tr>
<tr>
<td>( \psi^2H^2D )</td>
<td>( O_{\psi^2H^2D} )</td>
</tr>
</tbody>
</table>

Table 3.1: Table from [11]. The 59 independent dimension six operators built from SM fields which conserve baryon number, as given in [84]. The operators are divided into eight classes: \( X^3, H^6 \), etc. Operators with +H.c. in the table heading also have Hermitian conjugates, as does the \( \psi^2 H^2D \) operator \( O_{H\text{ud}} \). The subscripts \( p, r, s, t \) are flavor indices.
To have the most general power counting, not knowing the exact behavior of the underlying UV theory, would require the power counting to both account for a weakly and strongly coupled underlying theory. A theory which is itself allowed to be an EFT. Such a power counting system, unfortunately, does not exist [88]. Therefore, in a weakly coupled EFT with no assumptions on the underlying UV theory, the most general expansion of the Lagrangian occurs in terms of the canonical dimension of operators. Higher dimensional operators are assigned with inverse powers of the cutoff $\Lambda$. Implementing power counting tools such as NDA or tree-loop suppression would remove this generality of the Lagrangian.

3.5 Constraints on Operators through Electroweak Precision Data

From a theoretical perspective, it is important to live up to the precision obtained by experimental measurements, through calculating possible effects of new physics, implementing a convenient approach to reveal how such new physics might manifest itself. Eventually, a full analysis of the data, taking theoretical calculations into account has to be performed. The task is to properly account for the constraints provided by data, in light of theoretical calculations. EW observables are being measured at high precision by experiments. Likewise, these observables need to be calculated at the same level of precision in theory, where new physics is incorporated through expanding the SM in the SMEFT.

The purpose of electroweak precision tests (EWPT) is to constrain EW observables. Experiments at LEP1 and LEP2 have provided accurate measurements on the Fermi constant $G_F$ in muon decays, the fine-structure constant $\alpha_{em}$ and on the $Z$ mass [89, 90]. These parameters themselves fix the parameters $g_1, g_2$ and $v$, the gauge couplings and the Higgs VEV. Furthermore, these experiments, together with the Tevatron, have measured the $Z$-pole, the $W$-mass and $e^+e^- \rightarrow \psi^+\psi^-$ cross sections.

EWPD are mostly sensitive to corrections to gauge boson propagators, the so-called oblique corrections. Such deviations are parametrized by the $S, T, Y$ and $W$ parameters [91]. The measurements of these parameters were originally intended as a probe of the Higgs mechanism through two-point effective couplings yielding masses and wave-functions of massive gauge bosons. The Lagrangian contains these parameters in the following way,

$$L_{EWPT} = \frac{T}{2} \frac{m_Z^2}{2} Z_\mu Z^\mu - \frac{S}{4 m^2_w} \frac{v^2}{2} (W^3_{\mu\nu} B_{\mu\nu}) - \frac{W}{2 m^2_w} (\partial^\mu W^3_{\mu\nu})^2 - \frac{Y}{2 m^2_w} (\partial^\mu B_{\mu\nu})^2.$$  \hspace{1cm} (3.17)

Any kinetic mixing between $W_3$ and $B$ contributes to the $S$ parameter,

$$S = \frac{16 \pi^2}{g_1 g_2} \frac{\partial}{\partial q^2} A(W_3 \rightarrow B) |_{q^2=0},$$  \hspace{1cm} (3.18)

and a mass difference between the $W$ and $Z$ contributes to the $T$ parameter,

$$T = \frac{1}{\alpha} \left( \frac{m^2_w}{m^2_Z} - 1 \right) = \frac{1}{\alpha} (\rho - 1).$$  \hspace{1cm} (3.19)
Before its discovery, the measurements of these two parameters indicated that a light Higgs boson would induce a consistent fit with data, as seen in figure 3.12. These oblique parameters therefore acted as probes for expected new physics, before the discovery of the Higgs.

After the Higgs discovery, the purpose of measuring these parameters has shifted into discovering unknown physics. This is done through analyzing the effects of higher dimensional operators on the $S$ and $T$ parameters. In the Warsaw basis, at tree-level, the gauge boson self energies are affected by the operators $O_{HWB}$ and $O_{HD}$. At loop level, also the operators $O_W$, $O_{HB}$ and $O_{HW}$ contribute. The operators contributing at tree-level will absorb the divergences produced by operators at loop level through RGEs.

Corrections from the dimension six operators $O_{HWB}$ and $O_{HD}$ come in as

$$ O_S = g_1 g_2 \text{Tr} \left( H T_3 B_{\mu \nu} H^\dagger T_a W^a_{\mu \nu} \right), $$

$$ O_T = e^2 \left[ \text{Tr} (T_3 H^\dagger D_{\mu} H) \right]^2. $$

(3.20)

Expanding these operators, $O_{HWB}$ is found to contribute directly to the $S$ parameter in the following way,

$$ C_{HWB} = - \frac{1}{8 \pi} \frac{\Lambda^2}{v^2} S. $$

(3.21)

Furthermore, when the Higgs acquires a VEV, the operator $O_{HD}$ contributes with the following mass term for the $Z$ boson in the effective Lagrangian,

$$ \frac{1}{16} C_{HD} (g_1^2 + g_2^2) v^4 Z_{\mu} Z^\mu. $$

(3.22)

Since it shifts the $Z$ mass and leaves the $W$ mass untouched, it generates a change in the $\rho$-parameter, therefore inducing a contribution to the $T$ parameter.
3.5. Contraints on Operators through Electroweak Precision Data

After the Higgs discovery and the confirmation of the predictions of the $S$ and $T$ parameter analyses, it is however, necessary to zoom out again and consider the effects new physics could have in a broader perspective including all observables in the SM. Assuming all other dimension six operator coefficients to vanish, strict bounds can be put on the $S$ and $T$ parameters [92]. However, these are greatly relaxed if other operators are included in the fit [93, 94]. Therefore, it is important to take a more general approach when calculating in the SMEFT, implementing all necessary operators to study certain interactions. This is, however, not an easy task, as will be explained in more detail in the next section.

Other operators seem to undergo heavy constraints from experiments in the same way. The most precise measurement carried out at LEP1 is the $Z$ width, $\Gamma_Z$, with a precision given by

$$\left(\frac{\Delta \Gamma_Z}{\Gamma_Z}\right)_{\text{exp}} \sim 0.1\%, \quad \left(\frac{\Delta \Gamma_Z}{\Gamma_Z}\right)_{\text{th}} \sim 0.02\%.$$ (3.23)

This measurement therefore provides constraints on, amongst many others, the dimension six operator,

$$(H^\dagger D^\mu H)(\bar{e}_p \gamma^\mu e_r),$$ (3.24)

contributing to the $Z$ width, since it shifts the $Z$ coupling to fermions for example with a contribution as

$$\frac{v^2}{\Lambda^2} Z_{\mu} \bar{e} \gamma^\mu e.$$ (3.25)

Flavours changing processes are highly constrained in the SM. Therefore, pure fermionic operators also seem to be particularly constrained by EWPD. The four-fermion operators contribute to the $K - \bar{K}$ mass difference and are therefore suppressed by a scale above $10^3$ TeV. The operators contributing to the electric dipole moment of the electron and the $\mu \rightarrow e \gamma$ decay are suppressed at the order $10^4$ TeV, [80]. The concept of minimal flavour violation (MFV) is therefore encouraged, where only flavor conserving processes are allowed [95]. In MFV, the flavour symmetry $U(3)^5 = U(3)_{QL} \otimes U(3)_{dR} \otimes U(3)_{uR} \otimes U(3)_{L_L} \otimes U(3)_{e_R}$ gets violated only by the Yukawa coupling matrices $Y_e$, $Y_u$ and $Y_d$. Since the dimension six operator part of the Lagrangian is assumed to respect MFV, also the RG evolution must preserve MFV [11]. Assuming MFV, the 2499 parameters get reduced to 76. Removing operators inducing flavour changing processes reduces the number of operators to be considered, as well as the heavy constraints.

There are only poor bounds on operators including only quarks and/or gluons. This results from the fact that the precision of hadron experiments falls behind the precision of $e^+ e^-$ machines [80].

The dipole operators $O_{\psi W}$ and $O_{\psi B}$, where $\psi = e, u, d$, contribute to the muon magnetic moment, $(g-2)_\mu$, which has also been measured at very high precision. Assuming the vanishing of all other operators, these dipole operators also become heavily constrained, suppressed by a scale above 100 TeV.

Corrections from QCD are important to take into consideration. When analysing the effects of higher dimensional operators, the effect of these contributions can however be avoided, when the contribution to the SM amplitude and to new physics corrections are identical. That is,
when they have the same operator form. In this case, the ratio of the modified production or
decay channel to the SM one can be taken and QCD corrections cancel in this ratio [96].

EWPD have been doing a great job providing increased precision on observables, in turn
providing great opportunities for constraining the higher dimensional operators, as described
above. However, all of the mentioned heavy constraints rely on setting all coefficients of higher
dimensional operators to zero, besides the ones being addressed, contributing directly to the
specific observable. As will be discussed, constraints only prove to be valid if a global analysis
is carried out, not neglecting any coefficients of higher dimensional operators. Operators are
not independent in the SMEFT, as will be shown explicitly in this thesis. Moreover, such
a global analysis requires the inclusion of theoretical errors, accounting for the calculations
within the SMEFT, which still need to be carried out. Following the logic of [93], this will be
addressed in the next section, where it will be argued how theoretical errors are important to
properly account for, to extract the correct constraints from experiments.

3.6  The Challenge Behind Global Fits to Electroweak Precision Data

Introducing higher dimensional operators produces a direct effect both on theoretical calcu-
lations and experimental measurements. To reveal any underlying new models, beyond the
known SM, it is necessary to obtain independent constraints on operators within the SMEFT.
A global analysis constraining operators at the dimension six level can prove to be an impor-
tant guide in the search for new physics. Furthermore, if an event at a higher energy level was
to be discovered, such an analysis would still provide a consistency check [93]. However, with
the amount of parameters existing already at the dimension six level, such a global constraint,
accounting for both experimental and theoretical errors turns out to be a formidable task.

As has been shown, EWPD has been very succesful in providing heavy constraints on measured
observables, limiting experimental errors considerably. Especially in the $h \to \gamma \gamma$ decay, the
LHC has provided a precision at $\mathcal{O}(10\%)$ and is expected to increase this precision to $\mathcal{O}(1\%)$,
during this Run II.

Improvement of theoretical errors, when taking the SMEFT into account, is falling behind
and will do so even more in the very near future. As a first approach to compensate for the
delayed theoretical improvements in this area, the present situation requires the inclusion of
all theoretical errors, when constraining operators at the dimension six level. In the following,
a few of these theoretical errors will be listed, to show the effects of including such theoretical
errors.

Loop corrections, involving higher dimensional operators, which also make up the main focus
of this thesis, give a theoretical error of the form

\[
\Delta_{\mathcal{L}_6}^{1\text{-loop}} \approx \frac{g^2_{1,2,3}}{16\pi^2} \left( a + b \log \left( \frac{\mu_1^2}{\mu_2^2} \right) \right) \frac{v^2}{\Lambda^2},
\]  

(3.26)

where $\mu_1$ and $\mu_2$ are the scales being run between in the renormalization group evolution.
Typically, $\mu_1 = \Lambda$ and $\mu_2 = v$. In this case, setting $a = b = 1$ and $g = 0.65$ theoretical errors
are found from these contributions of the sizes
3.6. The Challenge Behind Global Fits to Electroweak Precision Data

\[ \Delta_{\mathcal{L}_6}^{1\text{-loop}} \simeq 0.02 \frac{v^2}{\Lambda^2}, \quad \Lambda = 3 \text{ TeV}, \]
\[ \Delta_{\mathcal{L}_6}^{1\text{-loop}} \simeq 0.01 \frac{v^2}{\Lambda^2}, \quad \Lambda = 1 \text{ TeV}. \]  

(3.27)

Including dimension six operators, the RGE is needed to run these operators down to the level of measurements. In the same way, some input parameters in the SM are measured at some energy level, such as the Fermi coupling, \( G_F \) being measured from the \( \mu^- \rightarrow e^- + \nu_e + \nu_\mu \) decay at \( \sim 10 \text{ GeV} \). When used as input for measurements, for example at LEP with \( \sqrt{s} \sim 190 \text{ GeV} \), the running effects in this parameter from higher dimensional operators has to be implemented as well. The correction is similar to the above perturbative correction, namely

\[ \Delta_{\text{RG}} \simeq \frac{g_{1,2,3}^2}{16\pi^2} \left( a + b \log \left( \frac{\mu_1^2}{\mu_2^2} \right) \right) \frac{v^2}{\Lambda^2}, \]  

(3.28)

now with \( \mu_1 = 10 \text{ GeV} \) and \( \mu_2 = v \), so that

\[ \Delta_{\text{RG}} \simeq 0.02 \frac{v^2}{\Lambda^2}. \]  

(3.29)

Two-loop corrections involving dimension six operators are much more suppressed, but could also become relevant to account for future experimental data. The estimate goes as

\[ \Delta_{\mathcal{L}_6}^{2\text{-loop}} \simeq \left( \frac{g_{1,2,3}^2}{16\pi^2} \right)^2 \left( a + b \log \left( \frac{\mu_1^2}{\mu_2^2} \right) \right) \frac{v^2}{\Lambda^2}, \]  

(3.30)

where it can be found,

\[ \Delta_{\mathcal{L}_6}^{2\text{-loop}} \simeq 10^{-5} \frac{v^2}{\Lambda^2}, \quad \Lambda = 3 \text{ TeV}. \]  

(3.31)

Besides taking loop corrections into account when making an estimate on the theoretical error from dimension six operators, it is also necessary to account for subleading terms in the EFT power counting, the dimension eight operators. The effect of these contributions is given by

\[ \Delta_{\mathcal{L}_8} \simeq \frac{v^4}{\Lambda^4} \]  

(3.32)

which become more important, in the case of having a smaller cutoff scale.

These theoretical errors of the SMEFT can be dominant in observables, when the cutoff scale is below 3 TeV, especially in a decay measured as precisely as the \( h \rightarrow \gamma \gamma \) decay. Above this scale, a SMEFT analysis becomes irrelevant since experiments become insensitive to such corrections to the SM. Many of the theoretical errors have been neglected in the literature as discussed in [93]. Including theoretical errors associated with these subleading terms will relax the bounds on operators considerably when carrying out a global analysis. A fully general global fit of all the coefficients that are present in the contribution to the amplitude from higher dimensional operators, \( \mathcal{A}^{NP} \), does not yet exist.
3.6. The Challenge Behind Global Fits to Electroweak Precision Data

The estimation of theoretical errors within the SMEFT is a subtle task, but highly relevant to provide a correlation between experiments and theory. On the other hand, theoretical errors can be eliminated through direct calculations, in all of the areas listed above. Theoretical work is therefore needed both in the area of estimating theoretical errors, as well as limiting them through direct calculations. In this thesis, the latter approach is taken, where the aim is to decrease the theoretical errors arising from higher dimensional operators contributing through loop corrections.
Chapter 4

Ultraviolet Completions of the Standard Model Effective Field Theory

Undoubtedly, it would be very interesting to discover new physics in this or the next run of the LHC. The question is, how such new physics could manifest itself. A highly interesting outcome would be the discovery of new particles. In this case, underlying UV models can be mapped to the new particle content, as well as to how these new particles interact with SM fields. The approach of using an EFT would at this point still increase the knowledge regarding underlying new physics. The, in this case partly known, new physics, based on newly discovered particles, could be integrated out and the effects studied compared to calculated interactions within the SMEFT.

In the other scenario, no new particles are discovered up to $\sim 2$ TeV, which will be the energy level probed during this and the high luminosity run of the LHC. This would still leave room for new physics up to $\sim 3$ TeV and possibly also new physics below $\sim 2$ TeV, which for some other reason is beyond reach of the LHC. In this scenario, the preciseness of experiments and theory is of utmost importance, as already argued. Small deviations from SM expectations will be the only window to new physics, and such deviations could have a highly predictive effect.

In both scenarios, it would be interesting to reveal whether the underlying new physics, expecting its existence, is weakly or strongly interacting. The manner in which the underlying new physics couples to fields in the SM can, amongst other things, give hints towards the power counting within the EFT.

Several models have been developed to draw a picture of possible underlying UV theories, which could account for what we see at low energy today. Some of these models, such as SUSY, make use of the underlying theory coupling weakly to SM particles. Others make use of having underlying strongly coupled theories. These include models implementing the ideas of composite Higgs models [97, 98], as well as technicolor models.

Since this thesis adapts a more general approach to revealing underlying new physics, none of these models will be addressed in detail. In the following, the aim is to describe the two scenarios of having an underlying weakly coupled or strongly coupled sector responsible for new physics. The two cases allow for different assumptions regarding the power counting methods,
as described in section 3.4. Therefore, is not possible to assume a weakly coupled renormal-
izable underlying theory, going into the strongly coupled regime by letting the coupling grow,
$g \rightarrow 4\pi$. Such a statement would attempt to relate two different underlying theories by a
tunable parameter. Therefore, it is necessary to make clear, what assumption is being made
regarding the underlying theory. The following sections will clarify what power counting tools
each area allow and forbid within a SMEFT approach.

4.1 Weakly Coupled Ultraviolet Sector

A weakly coupled theory can be constructed as a perturbative expansion in a small coupling
$g$. Amplitudes are expanded in loops, which scale as $\frac{g^2}{16\pi^2}$. In the scenario of the SM being
an EFT, with the underlying new physics weakly interacting and renormalizable, such as
within SUSY, a classification of the operators can be carried out based on whether they are
generated at tree or loop level [99–101]. This depends on whether the fields at low energy
couple to the heavy fields through loop level or tree level interactions in the underlying UV
theory. Assuming the symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ of the low energy EFT to be
unbroken, the effective Lagrangian can be matched onto the high energy theory to determine
the strength of the couplings.

A four-fermion operator can be produced at tree level by any underlying theory, where some
heavy gauge field couples to the SM fermions, see figure 4.1. When the gauge field is integrated
out, the four-fermion non-renormalizable term is obtained. The diagram in the UV could on
the other hand also be a loop level interaction. This depends on exact knowledge of the
underlying UV theory. Since tree-level interactions are possible, it can be assumed that this
four-fermion operator does not receive a loop level suppression.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.1.pdf}
\caption{UV interaction between SM fermions and a heavy gauge field of the UV sector. When
the heavy gauge field has been integrated out, this leaves behind an effective four-fermion
coupling at low energy.}
\end{figure}

An analysis of the classification of operators can be carried out, as was done in [87]. All possible
tree level diagrams can be drawn in the full theory, which are expected to contribute to the
dimension six effective operators. These diagrams are composed of light external particles and
heavy internal particles. Possible vertices are shown in figure 4.2.

The SM gauge, scalar and fermion fields are represented by $A_{\mu}^a$, $\phi_l$ and $\psi_l$ respectively, whereas
the heavy gauge, scalar and fermion fields are represented respectively by $X_{\mu}^a$, $\phi_h$ and $\psi_h$. This
results in the following vertices being forbidden under the gauge symmetry,

$$
X A A, \quad A A A X, \\
\phi_l \phi_l \phi_l, \\
\phi_l \psi_h A, \\
\phi_l \phi_h A A, \quad \partial \phi_l \phi_h A, \quad \phi_l A X, \quad \phi_l A A, \quad \phi_h A A, \quad \phi_h A X, \quad \phi_l X X. \quad (4.1)
$$
4.2 Strongly Coupled Ultraviolet Sector

As can be seen from this analysis, an operator such as $O_{HB} = g_1^2 H^\dagger H B_{\mu \nu} B^{\mu \nu}$ will be loop suppressed, since the vertices $XAA$ and $\phi_BAA$ are not allowed by the symmetry. As has been outlined in this section, such a statement requires the fields to be coupled weakly. Furthermore, the individual fields are not themselves allowed to be composite fields, namely, the couplings cannot represent effective vertices.

Assuming the underlying theory to be weakly coupled, which could allow for this power counting tool in terms of classifying operators as loop or tree level, provides a possible approach, when looking for a UV completion of the SM. The most general approach is, however, to also account for the possibility that the underlying theory could be non-renormalizable and effective in itself. Such a scenario would not allow for the above described power counting tool.

4.2 Strongly Coupled Ultraviolet Sector

A strongly coupled theory such as $\chi{\text{PT}}$, the low energy limit of QCD, as described in section 2.3, does not, as a weakly coupled theory, undergo a perturbative expansion around the small coupling. Therefore, an expansion of the amplitudes in terms of loops is not possible, as can also be inferred from the expansion parameter, $g^2/16\pi^2$, as $g \to 4\pi$. An underlying strongly coupled UV sector is itself an effective non-renormalizable field theory.

SSB in QCD has been the important inspirational source of the construction of technicolor and in turn composite Higgs models, which fall into the category of strongly coupled underlying UV theories. The weak interaction gauge symmetry is spontaneously broken by an underlying strongly interacting sector. Such models make use of the same method as within $\chi{\text{PT}}$. Unlike in the case of the Higgs mechanism, where the dynamics responsible for the symmetry breaking is unknown and weakly coupled, the dynamics responsible for the SSB in these models consists of a condensate, generated by new interactions, which couple strongly at the EW scale. This approach corresponds to how chiral symmetry is broken in QCD, and can introduce the light scalar field, or the Higgs, as a pseudo Goldstone Boson of the spontaneously broken symmetry group [48, 72, 102, 103]. Following the behavior of the pions in $\chi{\text{PT}}$, the Higgs is a composite field with a light mass [97]. Since the Higgs is light, the bound states need to have a binding energy $\sim \Lambda$, they have to be strongly coupled. As is the case for $\chi{\text{PT}}$, a new strongly interacting sector arises at the compositeness scale, the strong scale, $\Lambda \sim 4\pi f$, where $f > v_{EW}$, so that $\Lambda > 3$ TeV. The Higgs interacts with this strong sector with a coupling constant $g_p$. Therefore, the Higgs dynamics is not perturbative and the scalar sector is best described through a non-linear parametrization, as explained in section 2.3. This means that the method of NDA, encountered in section 3.4, can be implemented to some extent [104].

The most general Lagrangian following $\chi{\text{PT}}$, which describes Higgs interactions, is given by
the following EW chiral Lagrangian, with a non-linear parametrization [35–38, 40–44, 46, 47, 49, 50],

\[ \mathcal{L} = \frac{1}{2} (\partial_\mu h)^2 - V(h) + \frac{v^2}{4} \text{Tr} \left[ (D_\mu \Sigma)^\dagger D^\mu \Sigma \right] (1 + 2a \frac{h}{f} + b \frac{h^2}{f^2} + b_3 \frac{h^3}{f^3} + ...) \\
+ v, \sum_{i,j} \left( Y'_{ij} \bar{u}_i L + Y''_{ij} \bar{d}_i R \right) \left( 1 + c \frac{h}{f} \right) + H.c. + ... \] (4.2)

In the SM, \( a = b = c = 1 \) and \( v = f \). In the case where \( f \gg v \), the Higgs is strongly coupled. The parameter

\[ \xi = \left( \frac{v}{f} \right)^2 \neq 0, \] (4.3)

the ratio of the scale of the new strong sector and of the EW sector, gives a measure of the non-linearity of the Higgs dynamics. \( \xi \to 0 \) results in the decoupling limit, where the Higgs is close to the SM Higgs. The linear regime reigns, the Higgs unitarizes the physical amplitudes, its interactions are renormalizable and the heavy new states are infinitely heavy. If \( \xi \to 1 \), the non-linear strongly coupled regime takes over. Between these two limits, there is a great amount of possibilities, where both the heavy resonances and the light Higgs can induce unitarity. This parameter therefore gives a measure of the deviations in the Higgs couplings from those expected for the SM.
When extending the SM in the SMEFT approach, renormalization has to be accounted for within this extended theory. Bare parameters of the original EFT Lagrangian differ from the physical quantities by UV divergent contributions, to be accounted for through counterterms. In renormalizable theories, these UV divergences cancel in relations amongst physical observables [27, 52, 53, 105–109]. The SMEFT is renormalizable order by order in the expansion in $\frac{1}{\Lambda}$. Within each order, the same approach as for a renormalizable theory can therefore be used. In the following, a revision is presented regarding renormalization in the SM to prepare for the outline following on the renormalization of this general extension of the SM.

### 5.1 Renormalized Perturbation Theory

A renormalizable field theory contains divergent amplitudes, which never show up in observable quantities but are renormalized away. A $\phi^4$ scalar Lagrangian is written up in terms of bare unphysical values,

$$L_0 = \frac{1}{2} (\partial_\mu \phi_0)^2 - \frac{1}{2} m_0^2 \phi_0^2 - \frac{\lambda_0}{4!} \phi_0^4. \quad (5.1)$$

The theory described by this Lagrangian contains the divergent amplitudes in figure 5.1. These divergences can be removed by redefining the bare fields, masses and couplings. $m_0^2 \phi_0^2$ and $\partial_\mu \phi_0 \partial^\mu \phi_0$ get redefined in the free Lagrangian, merely shifting parameters of the path integral involving this Lagrangian in the action, namely

$$- \int d^4x \phi_0 (\partial^2 + m_0^2) \phi_0, \quad (5.2)$$

which modifies the free scalar propagator

$$P_0(p^2) = \frac{1}{p^2 - m_0^2}. \quad (5.3)$$
These redefinitions are given in terms of renormalization constants,

\begin{align}
\phi_0^2 &= Z_\phi \phi_r^2 = (1 + \delta Z_\phi) \phi_r^2, \\
m_0^2 &= \frac{1 + \delta Z_m}{Z_\phi} m_r^2 = (1 + \delta Z_m - \delta Z_\phi) m_r^2, \\
\lambda_0 &= \frac{1 + \delta Z_\lambda}{Z_\phi^2} \lambda = (1 + \delta Z_\lambda - 2 \delta Z_\phi) \lambda, 
\end{align}

(5.4)
defined as follows,

\begin{align}
Z_\phi &= 1 + \delta Z_\phi + \mathcal{O}(g_0^4), \\
Z_m &= 1 + \delta Z_m - \delta Z_\phi + \mathcal{O}(g_0^4), \\
Z_\lambda &= 1 + \delta Z_\lambda - 2 \delta Z_\phi. 
\end{align}

(5.5)

The renormalized Lagrangian is therefore given by,

\begin{align}
\mathcal{L} &= \frac{1}{2} Z_\phi (\partial_\mu \phi_r)^2 - \frac{1}{2} Z_m Z_\phi m_r^2 \phi_r^2 - Z_\lambda Z_\phi^2 \frac{\lambda}{4!} \phi_r^4 \\
&= \frac{1}{2} (\partial_\mu \phi_r)^2 - \frac{1}{2} m_r^2 \phi_r^2 - \frac{\lambda}{4!} \phi_r^4 + \frac{1}{2} \delta Z_\phi (\partial_\mu \phi_r)^2 - \frac{1}{2} \delta Z_m m_r^2 \phi_r^2 - \frac{\delta Z_\lambda \lambda}{4!} \phi_r^4, 
\end{align}

(5.6)

where the last three terms are counterterms to absorb the infinities. The renormalization constants are determined from the requirement that the Green functions, the time ordered products of fields, are finite when expressed in terms of these renormalized quantities.

In the BFM, the effective action is invariant under background gauge transformations. Therefore, relations amongst the renormalization constants following the Ward identities, gauge invariance in disguise, are maintained. They follow the relations obtained from Ward identities in the classical Lagrangian [62].

Gauge invariance is introduced in the fermionic sector, through the Abelian covariant derivative, \( D_\mu = \partial_\mu + i e A_\mu \), such that the renormalized kinetic fermionic terms of the Lagrangian are

\[ Z_\psi Z_D \bar{\psi}_r i \not{D} \psi_r = Z_\psi \bar{\psi}_r i \not{\partial} \psi_r - e \sqrt{Z_e} Z_\psi \sqrt{Z_A} \bar{\psi} \gamma_\mu \psi_r A_\mu^\mu. \]

(5.7)

To insure gauge invariance, the following relation needs to be satisfied,

\begin{align}
\delta Z_D + \delta Z_\psi &= \delta Z_e - \frac{1}{2} \delta Z_e + \delta Z_\psi + \frac{1}{2} \delta Z_A, \\
\delta Z_A &= - \delta Z_e. 
\end{align}

(5.8)
In the non-Abelian case, the neutral gauge fields are renormalized through a renormalization constant matrix,

$$
\begin{pmatrix}
Z_r \\
A_r
\end{pmatrix} = 
\begin{pmatrix}
Z_{ZZ} & Z_{AZ} \\
Z_{ZA} & Z_{AA}
\end{pmatrix}
\begin{pmatrix}
Z_0 \\
A_0
\end{pmatrix} .
$$

(5.9)

The two-point $A_\mu - Z_\mu$ function is given by

$$
\Gamma^{AZ}_{\mu\nu}(p, -p) = i (-g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2}) \Sigma_T^{AZ}(p^2) - i \frac{p_\mu p_\nu}{p^2} \Sigma_L^{AZ}(p^2) .
$$

(5.10)

where $\Sigma_T^{AZ}(p^2)$ and $\Sigma_L^{AZ}(p^2)$ are the transverse and longitudinal self-energies respectively, such that the Ward identity results in

$$
p^\mu \Gamma^{AZ}_{\mu\nu}(p, -p) = 0, \Rightarrow
\Sigma_L^{AZ}(p^2) = 0 .
$$

(5.11)

The vanishing of the longitudinal self-energy of the two-point function results in the vanishing of the renormalization constant $Z_{ZA}$, which acts to renormalize this self-energy,

$$
\Sigma_L^{rAZ}(p^2) = \Sigma_L^{AZ}(p^2) - p^2 \delta Z_{ZA} = -p^2 \delta Z_{ZA} = 0 , \Rightarrow
\delta Z_{ZA} = 0 .
$$

(5.12)

In the non-Abelian case, the reasoning for the covariant derive follows as in the Abelian case. Extending the covariant derivative to include the non-Abelian gauge fields, $D_\mu = \partial_\mu + i g_2 W_\mu^I T^I + i g_1 B_\mu Y$, the Ward identities follow again from inspecting the renormalized kinetic fermion term in the non-Abelian case,

$$
Z_\psi Z_{D \psi r} i D \psi r = Z_\psi \bar{\psi} r i \psi r - g_2 \sqrt{Z_{g_2} Z_\psi} \sqrt{Z_W T^I} \bar{\psi} \gamma_\mu \psi r W_{r I} T^I
- g_1 \sqrt{Z_{g_1} Z_\psi} \sqrt{Z_B} \bar{\psi} \gamma_\mu \psi r B_{I}^\mu Y .
$$

(5.13)

This leads to the following relations,

$$
Z_{g_2} Z_W = 1 , \quad Z_{g_1} Z_B = 1 .
$$

(5.14)

Using that $\delta Z_{ZA} = 0$, the Ward identity can be found to give a relation between the photon and the electromagnetic coupling, as was seen in the Abelian case, when rotating into the mass eigenstates,

$$
Z_\psi Z_{D \psi r} i D \psi r = Z_\psi \bar{\psi} r i \psi r - g_2 \sqrt{Z_{g_2} Z_\psi} \sqrt{Z_{sw} s_w} \sqrt{Z_A} \bar{\psi} \gamma_\mu \psi r A^\mu r T^3
- g_1 \sqrt{Z_{g_1} Z_\psi} \sqrt{Z_{cw} c_w} \sqrt{Z_A} \bar{\psi} \gamma_\mu \psi r A^\mu r Y + ... \\
= Z_\psi \bar{\psi} r i \psi r - \sqrt{Z_c} Z_\psi \sqrt{Z_A} \bar{\psi} \gamma_\mu \psi r A^\mu r (T^3 + Y) + ... .
$$

(5.15)
5.2. Dimensional Regularization

namely,

\[ Z_A Z_e = 1. \]  

(5.16)

In the BFM, with the maintenance of the Ward identities, the following relations,

\[ Z_A Z_e = 1, \quad Z_W Z_{g_2} = 1, \]  

(5.17)

can therefore be exploited during calculations, when accounting for renormalizations [62]. This causes technical simplifications in loop calculations, as will be evident later.

5.2 Dimensional Regularization

Diverging integrals, such as the amplitudes of figure 5.1 can be calculated using dimensional regularization (DREG) [53]. This means promoting the dimension of energy-momentum space to \( d = 4 - 2\epsilon \), where the metric is now defined as

\[ g^{00} = -g^{ii} = 1, \quad i = 1, 2, \ldots, d - 1, \]  

(5.18)

and the internal momenta as

\[ k^\alpha = (k^0, k^1, \ldots, k^{d-1}), \]  

(5.19)

\[ k^2 = (k^0)^2 - \sum_{i=1}^{d-1} (k^i)^2. \]  

(5.20)

The singularities of the diagrams become of the form \( \frac{1}{\epsilon^i} \) where \( i = 1, \ldots, L \) and \( L \) is the number of loops. DREG preserves gauge invariance, naturally introduces the modified minimal subtraction (\( \overline{\text{MS}} \)) scheme and regularizes the infrared (IR) divergences in a massless theory.

Within DREG, the couplings become dimensionful, as can be seen through a dimensional analysis. The \( \mu \) parameter is therefore introduced such that the renormalized couplings are dimensionless. The fields, masses and couplings of the Lagrangian are in DREG renormalized as before, with a modification of the coupling renormalization,

\[ \phi_r = \frac{1}{\sqrt{Z_\phi}} \phi_0, \quad m_r = \frac{1}{\sqrt{Z_m}} m_0, \quad \lambda = \frac{1}{Z_\lambda} \lambda_0 \mu^{-\epsilon}. \]  

(5.21)

The renormalized Lagrangian of the \( \phi^4 \) theory becomes

\[
\mathcal{L} = \frac{1}{2} Z_\phi (\partial_\mu \phi_r)^2 - \frac{1}{2} Z_m Z_\phi m_r^2 \phi_r^2 - Z_\lambda Z_\phi^2 \frac{\lambda}{4!} \phi_r^4 + \text{counterterms}.
\]  

(5.22)
5.3 Renormalization Scheme

The renormalization scheme employed to renormalize a theory is arbitrary. The renormalization scheme defines the choice of independent input parameters as well as how these bare parameters are separated into renormalized parameters and counterterms. The renormalization scheme fixes the choice of the renormalization conditions, which in turn fixes the counterterms. Different choices of schemes will eventually lead to the same physical results.

In the minimal subtraction (MS) scheme, only the pole part of the bare parameters are subtracted by the renormalization constants. In DREG, the renormalization constants are therefore given by

\[ Z = 1 + \delta Z = 1 + \sum_{i=1}^{L} \frac{c_i}{\epsilon^i}, \]

where \( c_i \) are the counterterm coefficients in the renormalization constants and \( L \) is the number of loops.

In the \( \overline{\text{MS}} \) scheme, the MS scheme is implemented, supplemented with redefining \( \mu^2 \to \mu^2 \frac{\sqrt{\epsilon}}{4\pi} \) [110]. The subtraction therefore includes the pole part with a finite term,

\[ \Gamma(\epsilon - 1) = -\frac{1}{\epsilon} + \gamma_E - 1 + \mathcal{O}(\epsilon). \]

The \( \overline{\text{MS}} \) scheme within DREG will be implemented throughout calculations in this thesis.

5.4 Renormalization at the Finite Level

Renormalization conditions dictate how external fields and couplings are renormalized, also at the finite level. Computing the S matrix element corresponds to computing the Green function,

\[ G^{(n)}(x_1, \ldots, x_n) = \langle 0 | T \{ \phi_1(x) \ldots \phi_n(x) \} | 0 \rangle, \]

taken to momentum space,

\[ \langle f | S | i \rangle = (p_f^2 - m_f^2) \ldots (p_i^2 - m_i^2) G^{(n)}(p_i, \ldots, p_f). \]

External lines have been removed through a Lehmann-Symanzik-Zimmermann (LSZ) reduction [111, 112]. With the external states, \( | i \rangle \) and \( | f \rangle \) put on-shell, \( p_f^2 = m_f^2 \) and \( p_i^2 = m_i^2 \), their respective propagators diverge. The LSZ reduction isolates these asymptotic states of the Green function, multiplying the Green function with the poles, as seen in equation 5.26. The one-particle states are projected out, multiplying with \( p^2 - m^2 \), or in the case of fermions, with \( \not{p} - m \).

The S matrix is therefore zero, unless there are poles in the Green function, cancelling the poles factored out. Physical one-particle states are required to have poles at their physical
5.4. Renormalization at the Finite Level

\[ h p + h p = \Gamma^r,h(p) = i(p^2 - m^2_h) + i \Sigma^r,h(p^2) \]

\[ \gamma p, \nu = \gamma p, \nu = i(-g_{\mu \nu} p^2 + p_\mu p_\nu) - i g_{\mu \nu} \frac{p_\nu p_\mu}{p^2} \Sigma^r,\gamma(p^2) \]

\[ \lim_{p^2 \rightarrow m^2_h} \frac{1}{p^2 - m^2_h} \Re \Gamma^r,h(p) = i, \]

\[ \Gamma^r,\gamma(p) e^{\nu}(p)|_{p^2=0} = 0, \quad \lim_{p^2 \rightarrow 0} \frac{1}{p^2} \Re \Gamma^r,\gamma(p) e^{\nu}(p) = -i \epsilon_\mu(p). \]

Figure 5.2: Renormalized 1PI two-point functions of the Higgs field.

mass. Therefore, the projection concerns renormalized masses, \( m_r \), and the renormalized Green functions have poles at \( p^2 = m_r^2 \), or for fermions, \( \not{p} = m_r \). Another requirement is to have the residue of the pole of the propagator be equal to 1 or \( i \). These requirements define the on-shell renormalization scheme, as well as the renormalization conditions for the external fields, leading to the so-called R factors. They force the physical masses and the residues of the propagator to be constant at all orders in perturbation theory [113]. The LSZ reduction therefore forces external lines to be on-shell and the finite renormalized fields and couplings correspond to the physical parameters [114].

The R factors contribute to the finite part of the renormalization constants and are therefore fixed in a similar way through renormalization conditions. The conditions are expressed for one-particle irreducible (1PI) two-point functions, when dealing with fields, masses and mixing matrices. When dealing with couplings, the renormalization conditions concern three point functions. In the case of the electromagnetic coupling, it concerns the three point \( e e \gamma \) vertex. Working in the BFM, the coupling renormalization is related to the field renormalization through the Ward identities, which remain unbroken in the BFM, as seen in section 5.1. Therefore, these coupling renormalizations can instead be found through field renormalizations. The Ward identity relating the renormalization for the electromagnetic coupling and the photon field, therefore leads to

\[ \delta R_e = -\delta R_A. \] (5.27)

The renormalized 1PI two-point functions of the Higgs and the photon are defined and shown schematically in figure 5.2. The propagators of the Higgs field and the photon field are subsequently obtained through the inverse of these two-point functions.

The renormalized mass parameters of the external physical particles are fixed by requiring that they are equal to the physical masses, as described above [114]. These are given by the real parts of the poles of the propagators, and therefore by the zeros of the 1PI two-point functions, in the onshell case, where \( p^2 = m^2 \). The matrices of the renormalized 1PI two-point functions are diagonal if the external lines are on their mass shell, simplifying things. The renormalization conditions for the Higgs and photon two-point functions in the case of having on-shell external physical fields are therefore,

\[ \text{Re} \Gamma^r,h(p)|_{p^2=m^2_h} = 0, \quad \lim_{p^2 \rightarrow m^2_h} \frac{1}{p^2 - m^2_h} \text{Re} \Gamma^r,h(p) = i, \]

\[ \Gamma^r,\gamma(p) e^{\nu}(p)|_{p^2=0} = 0, \quad \lim_{p^2 \rightarrow 0} \frac{1}{p^2} \text{Re} \Gamma^r,\gamma(p) e^{\nu}(p) = -i \epsilon_\mu(p). \] (5.28)
The above renormalization conditions for the two-point functions lead to the following conditions for the self energy functions,

\[ \text{Re} \Sigma^{\gamma, h}(m_h^2) = 0, \quad \text{Re} \left. \frac{\partial \Sigma^{\gamma, h}(p^2)}{\partial p^2} \right|_{p^2 = m_h^2} = 0, \]

\[ \Sigma^{\gamma, h}(0) = 0, \quad \text{Re} \left. \frac{\partial \Sigma^{\gamma, h}(p^2)}{\partial p^2} \right|_{p^2 = 0} = 0. \] (5.29)

The R factors of the external fields are as a result given by the finite residue of the poles of the external fields in their self energies fixed via the above on shell renormalization condition of these fields. They are expressed in terms of the unrenormalized self energies, defined on shell. For the Higgs and photon field, as well as for the electromagnetic coupling, defined through the Ward identity in the BFM, the R factors are given by

\[ \delta m_h^2 = \text{Re} \Sigma^h(m_h^2), \]

\[ \delta R_h = -\text{Re} \left. \frac{\partial \Sigma^h(p^2)}{\partial p^2} \right|_{p^2 = m_h^2}, \]

\[ \delta R_A = -\frac{1}{2} \text{Re} \left. \frac{\partial \Sigma_A^h(p^2)}{\partial p^2} \right|_{p^2 = 0}, \]

\[ \delta R_e = \frac{1}{2} \text{Re} \left. \frac{\partial \Sigma_e^h(p^2)}{\partial p^2} \right|_{p^2 = 0}. \] (5.30)

5.5 Renormalization Group Equations

Scale invariance is a broken symmetry within the strength of interactions, the couplings. This is evidenced through the running of the EW and strong coupling constants [110, 115–117]. However, physical observables are required to be unchanged under these scale variations, also known as renormalization group transformations. When a theory is required to be independent of changes, various differential equations can be set up with respect to these changes. These make up the so-called renormalization group equations (RGE), which express the conservation of observables under changes in the way things are being calculated. The solution to these RGEs gives a trajectory in the space of theories, conserving the observables.

The generating functional for a connected Green’s function, to which correspond connected 1PI Feynman diagrams, is given by,

\[ W[J, g, m] = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4 x_1 d^4 x_2 \ldots d^4 x_n \ G^{(n)}_{\text{conn}}(x_1, x_2, \ldots, x_n) \ J(x_1) J(x_2) \ldots J(x_n). \] (5.31)

It attains a finite form when it is expressed in terms of the renormalized parameters,

\[ J_r = \left. \frac{1}{\sqrt{Z_3}} \right| J, \quad g_r = \left. \frac{1}{Z_g} \right| g, \quad m_r^2 = \left. \frac{1}{Z_m} \right| m^2, \] (5.32)

the renormalized current/propagator, coupling and mass. In a different renormalization scheme, these acquire different values,

\[ J'_r = \left. \frac{1}{\sqrt{Z'_3}} \right| J, \quad g'_r = \left. \frac{1}{Z'_g} \right| g, \quad m'_{r^2} = \left. \frac{1}{Z'_m} \right| m^2. \] (5.33)
5.6. The Renormalization Group Equation of Standard Model Couplings

However, the physical prediction is still the same for arbitrary renormalization schemes, that is, the generating functional does not change,

\[ W'_r[J'_r, g'_r, m'_r, \mu'] = W_r[J_r, g_r, m_r, \mu]. \] (5.34)

Any finite renormalization will transform the observables from one scheme to the other,

\[ J'_r = \sqrt{\frac{Z_3}{Z_3}} J_r, \quad g'_r = \frac{Z_g}{Z'_g} g_r, \quad m'_r = \frac{Z_m}{Z'_m} m_r^2. \] (5.35)

In a specific subtraction scheme, \( g_r \) and \( m_r \) depend on \( \mu \), the scale that defines the subtraction procedure,

\[ g_r(\mu) = \frac{1}{Z_g(\mu)} g_0, \quad m_r(\mu) = \frac{1}{\sqrt{Z_m(\mu)}} m_0, \] (5.36)

where the scale has been absorbed into the renormalization constant. Therefore, two different subtraction schemes are related by

\[ g_r(\mu') = Z_g(\mu', \mu) g_r(\mu), \quad m_r(\mu') = Z_m(\mu', \mu) m_r(\mu), \] (5.37)

where \( Z_g(\mu', \mu) = \frac{Z_g(\mu)}{Z'_g(\mu')} \) and \( Z_m(\mu', \mu) = \sqrt{\frac{Z_m(\mu)}{Z'_m(\mu')}} \) transform the parameters from one scheme to the other. The different subtraction procedures therefore have group properties, so that

\[ Z_g(\mu'', \mu') Z_g(\mu', \mu) = Z_g(\mu'', \mu), \] (5.38)

changes \( g_r(\mu) \) through \( \mu \to \mu' \to \mu'' \). The inverse of a transformation is

\[ Z_g^{-1}(\mu', \mu) = Z_g(\mu, \mu'), \] (5.39)

and \( Z_g(\mu, \mu) = 1 \). The subtraction schemes are therefore part of an Abelian group, the renormalization group. Infinitesimal changes in \( \mu \) lead to these RGEs, independent of the perturbation theory. They describe the running of parameters, as will be shown in the next section.

5.6 The Renormalization Group Equation of Standard Model Couplings

The RGE of a general coupling can be set up using DREG. The scale symmetry is broken by the renormalization scale \( \mu \). As shown previously, \( \mu \) is introduced to get dimensionless couplings in the Lagrangian, when extended to \( d \) dimensions in DREG. In \( d = 4 - 2 \epsilon \), the coupling is renormalized in the following way,

\[ g_0 = \mu^{\epsilon} Z_g(\mu) g(\mu). \] (5.40)
5.6. The Renormalization Group Equation of Standard Model Couplings

Since the bare coupling \( g_0 \) does not depend on \( \mu \), it must satisfy the differential equation, the RGE,

\[
\mu \frac{d}{d\mu} \left( \mu^\epsilon g(\mu) Z_g(\mu) \right) = \mu^\epsilon g(\mu) Z_g(\mu) \left( \epsilon + \frac{\mu}{g(\mu)} \frac{\partial g(\mu)}{\partial \mu} + \frac{\mu}{Z_g(\mu)} \frac{\partial Z_g(\mu)}{\partial \mu} \right) = 0. \tag{5.41}
\]

The renormalized coupling \( g(\mu) \) is not invariant under scale variations, manifesting its running. The running of this coupling, the \( \beta \) function, can therefore be defined as

\[
\beta(g(\mu)) = \mu \frac{d}{d\mu} g(\mu) = \mu \frac{\partial g(\mu)}{\partial \mu} (\mu^\epsilon Z_g^{-1}(\mu) g_0) = -\epsilon g(\mu) - \frac{g(\mu)}{Z_g(\mu)} \mu \frac{\partial g(\mu)}{\partial \mu} Z_g(\mu) = -\epsilon g(\mu) + g(\mu) \gamma_g(\mu), \tag{5.42}
\]

where the anomalous dimension of the RGE is defined as

\[
\gamma_g = -\frac{1}{Z_g(\mu)} \frac{\partial Z_g(\mu)}{\partial g} = -\frac{1}{Z_g(\mu)} \frac{\partial Z_g(\mu)}{\partial g} \frac{\partial g(\mu)}{\partial \mu} = -\frac{1}{Z_g(\mu)} \frac{\partial Z_g(\mu)}{\partial g} \beta(g(\mu)). \tag{5.43}
\]

The \( \beta \) function and the anomalous dimension therefore enter the RGE in the following way, dropping the signature of \( \mu \) dependence \((\mu)\) from here on,

\[
\mu^\epsilon g Z_g \left( \epsilon + \frac{1}{g} \beta(g) - \gamma_g \right) = 0. \tag{5.44}
\]

In DREG implementing the \( \overline{\text{MS}} \) scheme, the renormalization constant is expanded in terms of the poles,

\[
Z_g = 1 + \delta Z_g = 1 + \sum_{k=1}^{L} \frac{c_k^L}{\epsilon^L} + \sum_{i=1}^{k} \frac{c_i^k}{\epsilon^i} = 1 + g^2 \frac{c_1^1}{\epsilon} + g^4 \left( \frac{c_2^2}{\epsilon^2} + \frac{c_3^3}{\epsilon^3} \right) + \text{O}(g^6), \tag{5.45}
\]

where \( c_i^k \) are the counterterm coefficients in the renormalization constants. The anomalous dimension can be expressed in terms of these coefficients in the following way,

\[
\gamma_g = -\frac{1}{Z_g(\mu)} \frac{\partial Z_g(\mu)}{\partial g} \beta(g),
\]

\[
= -\frac{1}{Z_g(\mu)} \frac{\partial}{\partial g} \sum_{k=1}^{L} g^{2k} \sum_{i=1}^{k} \frac{c_k^i}{\epsilon^i} \beta(g). \tag{5.46}
\]

Expanding the anomalous dimension in orders of the coupling,

\[
\gamma(g) = \gamma^{(0)}(g^2) + \gamma^{(1)}(g^4) + \text{O}(g^6), \tag{5.47}
\]

and plugging in the expansion of the renormalization constant, the anomalous dimension at each order in the coupling can be matched with its corresponding counterterm,
### 5.7. Renormalizing Composite Operators

\[
Z_g \gamma_g = (1 + g^2 c_1 \frac{1}{\epsilon} + g^4 (\frac{c_2^2}{\epsilon^2} + \frac{c_1^2}{\epsilon}) + O(g^6)) \gamma_g = -\beta_0(g) \frac{\partial Z_g}{\partial g}
\]

\[
= (g - g \gamma(g)) (2 g \frac{c_1}{\epsilon} + 4 g^3 (\frac{c_2^2}{\epsilon^2} + \frac{c_1^2}{\epsilon}) + O(g^5)).
\]  

(5.48)

Setting \(\epsilon \to 0\), the part proportional to \(\epsilon_0\) becomes

\[
\gamma(g) = -2 c_1^2 g^2 - 4 c_1^2 g^4 + O(g^6).
\]  

(5.49)

Therefore, it is found that \(\gamma^{(0)} = -2 c_1^2\), \(\gamma^{(1)} = -4 c_1^2\), etc, establishing the relation between anomalous dimensions and counterterms.

### 5.7 Renormalizing Composite Operators

The renormalization within an EFT follows a power counting structure, where coefficients of operators of a certain dimension get renormalized by coefficients of operators of all dimensions at or below this dimension, following [88],

\[
\mu \frac{d\{C_5\}}{d\mu} = A_1\{C_4\}\{C_5\} \quad \text{and} \quad \mu \frac{d\{C_6\}}{d\mu} = A_2\{C_4\}\{C_6\} + A_3\{C_4\}\{C_5\}^2
\]  

(5.50)

where \(C_d\) are the Wilson coefficients of the higher dimensional operators and \(d\) is the dimension of the operator. The EFT is non-renormalizable, since one can expand the theory to infinitely high dimensions and therefore needs renormalization from coefficients of these infinitely many dimensions. The SM is a special renormalizable case, in which operators of dimensions greater than four simply do not exist. This renormalizable case is recovered when \(\Lambda \to \infty\), and the SM theory is decoupled from new physics.

Higher dimensional operators originate from unknown interactions at the scale \(\Lambda\). The effects of these interactions can only be measured at low energy, at the scale of experiments, implementing for example the SMEFT. Therefore, RGEs are important in the SMEFT, accounting for the running of the parameters associated with the interactions at the high energy scale, with respect to the energy. To match deviations found at the EW scale by experiments, it is necessary to run the values found at the high scale \(\Lambda\) down to this EW level.

The bare effective Lagrangian including dimension six operators is given by

\[
\mathcal{L}_{\text{eff}}^0 = \mathcal{L}_{\text{kin}}^0 + \mathcal{L}_{\text{mass}}^0 + \sum_i \frac{C_i^0}{\Lambda^2} \mathcal{O}_i^0.
\]  

(5.51)

For an independent composite operator, where the SM fields are renormalized in the usual way, as described, the renormalization follows simply as [32],

\[
\mathcal{O}^{(0)} = Z_0 Z_{\text{SM}} \mathcal{O}^{(r)}.
\]  

(5.52)
5.7. Renormalizing Composite Operators

The operator merely receives an additional renormalization through multiplication of the renormalization constants of the SM and the SMEFT respectively. The anomalous dimension of the composite operator is, as in the SM, given by

$$\gamma_O = -\mu \frac{d \ln Z_O}{d\mu}.$$  \hspace{1cm} (5.53)

In the case where multiple invariant local operators exist with the same quantum numbers, operator mixing occurs, such that the renormalization constant becomes instead a renormalization matrix,

$$O_i^{(0)} = Z_{ij} O_j.$$  \hspace{1cm} (5.54)

In the \textit{MS} scheme, the renormalization matrix is dimensionless. Operators of different dimensions do not mix and the theory can be renormalized order by order in dimension [32]. The S matrix element at each order in the expansion, \(\frac{1}{\Lambda^2}\), can therefore be reconstructed within the EFT. This makes it convenient to work in the \textit{MS} scheme, when renormalizing an EFT composed of higher dimensional operators. In the SMEFT, the RGE is set up in the same way, as described in the previous section. Letting \(O_i^{(0)}\) absorb the SM renormalizations, \(O_i^{(0)} \to Z_{SM} O_i^{(0)}\), ignoring these for now, the anomalous dimension is defined through

$$0 = \mu \frac{\partial}{\partial \mu} C_i^{(0)} = \left( \mu \frac{\partial}{\partial \mu} Z_{ij} \right) C_j + Z_{ij} \left( \mu \frac{\partial}{\partial \mu} C_j \right), \Rightarrow \mu \frac{\partial}{\partial \mu} C_j = -\left( Z_{jk}^{-1} \right) \left( \mu \frac{\partial}{\partial \mu} Z_{ki} \right) C_i = \gamma_{ij} C_i.$$  \hspace{1cm} (5.55)

One can choose to renormalize the coefficients instead, where

$$C_i^{(0)} = Z_{ij} C_j,$$  \hspace{1cm} (5.56)

such that \(Z_{ij} = \left( Z_{ji}^{(0)} \right)^{-1}\). The anomalous dimensions of the Wilson coefficients are found in the same way to be

$$\mu \frac{\partial}{\partial \mu} C_j \equiv \gamma_{ji} C_i = -Z_{jk}^{-1} \left( \mu \frac{\partial}{\partial \mu} Z_{ki} \right) C_i = (\gamma_{ij}^{(0)})^T.$$  \hspace{1cm} (5.57)

The anomalous dimension enters the RGE, as can be seen solving the differential equation for the coupling. Expanding the anomalous dimension, making in this case explicit the loop parameter, \(\frac{1}{16 \pi^2}\),

$$\gamma_{ij} = \frac{g^2}{16 \pi^2} \gamma_{ij}^{(0)} + \frac{g^4}{(16 \pi^2)^2} \gamma_{ij}^{(1)} + \ldots,$$  \hspace{1cm} (5.58)

the solution up to the level of two loops is given by

$$C_i(\mu) = C_i(\Lambda) - \frac{g^2}{16 \pi^2} \gamma_{ij}^{(0)} C_j(\Lambda) \log \left( \frac{\Lambda}{\mu} \right) - \frac{g^4}{(16 \pi^2)^2} \gamma_{ij}^{(1)} C_j(\Lambda) \log \left( \frac{\Lambda}{\mu} \right) + \mathcal{O}(g^6).$$  \hspace{1cm} (5.59)
5.7. Renormalizing Composite Operators

The precise mixing of the higher dimensional operators is therefore established through these RGEs. The mixing can be seen to be log enhanced, which could lead to considerable contributions, when calculating observables in the SMEFT.
Chapter 6

The One Loop $h \rightarrow \gamma \gamma$ Decay in the Standard Model Effective Field Theory

Contributions to interactions within the SM from new physics, manifesting themselves through higher dimensional operators, are expected to be very small. As experiments push up the scale of new physics discoveries, these contributions will most likely decrease further. The most suitable areas to look for such deviations within SM interactions, allowing for more noticeable contributions, are within SM interactions and couplings, which already take place at loop-level. Included in such interactions are $h \rightarrow \gamma \gamma$ and $h \rightarrow Z \gamma$. Moreover, these channels are easier to access for the LHC than other interactions, making the measurements of these interactions more precise. This is due to QCD background corrections not entering these particular channels. Therefore, these interactions are more relevant to address in the search for new physics at present time [118].

On account of these arguments, a decay such as the $h \rightarrow \gamma \gamma$ decay is highly suitable for an initial study of contributions from the SMEFT at loop order. Assuming a linear realization of the scalar sector, such that the Higgs is part of a SM doublet, it will in the following be investigated how higher dimensional operators contribute to the $h \rightarrow \gamma \gamma$ decay through finite contributions.

The logarithmically enhanced contributions entering through mixing of operators in the RGEs have already been calculated at one-loop [6, 7, 9–11]. The focus of this thesis is the step further, namely the calculation of the remaining finite contributions. If the scale of new physics is kept at a few TeV, such contributions are of the same order as the RGE contributions and therefore just as important to calculate. The calculation of these finite terms will be carried out in the broken theory, which introduces subtleties. As a check, the divergent parts are calculated in the broken phase and compared with the RGE results found in the unbroken phase. The full one-loop result of finite contributions to the $h \rightarrow \gamma \gamma$ decay, following [119, 120] will as a result be presented in the following sections.

Furthermore, an outline of the calculational methods will be given, through an explicit example, where these are implemented in a SM one-loop calculation. Renormalization will be accounted for, ensuring the finiteness of the result. The calculations are carried out in $R_\xi$ gauge, which eventually provides a check of the result. Intermediate steps of the calculation, such as the write up of the amplitudes and the full $\xi$ dependent results, are presented in the
appendices.

### 6.1 $h \to \gamma \gamma$ Decay in the Standard Model

The SM one-loop contributions to the $h \to \gamma \gamma$ decay have already been calculated, [96, 121–123]. These contributions will in this thesis be calculated in the $R_\xi$ gauge in the BFM. Diagrams contributing to this decay in the BFM at one-loop are shown in figure 6.1. The Feynman rules within the BFM have been derived and are listed in appendix C. Using these Feynman rules, the amplitudes for the diagrams in the BFM are written up in appendix E in the $\overline{\text{MS}}$ scheme using DREG.

![Diagram](image.png)

**Figure 6.1:** One loop diagrams contributing to the $h \to \gamma \gamma$ decay at one-loop in the SM. Arrows on propagators indicate charge flow. Here, $h$ is the Higgs field, $\phi_\pm$ are the charged Goldstone bosons, $\psi$ are the fermions, $u_\pm$ are the charged ghosts and $W$ and $\gamma$ are the gauge fields.

To illustrate the calculational approach used throughout the thesis, the fermionic diagram (e) of figure 6.1, with the amplitude,

$$i \mathcal{A}_e = 2 m_\psi e^2 Q_\psi^2 N_c \left( \frac{\mu^2 e^{\gamma_E}}{4 \pi} \right)^\epsilon \int \frac{d^4 q}{(2\pi)^d} c_\alpha(p_2) c_\beta(p_3) \frac{1}{q^2 - m_\psi^2} \frac{1}{(q - p_2)^2 - m_\psi^2} \frac{1}{(q - p)^2 - m_\psi^2} \left[ (q - m_\psi) \gamma_\alpha (q - p_2 - m_\psi) \gamma_\beta (q - p - m_\psi) \right],$$

will be evaluated in greater detail. The same methods will be used when proceeding to the SMEFT calculations, implementing the same tools, as will be described, to carry out the
The numerator contains the following trace,

\[ \mathcal{M}_{\alpha\beta}^{\text{Tr}} = \text{Tr} \left[ (\slashed{q} - m_\psi) \gamma_\alpha (\slashed{q} - \slashed{p}_2 - m_\psi) \gamma_\beta (\slashed{q} - \slashed{p} - m_\psi) \right], \quad (6.2) \]

which can be evaluated using the identities written up in Appendix D. Furthermore, the Ward identities, \( p_2^\alpha \epsilon_\alpha (p_2) = p_3^\beta \epsilon_\beta (p_3) = 0 \) are applied and the photons are put on-shell, \( p_2^2 = p_3^2 = 0 \), so that

\[ \mathcal{M}_{\alpha\beta}^{\text{Tr}} = -4 m_\psi \left( 4 q_\alpha q_\beta - 4 q_\alpha p_{2,\beta} - q_\beta p_{3,\alpha} + p_{3,\alpha} p_{2,\beta} + g_\alpha \beta \left( 2 q \cdot p_2 - q^2 - 2 p_2 \cdot p_3 + m_\psi^2 \right) \right). \quad (6.3) \]

The various form factors in the Passarino-Veltman decomposition can be projected out, as described in Appendix B.2. The full amplitude is therefore decomposed as,

\[ i A_c = i A_\epsilon^{\alpha\beta} e^{\alpha} (p_2) e^{\beta} (p_3) = \left( T_4 p_2^\beta p_3^2 + T_5 p_2 \cdot p_3 g^{\alpha\beta} \right) e^{\alpha} (p_2) e^{\beta} (p_3), \quad (6.4) \]

and the form factors are given by

\[
T_4 = i Q_{\alpha\beta} A_\epsilon^{\alpha\beta} = 8 m_\psi^2 \epsilon^2 Q_\psi^2 N_c \left( \frac{\mu^2 e^{\gamma_E}}{4 \pi} \right)^\epsilon \frac{1}{d - 2} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_\psi^2} \frac{1}{(q - p_2)^2 - m_\psi^2},
\]

\[
\times \left( \frac{8}{m_\psi^2} q^2 - \frac{16}{m_\psi^2} q \cdot p_2 q \cdot p_3 + \frac{8(d - 2)}{m_\psi^2} q \cdot p_2 + (2 - d) \right),
\]

\[
T_5 = i P_{\alpha\beta} A_\epsilon^{\alpha\beta} = 8 m_\psi^2 \epsilon^2 Q_\psi^2 N_c \left( \frac{\mu^2 e^{\gamma_E}}{4 \pi} \right)^\epsilon \frac{1}{d - 2} \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_\psi^2} \frac{1}{(q - p_2)^2 - m_\psi^2},
\]

\[
\times \left( (d - 6) q^2 + \frac{16}{m_\psi^2} q \cdot p_2 q \cdot p_3 - 2(d - 2) q \cdot p_2 + \frac{d}{2} - 1 \right) \frac{1}{m_\psi^2} + (2 - d) m_\psi^2. \quad (6.5)
\]

The form factors are integrals composed of a numerator part with various scalar products of the internal and external momenta, and of denominators \( \sim \frac{1}{q^2 - m^2} \).

In the next step, these integrals are reduced to master integrals, using Passarino-Veltman reductions, as described in Appendix B.3. Such master integrals are subsequently more easy to solve. The above traces and reductions are carried out automatically using the tool FORM, as described in Appendix B.1.

The Feynman parametrization is carried out in Mathematica for this one loop case. The reduced integrals are shifted in the following way, where \( N \) is the number of propagators,

\[
\frac{1}{\prod_{i=1}^N ((q + p_i)^2 - m_i)} : \quad q \to l - \delta (1 - \sum_{i=1}^N x_i) \sum_{i=1}^N x_i p_i,
\]

\[
\Delta = l^2 - \delta (1 - \sum_{i=1}^N x_i) \sum_{i=1}^N x_i ((q + p_i)^2 - m_i), \quad (6.6)
\]
and eventually take the form
\[ \int \frac{d^d l}{(2\pi)^d} \frac{(l^2)^\alpha}{(l^2 - \Delta)^n}, \tag{6.7} \]

which can be evaluated using the steps of appendix B.5.

After the final integrations over Feynman parameters and expanding in \( \epsilon \), the result is recovered. For the SM \( h \to \gamma \gamma \) amplitude, the following complete result has been verified \([96, 121–123, 123]\),

\[ A_{SM}(h \to \gamma \gamma) = \frac{\Gamma^\gamma e^2 A_{1}^{h \gamma \gamma}}{8\pi^2 v}, \tag{6.8} \]

where

\[ \Gamma^\gamma = A_1(\tau_p) + N_c Q^2 A_{1/2}(\tau_i), \tag{6.9} \]

is given in terms of the functions,

\[ A_1(\tau_p) = 2 + 3 \tau_p (1 + (2 - \tau_p) f(\tau_p)), \tag{6.10} \]
\[ A_{1/2}(\tau_p) = -2 \tau_p (1 + (1 - \tau_p) f(\tau_p)), \tag{6.11} \]

where \( \tau_p = 4 m_p^2/m_h^2 \) and

\[ f(\tau_p) = \begin{cases} \arcsin^2 \sqrt{1/\tau_p}, & \tau_p \geq 1, \\ -\frac{1}{4} \left[ \log \left( \frac{1 + \sqrt{1 - \tau_p}}{1 - \sqrt{1 - \tau_p}} \right) - i\pi \right]^2, & \tau_p < 1. \end{cases} \tag{6.12} \]

### 6.2 Dimension Six Operators Contributing to the \( h \to \gamma \gamma \) Decay in the Standard Model Effective Field Theory

The SM Lagrangian is extended, including the operators contributing to the \( h \to \gamma \gamma \) decay, namely by

\[ \mathcal{L}_6^{(0)} = C_{HB}^{(0)} \mathcal{O}_{HB}^{(0)} + C_{HW}^{(0)} \mathcal{O}_{HW}^{(0)} + C_{HWB}^{(0)} \mathcal{O}_{HWB}^{(0)} + C_{W}^{(0)} \mathcal{O}_{W}^{(0)} + C_{H}^{(0)} \mathcal{O}_{H}^{(0)} + C_{H^{\square}}^{(0)} \mathcal{O}_{H^{\square}}^{(0)} \\
+ C_{HD}^{(0)} \mathcal{O}_{HD}^{(0)} + C_{eW}^{(0)} \mathcal{O}_{eW}^{(0)} + C_{eB}^{(0)} \mathcal{O}_{eB}^{(0)} + C_{uW}^{(0)} \mathcal{O}_{uW}^{(0)} + C_{uB}^{(0)} \mathcal{O}_{uB}^{(0)} + C_{dW}^{(0)} \mathcal{O}_{dW}^{(0)} + C_{dB}^{(0)} \mathcal{O}_{dB}^{(0)} \\
+ C_{eH}^{(0)} \mathcal{O}_{eH}^{(0)} + C_{uH}^{(0)} \mathcal{O}_{uH}^{(0)} + C_{dH}^{(0)} \mathcal{O}_{dH}^{(0)} + H.c., \tag{6.13} \]

where the Wilson coefficients are dimensionfull, including a factor of \( 1/\Lambda^2 \). The bare operators, defined in terms of bare parameters with the \( (0) \) labels suppressed on the right hand side, are
as follows, following the operator notation of \[84\].

\[
\begin{align*}
\mathcal{O}_{HB}^{(0)} &= g_1^2 H^* H B_{\mu \nu} B^{\mu \nu}, \\
\mathcal{O}_{HW}^{(0)} &= g_1 g_2 H^* H W_{\mu \nu}^I B^{\mu \nu}, \\
\mathcal{O}_{dW}^{(0)} &= g_2 \bar{q}_{r, a} \sigma^{\mu \nu} d_s \tau_{ab} H_b W_{\mu \nu}^I, \\
\mathcal{O}_{uW}^{(0)} &= g_2 \bar{q}_{r, a} \sigma^{\mu \nu} u_s \tau_{ab} H_b W_{\mu \nu}^I, \\
\mathcal{O}_H^{(0)} &= \lambda (H^I H)^3, \\
\mathcal{O}_{H^D}^{(0)} &= (H^I D_{\mu} H)^* (H^I D^{\mu} H), \\
\mathcal{O}_{uH}^{(0)} &= Y_u H^I H (\bar{q}_u u_r H), \\
\mathcal{O}_{dH}^{(0)} &= Y_d H^I H (\bar{q}_d d_r H).
\end{align*}
\]

\(\tau^I\) are the weak isospin Pauli matrices, \(g_i\) are the canonically normalized SM gauge couplings and \(r, s\) are flavour indices. These operators are expanded and the fields rotated into their mass eigenstates. This eventually leads to the effective Lagrangian, which will be written up in the following. From the effective Lagrangian, the three-point and four-point effective vertices are found, which contribute to the \(h \to \gamma \gamma\) decay. The Feynman rules for these are derived and listed in appendix 6H.

Operators are divided into two groups, depending on how they enter the \(h \to \gamma \gamma\) decay. Operators leading to direct contributions (DC) enter directly with an effective interaction, leading to an effective vertex in a diagram contributing to the decay. Such diagrams need not already exist in the SM. Operators leading to indirect contributions (IC) contribute through quadratic terms already present in the SM Lagrangian. Such terms are accounted for through a canonical normalization. This leads to a redefinition of SM fields and parameters. These operators therefore enter indirectly through diagrams already existing in the SM. It is possible that operators contribute both directly and indirectly. These operators redefine the SM fields and parameters. After canonical normalization, the same operators enter the Lagrangian through new interactions. The DCs and ICs will be addressed in the following two sections, named accordingly.

### 6.3 Direct Contributions

DCs enter the effective Lagrangian through expanding the dimension six operators, \(\mathcal{O}_{HB}, \mathcal{O}_{HW}, \mathcal{O}_{dW}, \mathcal{O}_{uW}\), and \(\mathcal{O}_{\psi B}\), where \(\psi = e, d, u\), and rotating the fields into their mass eigenstates. Through the effective Lagrangian, these operators enter the \(h \to \gamma \gamma\) decay via the diagrams of figure 6.2, where the contributions can be seen as rectangles. They contribute with new interactions amongst SM fields.

For the \(h \to \gamma \gamma\) decay, the relevant part of the effective Lagrangian from these direct contributions is derived to be,
6.4. Indirect Contributions from Dimension Six Operators

\[ \mathcal{L}_{\text{eff}}^{\text{Exp}} = \frac{1}{2} \left( h^2 + 2h \sqrt{Z_h} (v \sqrt{Z_v} + \delta v) + \phi^2 \right) \left( N_{HB} + N_{HW} - N_{HWB} \right) e^2 A_{\mu \nu} A^{\mu \nu} + (\phi_+ \phi_-) (C_{HB} + C_{HW} + C_{HWB}) e^2 A_{\mu \nu} A^{\mu \nu} + (2h v) C_{HW} g^2 W^+_{\mu} W^\mu_{\nu} \]  
(6.15)

\[ - 2i e g^2_2 (2h v + v^2) \left[ C_{HW} \left( A_{\mu} W^\nu_{\nu} - A_{\nu} W^\mu_{\mu} \right) \right] + 2i e g^2_2 (2h v + v^2) \left[ \left( C_{HW} - \frac{C_{HWB}}{2} \right) \left( A_{\mu} W^\mu_{\mu} A_{\nu} \right) \right] + 2 e^2 g^2_2 (2h v + v^2) C_{HW} \left( W^+_{\mu} W^-_{\mu} A_{\nu} - W^+_{\nu} W^-_{\nu} A_{\mu} \right) + i e g^2_2 (v + h) C_{HWB} \left( A_{\mu} \phi^+ W^-_{\mu} - \phi^- W^+_{\mu} \right) + C_{HWB} v e^2 g^2 \left( A_{\mu} (\phi^+ W^-_{\mu} - \phi^- W^+_{\mu}) - (\phi^+ W^-_{\mu} + \phi^- W^+_{\mu}) A_{\nu} \right) A^{\mu \nu} + 3i s_w g^2_2 C_{W} A_{\mu} \left( W^+_{\rho} W^-_{\rho} - W^+_{\mu} W^\mu_{\mu} \right) + 3 e s_w g^2_2 C_{W} A_{\mu} \left( W^\mu_{\rho} (A^\mu_{\mu} W^-_{\rho} - A_{\nu} W^\mu_{\nu}) + W^\rho_{\mu} (A^\mu_{\mu} W^\rho_{\rho} - A_{\nu} W^\rho_{\nu}) \right) + W^\nu_{\mu} (A^\mu_{\mu} W^\rho_{\rho} - A_{\nu} W^\rho_{\nu}) + W^\rho_{\nu} (A^\mu_{\mu} W^\rho_{\rho} - A_{\nu} W^\rho_{\nu}) - \frac{1}{\sqrt{2}} e g^2_2 \sigma^{\mu \nu}(h + v) \left( C_{eW} e_{e} - C_{eB} e_{e} \right) A_{\mu \nu} + (e \leftrightarrow \mu) + (e \leftrightarrow \tau) + H.c. + \frac{1}{\sqrt{2}} e g^2_2 \sigma^{\mu \nu}(h + v) \left( C_{uW} u_{u} + C_{uB} u_{u} \right) A_{\mu \nu} + (u \leftrightarrow c) + (u \leftrightarrow t) + H.c. - \frac{1}{\sqrt{2}} e g^2_2 \sigma^{\mu \nu}(h + v) \left( C_{dW} d_{d} - C_{dB} d_{d} \right) A_{\mu \nu} + (d \leftrightarrow s) + (d \leftrightarrow b) + H.c. \]  
(6.16)

The Feynman rules can be found from this effective Lagrangian, and they are given in appendix H. Subsequently, the diagrams in figure 6.2 can be calculated, giving the contributions to the \( h \rightarrow \gamma \gamma \) decay from these direct operators. The amplitudes are written up in appendix I, from which the results will be calculated.

6.4 Indirect Contributions from Dimension Six Operators

The ICs arise from operators entering indirectly through redefining fields and parameters within the SM, when canonically normalizing the Lagrangian. This leads to vertices within the SM being redefined [124, 125]. In the \( h \rightarrow \gamma \gamma \) decay, the operators contributing indirectly are \((O^0_H, O^0_{HW}, O^0_{HWB}) \). The first three operators contribute to the Higgs and Goldstone boson kinetic terms as well as to the Higgs mass. These contributions are absorbed into the SM kinetic and mass terms to acquire a canonically normalized Lagrangian, leading to redefinitions of the Higgs and Goldstone fields, as well as the VEV and the coupling \( \lambda \). These redefinitions enter the Higgs potential, affecting interactions amongst Higgs and Goldstone bosons. The fermionic operators contribute to interactions amongst fermions and the Higgs boson, leading to a redefinition of the Yukawa coupling. The last two operators, which have already been seen to enter directly in the effective Lagrangian also contribute indirectly. \( O_{HWB} \) causes a \( W^\pm_{\mu \nu} - B_{\mu \nu} \) kinetic mixing, which eventually affects the rotation of the gauge fields into their mass eigenstates. \( O_{HW} \) redefines the gauge fields \( W^\pm \) and the coupling associated with them, \( g_2 \).
6.4. Indirect Contributions from Dimension Six Operators

Besides redefining parameters, the six scalar and fermion operators also enter the Lagrangian with DCs causing interactions not already seen in the SM. However, this only occurs at the four-point vertex level, as will be shown. Therefore, these DCs from indirect operators will not affect the one loop calculation of the $h \to \gamma \gamma$ decay.
6.4. Indirect Contributions from Dimension Six Operators

6.4.1 Canonical Normalization

The canonical normalization of the Lagrangian, causing an absorption of the indirect operators into kinetic and mass terms will be outlined in the following. The dimension six operators $O_{H^0}$ and $O_{HD}$ contribute to the kinetic part of the Higgs Lagrangian, $L_{kin}$, as can be seen through expansion,

$$L_{kin}^{H^0+HD} = \frac{1}{4} C_{H^0} (2 \phi_+ \phi_- + h^2 + v^2 + 2h v + \phi_0^2) \partial^2 (2 \phi_+ \phi_- + h^2 + v^2) + 2h v + \phi_0^2 + \frac{1}{4} C_{HD} (2 \phi_- \partial_\mu \phi_+ + (h + v + i \phi_0) \partial_\mu(h + v + i \phi_0))^* \times (2 \phi_+ \partial_\mu \phi_+ + (h + v + i \phi_0) \partial_\mu(h + v + i \phi_0)) = C_{H^0} \left(-2 \phi_+ \phi_- \partial_\phi^+ \partial_\phi^- - (h + v)^2 (\partial h)^2 - \phi_0^2 (\partial \phi_0)^2 \right) - (\phi_-)^2 (\partial \phi_+)^2 - (\phi_+)^2 (\partial \phi_-)^2 - 2(h + v) \left(\phi_- \partial_\mu \phi_+ + \phi_+ \partial_\mu h \partial_\mu \phi_- + \phi_0 \partial_\mu \phi_0 \partial_\mu h - 2 \phi_0 \phi_- \partial_\mu \phi_0 \partial_\mu \phi_+ - 2 \phi_0 \phi_+ \partial_\mu \phi_0 \partial_\mu \phi_- \right) + \frac{1}{4} C_{HD} \left((h + v)^2 + \phi_0^2)((\partial_\mu h)^2 + (\partial_\mu \phi_0)^2) + 4\phi_- \phi_+ \partial_\mu \phi_+ \partial_\mu \phi_- + 2 \phi_+ (h + v + i \phi_0) \partial_\mu \phi_- (\partial_\mu h - i \partial_\mu \phi_0) + 2 \phi_- (h + v - i \phi_0) \partial_\mu \phi_+ (\partial_\mu h + i \partial_\mu \phi_0) \right), \quad (6.17)$$

where integration by parts has been implemented in the last equality. To canonically normalize the kinetic part of the Lagrangian, the kinetic terms are rewritten as follows,

$$\frac{1}{2} (\partial_\mu h)^2 \left(1 - 2(h + v)^2 \left(C_{H^0} - \frac{1}{4} C_{HD} \right) \right) = \frac{1}{2} \left(\partial_\mu \left[1 - v^2 \left(C_{H^0} - \frac{1}{4} C_{HD} \right) \right] - v \left(C_{H^0} - \frac{1}{4} C_{HD} \right) \right) h^2$$

$$- \frac{1}{3} \left(C_{H^0} - \frac{1}{4} C_{HD} \right) h^3 \right)^2, \quad (6.18)$$

$$\frac{1}{2} (\partial_\mu \phi_0)^2 \left(1 - 2 \phi_0^2 \left(C_{H^0} - \frac{1}{4} C_{HD} \right) \right) = \frac{1}{2} \left(\partial_\mu \left[\phi_0 - \frac{1}{3} \left(C_{H^0} - \frac{1}{4} C_{HD} \right) \phi_0^3 \right] \right)^2, \quad (6.18)$$
\[ \partial_{\mu} \phi^+ \partial^\mu \phi^- \left( 1 - 2 \phi_+ \phi_- \left( C_{H^\square} - \frac{1}{2} C_{HD} \right) \right) \]
\[ = \left( \partial_{\mu} \left[ \phi^+ - \frac{1}{2} \phi_- \left( \phi_+^2 \right) \left( C_{H^\square} - \frac{1}{2} C_{HD} \right) \right] + \frac{1}{2} \left( C_{H^\square} - \frac{1}{2} C_{HD} \right) \phi_+^2 \partial_{\mu} \phi^- \right) \]
\times \left( \partial_{\mu} \left[ \phi^- - \frac{1}{2} \phi_+ \left( \phi_-^2 \right) \left( C_{H^\square} - \frac{1}{2} C_{HD} \right) \right] + \frac{1}{2} \left( C_{H^\square} - \frac{1}{2} C_{HD} \right) \phi_-^2 \partial_{\mu} \phi_+ \right). \] (6.19)

The following redefinitions will therefore lead to a canonically normalized kinetic part of the Lagrangian,

\[ h \to h \left( 1 + \frac{1}{3} \left( C_{H^\square} - \frac{1}{4} C_{HD} \right) \phi_0^2 \right), \]
\[ \phi_0 \to \phi_0 \left( 1 + \frac{1}{3} \left( C_{H^\square} - \frac{1}{4} C_{HD} \right) \phi_0^2 \right), \]
\[ \phi_+ \to \phi_+ \left( 1 + \frac{1}{2} \left( C_{H^\square} - \frac{1}{2} C_{HD} \right) \phi_- \phi_+ \right), \]
\[ \phi_- \to \phi_- \left( 1 + \frac{1}{2} \left( C_{H^\square} - \frac{1}{2} C_{HD} \right) \phi_- \phi_+ \right). \] (6.20)

After this redefinition, the kinetic Lagrangian becomes, including new direct contributions from the above operators as a result of the redefinitions,

\[ \mathcal{L}_{\text{kin}} + \mathcal{L}_0^{H^\square+HD} \to \partial_{\mu} \phi^+ \partial^\mu \phi^- + \frac{1}{2} \left( \partial_{\mu} h \right)^2 + \frac{1}{2} \left( \partial_{\mu} \phi_0 \right)^2 \]
\[ - \frac{1}{2} \left( C_{HD} + \frac{1}{2} C_{HD} \right) \left( \phi_+^2 \left( \partial_{\mu} \phi_- \right)^2 + \phi_-^2 \left( \partial_{\mu} \phi_+ \right)^2 \right) \]
\[ - 2 \left( h + v \right) \left( \partial_{\mu} \phi_0 \partial h \partial \phi_0 \right) \]
\[ - 2 \left( C_{H^\square} - \frac{1}{4} C_{HD} \right) \left( \left( h + v \right) \partial h + \phi_0 \partial \phi_0 \right) \left( \phi_- \partial \phi_+ + \phi_+ \partial \phi_- \right) \]
\[ + \frac{1}{2} i C_{HD} \left( \phi_0 \partial h - h \partial \phi_0 \right) \left( \phi_+ \partial \phi_- - \phi_- \partial \phi_+ \right). \] (6.21)

Besides changes in the kinetic Lagrangian occurring from these redefinitions, changes will also appear in the potential. The operator \( \mathcal{O}_H \) contributes to the Higgs potential part of the Lagrangian, \( \mathcal{L}_{\text{pot}} \), in the following way

\[ \mathcal{L}_{\text{pot}} + \mathcal{L}_0^H = -\lambda \left( H^\dagger H - \frac{1}{2} v^2 \right)^2 + C_H \lambda (H^\dagger H)^3. \] (6.22)

The new SMEFT VEV, \( \tilde{v} \), is found through minimizing the potential, including the contribution from the dimension six operator,
6.4. Indirect Contributions from Dimension Six Operators

\[
\frac{\partial V(H)}{\partial (H^\dagger H)} = -\lambda v^2 + 2 \lambda H^\dagger H - 3 C_H \lambda (H^\dagger H)^2 \equiv 0, \quad \Rightarrow \\
\langle H^\dagger H \rangle \approx \frac{1}{2} v^2 \left( 1 + \frac{3}{4} C_H v^2 \right) \equiv \frac{1}{2} v^2. \tag{6.23}
\]

This redefines the SM VEV in the following way,

\[
v \to \bar{v} \left( 1 - \frac{3}{8} C_H \bar{v}^2 \right). \tag{6.24}
\]

From the above derivation, the new VEV can be seen to enter the Higgs doublet, defined in the SMEFT to be

\[
H = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \sqrt{2} i \phi^+ \\ h + \bar{v} + i \phi_0 \end{array} \right). \tag{6.25}
\]

On the other hand, the field content of this doublet is given by the SM fields without inclusion of effective operators and therefore needs to be redefined, as described above, when entering the Lagrangian. With the SMEFT VEV and redefinitions, suppressing the bar on the VEV \( \bar{v} \to v \) in what follows, the expansion of the potential can be carried out,

\[
V(H) = \lambda \left( 2 v h \phi_+ \phi_- + v h \phi_0^2 \right) \left( 1 - \left( \frac{3}{2} C_H - C_H \square + \frac{1}{4} C_{HD} \right) v^2 \right) \\
+ v^2 h^2 \left( 1 - \left( \frac{3}{2} C_H - 2 C_H \square + \frac{1}{2} C_{HD} \right) v^2 \right) \\
+ \left( (\phi_+)^2 (\phi_-)^2 + \phi_0^2 \phi_+ \phi_- + \frac{1}{4} \phi_0^4 \right) \left( 1 - \frac{3}{2} C_H v^2 \right) \\
+ \frac{1}{4} h^4 \left( 1 - \left( \frac{15}{2} C_H - \frac{14}{3} C_H \square + \frac{7}{6} C_{HD} \right) v^2 \right) \\
+ h^2 \phi_+ \phi_- \left( 1 - \left( \frac{15}{4} C_H - 4 C_H \square + C_{HD} \right) v^2 \right) \\
+ v h^3 \left( 1 - \left( \frac{5}{2} C_H - 5 C_H \square + \frac{5}{4} C_{HD} \right) v^2 \right) \\
+ \frac{1}{2} h^2 \phi_0^2 \left( 1 - \left( \frac{9}{2} C_H - 4 C_H \square + C_{HD} \right) v^2 \right) \tag{6.26}
\]
6.4. Indirect Contributions from Dimension Six Operators

\[ L_Y + L_Y^H = -\frac{1}{\sqrt{2}} Y_\psi \bar{\psi} \psi (h + v \pm i \phi_0) \left( 1 - \frac{1}{2} C_{\psi H} ( (h + v)^2 + \phi_0^2 + 2 \phi_+ \phi_- ) \right) + \ldots, \]  

(6.29)

where \( C_{\psi H} = (C_{\psi H}^{(s)}, C_{\psi H}^{(uH)}, C_{\psi H}^{(dH)}) \), and the \( \pm \) represents \( \psi = d/e \) or \( u \), respectively. To canonically normalize the fermion mass term, the redefinition

\[ Y_\psi \to Y_\psi (1 + \frac{1}{2} C_{\psi H}^{(s)} v^2) \]  

(6.30)

is performed to obtain

\[ L_Y + L_Y^H = \frac{1}{\sqrt{2}} Y_\psi \bar{\psi} \psi \left[ v - h \left( 1 - C_{\psi H}^{(s)} v^2 + C_{H} v^2 - \frac{1}{4} C_{HD} v^2 \right) \right] + i \phi_0 \left( 1 + \frac{1}{3} \left( C_{H} - \frac{1}{4} C_{HD} \right) \phi_0^2 \right) + \frac{1}{2} C_{\psi H}^{(s)} \left( 3 h^2 v + h^3 + (h + v) \phi_0^2 \right) + 2(h+v)\phi_+\phi_- \pm 2 i v h \phi_0 \pm i h^2 \phi_0 \pm i \phi_0^3 \pm 2 i \phi_0 \phi_+ \phi_- \right) + \ldots, \]  

(6.31)

where the Higgs and Goldstone boson field redefinitions have also been accounted for. With all of the above redefinitions, the total Lagrangian is up to sixth order in the field content, given by
6.4. Indirect Contributions from Dimension Six Operators

\[ \mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{pot}} + \mathcal{L}_Y + \mathcal{L}_B^{H + H C + H D} \]

\[ = \partial_\mu \phi^+ \partial^\mu \phi^- + \frac{1}{2} (\partial_\mu h)^2 + \frac{1}{2} (\partial_\mu \phi_0)^2 - 2 (h + v) C_{H \square} \phi_0 \partial h \partial \phi_0 \]

\[ - \frac{1}{2} \left( C_{H \square} + \frac{1}{2} C_{HD} \right) \left( \phi^2_+ (\partial_\mu \phi_-)^2 + \phi^2_- (\partial_\mu \phi_+)^2 \right) \]

\[ - 2 \left( C_{H \square} - \frac{1}{4} C_{HD} \right) \left( (h + v) \partial h + \phi_0 \partial \phi_0 \right) \left( \phi_- \partial \phi_+ + \phi_+ \partial \phi_- \right) \]

\[ + \frac{1}{2} i C_{HD} \left( \phi_0 \partial h - h \partial \phi_0 \right) \left( \phi_+ \partial \phi_- - \phi_- \partial \phi_+ \right) \]

\[ - \frac{1}{\sqrt{2}} Y \psi \bar{\psi} \left[ v + h \left( 1 - C_{\psi H} v^2 + C_{H \square} v^2 - \frac{1}{4} C_{HD} v^2 \right) \right] \]

\[ + i \phi_0 \left( 1 + \frac{1}{3} \left( C_{H \square} - \frac{1}{4} C_{HD} \right) \phi_0^2 \right) - \frac{1}{2} C_{\psi H} \left( h^3 + (h + v) \phi_0^2 \right) \]

\[ + 3 h^2 v + 2 (h + v) \phi_+ \phi_- + 2 i v h \phi_0 + i h^2 \phi_0 + i \phi_0^3 + 2 i \phi_0 \phi_+ \phi_- \right] \]

\[ - \lambda \left\{ v^2 h^2 + \frac{1}{4} h^4 \left( 1 - \left( 6 C_H - \frac{8}{3} C_{H \square} + \frac{2}{3} C_{HD} \right) v^2 \right) \right\} \]

\[ + (\phi_+)^2 (\phi_-)^2 \left( 1 - 2 C_{H \square} v^2 + \frac{1}{2} C_{HD} v^2 \right) \]

\[ + \frac{1}{4} \phi_0^4 \left( 1 - 2 C_{H \square} v^2 + \frac{1}{2} C_{HD} v^2 \right) \]

\[ + \phi_0^2 \phi_+ \phi_- \left( 1 - \left( \frac{9}{4} C_H - 2 C_{H \square} + \frac{1}{2} C_{HD} \right) v^2 \right) \]

\[ + 2 v h \phi_+ \phi_- \left( 1 - \left( C_{H \square} - \frac{1}{4} C_{HD} \right) v^2 \right) \]

\[ + v^3 \left( 1 - \left( C_H v^2 - 3 C_{H \square} + \frac{3}{4} C_{HD} \right) v^2 \right) \]

\[ + \frac{1}{2} h^2 \phi_0^2 \left( 1 - \left( 2 C_H - 2 C_{H \square} + \frac{1}{2} C_{HD} \right) v^2 \right) \]

\[ + v h \phi_0^2 \left( 1 - \left( C_{H \square} - \frac{1}{4} C_{HD} \right) v^2 \right) \]

\[ - C_H \left( \frac{1}{8} v^6 + \frac{3}{4} (h^4 + \phi_0^4) \phi_+ \phi_- + \frac{3}{2} (h^2 + 2 h v + \phi_0^2) (\phi_+) (\phi_-)^2 \right) \]

\[ + (\phi_+)^3 (\phi_-)^3 + 3 h^3 v \phi_+ \phi_- + \frac{3}{2} h^2 \phi_0^2 \phi_+ \phi_- + 3 h v \phi_0^2 \phi_+ \phi_- \]

\[ + \frac{1}{8} h^6 + \frac{3}{8} h^2 \phi_0^4 + \frac{3}{4} h^5 v + \frac{3}{8} h^4 \phi_0^2 + h^3 v \phi_0^2 + \frac{3}{4} h v \phi_0^4 + \frac{1}{8} \phi_0^6 \right\} \]  \hfill (6.32)
6.4. Indirect Contributions from Dimension Six Operators

\[ + \left( C_{H\Box} - \frac{1}{2} C_{HD} \right) \left( 2 \phi_+^3 \phi_-^3 + (h^2 + \phi_0^2 + 2 h v) \phi_+^2 \phi_-^2 \right) \]
\[ + \left( C_{H\Box} - \frac{1}{4} C_{HD} \right) \left( 2 v h^5 + \frac{1}{3} (h^6 + 2 (h^4 + \phi_0^4 + 4 h^3 v) \phi_+ \phi_- \right. \]
\[ + \left. (4 h^3 v + h^4) \phi_0^2 + (h^2 + 2 h v) \phi_0^4 \right) \right) + \ldots \]  
(6.33)

In the BFM, the fields entering the above Lagrangian are split into classical and quantum fields, which needs to be accounted for, when working out the Feynman rules from these interactions. Furthermore, terms from the gauge fixing in the BFM, equation (2.61), get redefined in the same way as described above. The above redefinitions will for this explicit calculation only affect the gauge fixing terms including a classical Higgs field or a classical photon field. The part of the gauge fixing Lagrangian relevant for this calculation is therefore,

\[ \mathcal{L}_{GF} = -\frac{1}{\xi} G^+ G^- + \ldots, \]  
(6.34)

where

\[ G^\pm = \partial^\mu W^\pm_\mu \pm i e A^\mu \left( (v + \hat{h}) \phi^\pm \right) + \ldots. \]  
(6.35)

The redefinitions caused by the pure scalar operators as seen in equation (6.20), therefore shift part of the gauge fixing Lagrangian as follows, listing here only the shifted terms,

\[ \mathcal{L}^{\text{shift}}_{GF} \rightarrow \frac{g_2}{2} \hat{h} \left( 1 + \left( C_{H\Box} - \frac{1}{4} C_{HD} \right) v^2 \right) \left( \phi^- \partial^\mu W^+_\mu + \phi^+ \partial^\mu W^-_\mu - \xi g_2 v \phi^- \phi^+ \right). \]  
(6.36)

The redefinitions of the Higgs field take place in a similar way in the ghost Lagrangian, which can be derived from equation (2.64), implementing the gauge transformations of equation (2.65). Including only terms relevant for the calculation, it is given by,

\[ \mathcal{L}_{gh} = -\frac{g_2^2}{2 c_{w}^2} \xi v \hat{h} \bar{u}_0 u_0 - \frac{g_2^2}{2} \xi v \hat{h} \bar{u}^\pm u^\pm \pm i e A^\mu \left( \bar{u}^\pm \partial^\mu u^\pm - (\partial^\mu \bar{u}^\pm) u^\pm \right) \]
\[ + e^2 A^\mu \bar{u}^\pm u^\pm + \ldots, \]  
(6.37)

from which the Feynman rules are derived in appendix C. Again, the terms shifted by the pure scalar indirect operators can be found,

\[ \mathcal{L}_{gh}^{\text{shift}} \rightarrow -\frac{g_2^2}{2 c_{w}^2} \xi v \hat{h} \left( 1 + \left( C_{H\Box} - \frac{1}{4} C_{HD} \right) v^2 \right) \bar{u}_0 u_0 \]
\[ - \frac{g_2^2}{2} \xi v \hat{h} \left( 1 + \left( C_{H\Box} - \frac{1}{4} C_{HD} \right) v^2 \right) \bar{u}^\pm u^\pm. \]  
(6.38)
6.4. Indirect Contributions from Dimension Six Operators

The redefinitions of equation (6.20) also take place in the kinetic Higgs term involving interactions with the gauge fields. This leads to the following contribution to the Higgs interactions with the $W$ fields, the only Higgs-gauge field interaction relevant for this calculation,

$$\frac{1}{2} g_2^2 v^4 W^+ W^- \to \frac{1}{2} g_2^2 v^4 \left(1 + \left(\frac{C_{H\Delta}}{C_{HD}}\right)^{1/2}\right) W^+ W^-.$$

(6.39)

The gauge field kinetic terms get contributions from the $v^2$ part of the operators $O_{HB}, O_{HW}$ and $O_{HWB}$ in the following way,

$$L_{SM}^{YM} + L_{v^2}^{(6)} = -\frac{1}{2} W_{\mu\nu}^{\mu} W_{\nu}^{\mu -} - \frac{1}{4} W_{\mu\nu}^{3} W_{\nu}^{\mu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{2} g_2^2 v^2 C_{HW} W_{\mu}^{I} W_{\nu}^{\mu}$$

$$+ \frac{1}{2} g_2^2 v^2 C_{HB} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} g_2 v^2 C_{HWB} W_{\mu}^{3} B^{\mu\nu} + \frac{1}{4} g_2^2 v^2 W^+ W^-$$

$$+ \frac{1}{8} v^2 (g_2 W_3^3 - g_1 B_\mu)^2.$$

(6.40)

To canonically normalize the gauge kinetic terms, the fields and couplings are shifted as,

$$W_\mu^\pm \to W_\mu^\pm (1 + C_{HW} g_2^2 v^2), \quad B_\mu \to B_\mu (1 + C_{HB} g_1^2 v^2),$$

$$g_2 \to g_2 (1 - C_{HW} g_2^2 v^2), \quad g_1 \to g_1 (1 - C_{HB} g_1^2 v^2),$$

(6.41)

such that the redefined Lagrangian becomes

$$L_{SM}^{YM} + L_{v^2}^{(6)} = -\frac{1}{2} W_{\mu\nu}^{\mu} W_{\nu}^{\mu -} - \frac{1}{4} W_{\mu\nu}^{3} W_{\nu}^{\mu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} g_2^2 v^2 g_1 g_2 C_{HWB} W_{\mu}^{3} B^{\mu\nu}$$

$$+ \frac{1}{4} g_2^2 v^2 W^+ W^- + \frac{1}{8} v^2 (g_2 W_3^3 - g_1 B_\mu)^2.$$

(6.42)

As can be seen, the operator $O_{HWB}$ affects the $W_{\mu\nu} - B_{\mu\nu}$ mixing, which is therefore redefined as follows [11],

$$\begin{pmatrix} W_3^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} 1 \\ -\frac{1}{2} g_1 g_2 v^2 C_{HWB} \end{pmatrix} \begin{pmatrix} c_w & s_w \\ -s_w & c_w \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

(6.43)

This subsequently requires the following angle redefinitions

$$s_w \to s_w \left(1 + \frac{1}{2} v^2 g_2 \left(c_w^2 - s_w^2\right) g_1 g_2 C_{HWB}\right),$$

$$c_w \to c_w \left(1 - \frac{1}{2} v^2 g_1 g_2 \left(c_w^2 - s_w^2\right) g_1 g_2 C_{HWB}\right).$$

(6.44)

The operator $O_{HWB}$ therefore accompanies any gauge field interaction, since it enters through the rotation of the gauge fields into their mass eigenstates. This modifies the relationship between the $SU(2)_L \times U(1)_Y$ gauge fields $W$ and $B$ and the mass eigenstate fields in the SMEFT.
The fact that the relation between the $W$ and $B$ fields and the physical propagating mass eigenstate fields changes order by order in the power counting of the theory, as do the mixing angles, has some interesting consequences. The Wilson coefficient $C_{HWB}$ causes mixing of the fields $A_\mu$ and $Z_\mu$ through the following terms,

$$
- \frac{c_w s_w}{\xi_B \xi_W} (\xi_B - \xi_W) - \frac{g_1 g_2 C_{HWB} v^2}{\xi_B \xi_W} \left( s_w^2 - c_w^2 \left( s_w^2 \xi_B + c_w^2 \xi_W \right) \right) \left( \partial^\mu A_\mu - \partial^\nu Z_\nu \right) \cdots (6.45)
$$

In the SM, the choice $\xi_B = \xi_W$ is usually made in t’Hooft gauge fixing, such that $A_\mu - Z_\mu$ mixing does not occur at tree level. The same choice in the SMEFT results in tree level $A_\mu - Z_\mu$ mixing. In the BFM, the mixing only takes place for the quantum fields. Therefore, it does not induce any effects in the loop calculations of the $h \to \gamma \gamma$ decay, when implementing the BFM. Without the implementation of the BFM or with quantum $A_\mu$ or $Z_\mu$ fields entering the calculations, the above mixing needs to be accounted for.

All of the redefinitions outlined in this section enter the effective Lagrangian of ICs, from which Feynman rules can be derived for the various interactions amongst SM fields.

### 6.4.2 The Effective Lagrangian of Indirect Contributions

The indirect operators only contribute to the $h \to \gamma \gamma$ decay through diagrams already in the SM, see figure 6.3. This follows from the derivation in the previous section, showing contributions only through redefining fields in interactions of the SM. These ICs arising through redefining SM interactions enter various parts of the effective Lagrangian, which for clarity can be split up into parts containing the gauge fixing, Yang-Mills, Higgs, ghost and Yukawa contributions, as follows,

$$
\mathcal{L}_{\text{eff}}^{\text{IC}} = \mathcal{L}_{\text{eff}}^{\text{GF}} + \mathcal{L}_{\text{eff}}^{\text{YM}} + \mathcal{L}_{\text{eff}}^{H} + \mathcal{L}_{\text{eff}}^{\text{gh}} + \mathcal{L}_{\text{eff}}^{Y} \quad (6.46)
$$

Using that all Higgs fields $h$ and all photon fields $A$ in the following are background classical fields, and all other fields are quantum fields, the various parts of the Lagrangian are found to be

$$
\mathcal{L}_{\text{eff}}^{\text{GF}} = C_{HWB} \left( \frac{1}{\xi} i e^3 v^2 (A_\mu W_\mu^\nu \partial_\nu W^\nu_\nu - A_\mu W_\mu^\nu \partial_\nu W_\nu^\nu) \right) + \frac{2}{\xi} e^4 v^2 W_\mu^+ W_\nu^\nu A_\nu A^\mu \\
+ \frac{1}{2} i e^3 g_2 (v + h) v^2 (\phi_+ A_\mu W_\mu^\mu - \phi_- A_\mu W_\mu^\mu) \right) + \frac{1}{2} \left( C_{H\Box} - \frac{1}{4} C_{HD} \right) \left( - g_2^2 v^3 \xi h \phi^- \phi^+ \\
+ g_2 v^2 \left( h \phi^- \partial^\mu W_\mu^+ + h \phi^+ \partial^\mu W_\mu^- \right) \right), \quad (6.47)
$$
\[ \mathcal{L}_{\text{eff}}^{\text{YM}} = C_{\text{HWB}} \left( i v^2 e^3 \left( A_\mu W_\nu^+ W_-^{\mu \nu} - A_\mu W_\nu^- W_+^{\mu \nu} \right) \\
+ i e^3 v^2 W_\mu^+ W_-^\mu A^{\mu \nu} \\
+ 2 e^4 v^2 \left( A_\mu A^\nu W_\nu^+ W_-^{\mu \nu} - A_\mu A_\nu W_+^{\mu \nu} W_-^{\nu \mu} \right) \\
+ C_{\text{HW}} \left( 2 i e g^2 v^2 \left( A_\mu W_\nu^- W_-^{\mu \nu} + A_\nu W_\mu^+ W_-^{\mu \nu} + W_-^\mu W_\nu^+ A^{\mu \nu} \right) \right) \right), \]

\[ \mathcal{L}_{\text{eff}}^{Y} = \frac{1}{\sqrt{2}} Y_\psi \bar{\psi} \psi \left( - C_{\psi H} v^2 + C_{\text{HD}} v^2 - \frac{1}{4} C_{\text{HD}} v^2 \right), \]

\[ \mathcal{L}_{\text{eff}}^{H} = C_{\text{HWB}} \left( \frac{1}{2} i e^3 g_2 v^4 (h + v) \left( \phi_+ A_\mu W_+^{\mu \nu} - \phi_+ A_\mu W_-^{\mu \nu} \right) \\
+ i e^3 v^2 \left( \phi_+ A_\mu \partial_\nu \phi_+ - \phi_+ A_\mu \partial_\nu \phi_- \right) \\
- 2 e^4 v^2 \phi_+ A_\mu A^\nu - \frac{1}{2} e^3 g_2 v^2 A_\mu \left( W_+^{\mu} \phi_+ + W_-^{\mu} \phi_+ \phi_0 \right) \right) \]

\[ \mathcal{L}_{\text{eff}}^{h} = C_{\text{HWB}} \left( e^4 v^2 \left( u^A + \frac{c_w}{s_w} u_0 \right) A_\mu \left( W_+^{\mu} \bar{u}^+ + W_-^{\mu} \bar{u}^- \right) \\
- \frac{1}{2} g_1^2 g_2^2 v^3 \xi u_0 u_0 h + \frac{1}{4} i e g_1 g_2^2 v^2 \xi u_0 h \left( \phi_+ u^- - \phi_- u^+ \right) \\
+ i e^3 v^2 \left( \bar{u}^+ A_\mu \partial_\mu u^+ - \bar{u}^- A_\mu \partial_\mu u^- \right) + i e^3 v^2 \partial_\mu A^\mu \left( \bar{u}^+ u^+ - \bar{u}^- u^- \right) \right) \]

\[ \mathcal{L}_{\text{eff}}^{h} = C_{\text{HWB}} \left( e^4 v^2 \left( u^A + \frac{c_w}{s_w} u_0 \right) A_\mu \left( W_+^{\mu} \bar{u}^+ + W_-^{\mu} \bar{u}^- \right) \right) \]

\[ \mathcal{L}_{\text{eff}}^{h} = C_{\text{HWB}} \left( \frac{g_1^2}{2 c_w^2} \xi v^3 h u_0 u_0 + \frac{g_1^2}{2} \xi v^3 \bar{h} \bar{u}^+ \right). \]  

6.5 Renormalizing the $h \to \gamma \gamma$ Decay in the Standard Model Effective Field Theory

The operators $O_{\text{HB}}$, $O_{\text{HW}}$ and $O_{\text{HWB}}$ contribute at tree-level to the $h \to \gamma \gamma$ decay. Mixing effects from the listed operators in section 6.2 entering the RGEs of these three operators lead to one-loop contributions. The divergences occurring from this mixing are canceled by the renormalization constant matrix $Z_{i, j}$, renormalizing the dimension six operators, as described in section 5.7,

\[ O_{i}^{(0)} = Z_{i, j} O_{j}^{(r)}. \]
6.5. Renormalizing the $h \rightarrow \gamma \gamma$ Decay in the Standard Model Effective Field Theory

\[ Z_{i,j} = \delta_{i,j} + \frac{Z_{i,j}}{16 \pi^2 \epsilon}. \] (6.50)

It contributes to the anomalous dimension of the Wilson coefficients, $C_i$, and has already been computed at one-loop in $d = 4 - 2 \epsilon$ dimensions in the $\overline{\text{MS}}$ scheme [6–11]. The calculation...

**Figure 6.3:** One loop diagrams contributing to the $h \rightarrow \gamma \gamma$ decay through indirect interactions from $\mathcal{L}_6^{H + H \phi + H D + H W B}$. Some of the diagrams appear also in figure 6.2, since they also have direct contributions. The naming is here the same as in figure 6.2. Arrows on propagators indicate charge flow.
of these anomalous dimensions, has taken place in the unbroken theory, which induces simplifications. At this point, SSB has not taken place, fields have not been rotated into their mass eigenstates and masses do not enter to complicate the calculations. Since the anomalous dimensions of gauge invariant operators do not depend on the gauge choice, the results are the same in the broken and the unbroken theory. The results from the unbroken theory can therefore be used in the following calculations, carried out in the broken theory.

On the basis where \( i = (O_{HB}, O_{HW}, O_{HWB}, O_W, O_{eB}/O_{eB}', O_{uB}/O_{uB}', O_{dB}/O_{dB}', O_{eW}/O_{eW}', O_{uW}/O_{uW}, O_{dW}/O_{dW}') \) and \( j = (O_{HB}, O_{HW}, O_{HWB}) \), the counterterm matrix, \( Z_{i,j} \), is given by

\[
Z_{i,j} = \begin{pmatrix}
\frac{g_i^2}{4} - \frac{9 g_i^2}{4} + 6 \lambda + Y \\
0 - \frac{3 g_i^2}{4} - \frac{5 g_i^2}{4} + 6 \lambda + Y \\
\frac{3 g_i^2}{2} & 0 \\
0 & -\frac{9 g_i^2}{4} + \frac{9 g_i^2}{4} + 2 \lambda + Y \\
-(y_i + y_e) Y_e & 0 & -\frac{9 g_i^2}{4} + \frac{9 g_i^2}{4} + 2 \lambda + Y \\
-(y_i + y_e) Y_e^{\dagger} & 0 & -\lambda Y_e \\
-N_c (y_q + y_u) Y_u & 0 & \frac{1}{2} N_c Y_u \\
-N_c (y_q + y_u) Y_u^{\dagger} & 0 & \frac{1}{2} N_c Y_u^{\dagger} \\
-N_c (y_q + y_d) Y_d & 0 & -\frac{1}{2} N_c Y_d \\
-N_c (y_q + y_d) Y_d^{\dagger} & 0 & -\frac{1}{2} N_c Y_d^{\dagger} \\
0 & -\frac{1}{2} Y_e & -(y_i + y_e) Y_e \\
0 & -\frac{1}{2} Y_e^{\dagger} & -(y_i + y_e) Y_e^{\dagger} \\
0 & -\frac{1}{2} N_c Y_u & N_c (y_q + y_u) Y_u \\
0 & -\frac{1}{2} N_c Y_u^{\dagger} & N_c (y_q + y_u) Y_u^{\dagger} \\
0 & -\frac{1}{2} N_c Y_d & -N_c (y_q + y_d) Y_d \\
0 & -\frac{1}{2} N_c Y_d^{\dagger} & -N_c (y_q + y_d) Y_d^{\dagger}
\end{pmatrix}
\]  

(6.51)

with

\[
Y = \text{Tr} \left[ N_c Y_u^{\dagger} Y_u + N_c Y_d^{\dagger} Y_d + Y_e Y_e^{\dagger} \right],
\]  

(6.52)

The resulting renormalized interactions, which contribute at tree-level to the \( h \to \gamma \gamma \) decay as seen in diagram (u), figure 6.4 are

\[
L_6^{(0)} = Z_{SM} Z_{i,j} C_i O_j^{(r)},
\]

\[
= Z_{SM} \left( N_{HB} O_{HB}^{(r)} + N_{HW} O_{HW}^{(r)} + N_{HWB} O_{HWB}^{(r)} \right),
\]  

(6.53)
6.6. Standard Model Renormalization of the $h \rightarrow \gamma\gamma$ Decay at Tree Level

where the renormalized couplings enter as

$$N_j = \frac{1}{16 \pi^2} C_i Z_{i,j}, \quad (6.54)$$

leading to the $h \rightarrow \gamma\gamma$ term of the effective Lagrangian at tree level,

$$L_{\text{tree}} = h v Z_{\text{SM}} (N_{HB} + N_{HW} - N_{HWB}) e^2 A_{\mu} A^{\mu}. \quad (6.55)$$

The renormalized couplings can be written out as

\begin{align*}
N_{HB} &= \frac{1}{16 \pi^2} \left\{ \left( 16 \pi^2 \epsilon + \frac{g_1^2}{4} - \frac{9 g_2^2}{4} + 6 \lambda + Y \right) C_{HB}(\Lambda) + \frac{3 g_2^2}{2} C_{HWB}(\Lambda) \right\}, \\
N_{HW} &= \frac{1}{16 \pi^2} \left\{ \left( 16 \pi^2 \epsilon - \frac{3 g_1^2}{4} - \frac{5 g_2^2}{4} + 6 \lambda + Y \right) C_{HW}(\Lambda) + \frac{g_1^2}{2} C_{HWB}(\Lambda) \right\}, \\
N_{HWB} &= \frac{1}{16 \pi^2} \left\{ \left( 32 \pi^2 \epsilon - \frac{g_1^2}{2} + \frac{9 g_2^2}{2} + 4 \lambda + 2 Y \right) C_{HWB}(\Lambda) + 2 g_1^2 C_{HB}(\Lambda), \right. \\
&\quad \left. + 2 g_2^2 C_{HW}(\Lambda) + 3 g_2^2 C_{W}(\Lambda) - [Y_c]_{sr} C_{cW}(\Lambda) + N_c[Y_u]_{sr} C_{uW}(\Lambda) - N_c[Y_d]_{sr} C_{dB}(\Lambda) \right\}.
\end{align*}

(6.56)

Evaluating the operator Wilson coefficients at the scale $\mu = \Lambda$, allows for a direct interpretation of any measured deviations in terms of the underlying beyond the standard model (BSM) physics sector.

6.6 Standard Model Renormalization of the $h \rightarrow \gamma\gamma$ Decay at Tree Level

The renormalization of SM fields and couplings is illustrated by the inclusion of the renormalization constant $Z_{\text{SM}}$ in equation (6.55). The couplings, which are absorbed into the $N_i$ in
\[ L_{\text{eff}} \] also depend on the SM counterterms. Additionally, the SM fields and couplings appearing in the effective Lagrangian through expanding the dimension six operators listed in equation (6.14), should all be multiplied by their respective renormalization constants. However, the latter two corrections are two loop order and therefore neglected in this one-loop calculation. Furthermore, using the relations given in equation (5.17) simplifies the SM renormalization, \( Z_{\text{SM}} \).

More explicitly, the SM renormalization in the specific decay, \( h \to \gamma \gamma \), is expected to include the renormalization of the Higgs field, the VEV, the photon field and the electromagnetic coupling. The Ward identities withheld in the BFM establishes a relation between the renormalization of the photon field and the electromagnetic coupling, namely \( Z_e Z_A = 1 \), canceling off their renormalizations in this decay. The renormalization therefore becomes

\[ Z_{\text{SM}} = \sqrt{Z_h (\sqrt{Z_v + \frac{\delta v}{v}})} \quad (6.57) \]

The tree level contribution to the \( h \to \gamma \gamma \) decay therefore becomes

\[ L_{\text{tree}} = h v \sqrt{Z_h (\sqrt{Z_v + \frac{\delta v}{v}})} (N_{HB} + N_{HW} - N_{HWB}) e^2 A_{\mu}A^{\mu}. \quad (6.58) \]

The Higgs field renormalization constant \( Z_h \) is found from calculating the Higgs self energy as done in appendix F in the BFM. The amplitudes are calculated in DREG, and the divergent parts are separately given for each diagram in the appendix. Summing over these divergences, the total divergent part of the Higgs self-energy is the following,

\[ i \Sigma_{\text{div}}^h(p^2) = \frac{i}{16 \pi^2} \left( 15 \lambda m_h^2 + \frac{3}{4} (g_1^2 + g_2^2) (3 + \xi^2) m_Z^2 + \frac{3 g_1^2}{2} (3 + \xi^2) m_w^2 \right. \]
\[ \left. - \frac{m_Z^2}{v^2} (9 m_Z^2 - 2 p^2) - \frac{1}{4} (g_1^2 + 3 g_2^2) (3 + \xi) p^2 - \frac{3}{8} \xi (g_1^2 + 3 g_2^2) m_h^2 \right), \quad (6.59) \]

where \( m_Z^2 = \frac{1}{2} Y v^2 \). The renormalization constant \( Z_h \) of the Higgs wavefunction is in the \( \overline{\text{MS}} \), as described in section 5.3, constructed to cancel the divergences in the above amplitude proportional to \( p^2 \),

\[ -i \Sigma^{r,h}(p^2) = -i \Sigma^h(p^2) + i p^2 \delta Z_h \quad (6.60) \]

Therefore, in the BFM,

\[ Z_h = 1 + \frac{3 + \xi}{64 \pi^2 \epsilon} (g_1^2 + 3 g_2^2) - \frac{Y}{16 \pi^2 \epsilon}. \quad (6.61) \]

The VEV renormalization \( (\sqrt{Z_v + \frac{\delta v}{v}}) \) can be calculated using the Higgs decay to two gluons, \( h \to g g \), which contains the same VEV structure. This decay, which only receives tree-level contributions from the dimension six operator,
6.6. Standard Model Renormalization of the $h \rightarrow \gamma \gamma$ Decay at Tree Level

$$\mathcal{O}^{(0)}_{HG} = H^\dagger H G^{\mu \nu} G^\mu G^\nu,$$  \hspace{1cm} (6.62)

which does not mix into other operators, allows for a simple extraction of the VEV renormalization. The S matrix element, including the insertion of this operator is renormalized as

$$\langle h | \mathcal{O}^0_{HG} | gg \rangle_{\text{loop}} = \frac{1}{Z_{GG} \sqrt{Z_h} (\sqrt{Z_v} + \frac{\delta v}{v})_{\text{div}}} \left[ iv + iv \langle h | \mathcal{O}^0_{HG} | gg \rangle \right].$$ \hspace{1cm} (6.63)

In the unbroken phase, the anomalous dimension of this operator has been found to be

$$\gamma_{HG} = \frac{Z_{HG}}{Z_{HG}} \frac{\partial Z_{HG}}{\partial \mu} = \frac{1}{16 \pi^2} \left( 12 \lambda + 2 Y - \frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 \right),$$ \hspace{1cm} (6.64)

so that

$$Z_{HG} = 1 + \frac{1}{16 \pi^2} \frac{1}{2} \gamma_{HG} = 1 + \frac{1}{16 \pi^2} (6 \lambda + Y - \frac{3}{4} g_1^2 - \frac{9}{4} g_2^2)$$ \hspace{1cm} (6.65)

The amplitude of the diagrams in figure 6.5 are calculated to be

$$\langle h | \mathcal{O}^0_{HG} | gg \rangle = \frac{1}{16 \pi^2 \epsilon} (6 \lambda + \frac{1}{4} \xi (g_1^2 + 3 g_2^2)) A_{\alpha \beta}^{h gg},$$ \hspace{1cm} (6.66)

where $A_{\alpha \beta}^{h gg} = -4 (p_2 \cdot p_3 g^{\alpha \beta} - p_2^\beta p_3^\alpha)$. Isolating $(\sqrt{Z_v} + \frac{\delta v}{v})_{\text{div}},$

$$(\sqrt{Z_v} + \frac{\delta v}{v})_{\text{div}} = 1 - \frac{1}{2} \delta Z_{HH} - \delta Z_{GG} + \langle h | \mathcal{O}^0_{HG} | gg \rangle = 1 + \frac{3 + 3 \xi}{128 \pi^2 \epsilon} (g_1^2 + 3 g_2^2) - \frac{Y}{32 \pi^2 \epsilon}.$$ \hspace{1cm} (6.67)

The renormalizations of the VEV and the Higgs field can therefore be seen to satisfy the following relation,

$$\left( \sqrt{Z_v} + \frac{\delta v}{v} \right)_{\text{div}} = \sqrt{Z_h}.$$ \hspace{1cm} (6.68)
6.7. The Finite Renormalization Contributions to the $h \rightarrow \gamma \gamma$ Decay

This is also a consequence of using the BFM, as described in [126]. In the BFM, the VEV is treated as a background field. Under SSB, the Higgs field gets shifted by the VEV as described in section 2.2, $h \rightarrow h + v$. The VEV therefore by definition has the same gauge transformation properties as the Higgs field, since the Lagrangian is still invariant under gauge transformations after this shift. It therefore follows from Ward identities that the Higgs and the VEV are renormalized as a combined unity. However, quantization requires gauge fixing, which explicitly breaks this gauge invariance. In the BFM, however, this breaking only occurs for the quantum fields. The background fields still satisfy gauge invariance, the Ward identity still holds and $(h + v)$ transforms as a whole.

Inserting all the tree-level renormalization contributions accounting both for the SM and the SMEFT renormalization, the divergences are found to be

$$i A_{\text{tree}}^{\text{div}} = i e^2 v A_{\alpha \beta}^{h \gamma \gamma} \left( 1 + \frac{1}{8 \pi^2} \epsilon \left( \frac{3 + \epsilon}{8} (g_1^2 + \frac{3}{2} g_2^2) - \frac{1}{2} Y \right) \right)$$

$$\times \frac{1}{16 \pi^2 \epsilon} \left[ \left( 16 \pi^2 \epsilon - \frac{3 g_1^2}{4} - \frac{9 g_2^2}{4} + 6 \lambda + Y \right) C_{HB}(\Lambda), \right.$$

$$+ \left( 16 \pi^2 \epsilon - \frac{g_1^2}{4} - \frac{9 g_2^2}{4} + 6 \lambda + Y \right) C_{HW}(\Lambda) - 9 g_2^4 C_W(\Lambda)$$

$$- \left( 16 \pi^2 \epsilon - \frac{g_1^2}{4} + \frac{9 g_2^2}{4} + 2 \lambda + Y + \frac{3 g_2^2}{2} + \frac{g_1^2}{2} \right) C_{HWB}(\Lambda)$$

$$+ \left( \frac{1}{2} - (y_l + y_e) \right) [Y_{e_{\text{sr}}} r_{s} \left( C_{eB}(\Lambda) - C_{eW}(\Lambda) \right)$$

$$- \left( \frac{1}{2} + (y_q + y_u) \right) N_c[Y_u_{\text{sr}} r_{s} \left( C_{uB}(\Lambda) + C_{uW}(\Lambda) \right)$$

$$+ \left( \frac{1}{2} - (y_q + y_d) \right) N_c[Y_d_{\text{sr}} r_{s} \left( C_{dB}(\Lambda) - C_{dW}(\Lambda) \right) + H.c. \right], \quad (6.69)$$

where gauge invariant interactions require

$$\left( y_l + y_e \right) = 2 Q_t + \frac{1}{2} = -\frac{3}{2}, \quad \left( y_q + y_u \right) = 2 Q_u - \frac{1}{2} = \frac{5}{6}, \quad \left( y_q + y_d \right) = 2 Q_d + \frac{1}{2} = -\frac{1}{6}. \quad (6.70)$$

The above divergences appearing from the RG mixing and the SM renormalization are expected to cancel the divergences of the diagrams. This will be shown to happen in the next sections through explicit calculation.

### 6.7 The Finite Renormalization Contributions to the $h \rightarrow \gamma \gamma$ Decay

The finite renormalization contributions to the $h \rightarrow \gamma \gamma$ decay arise from the renormalization condition fixing the minimum of the Higgs potential, causing a contribution to the VEV in the form $\delta v$, the renormalization conditions fixing the external two point functions for the Higgs
6.7. The Finite Renormalization Contributions to the $h \to \gamma \gamma$ Decay

($\delta R_h$) and the photon ($\delta R_A$) fields, as well as the definition of the electric coupling $e$, which fixes $\delta R_e$ [114, 127]. These contributions enter the $S$ matrix element as

$$\langle h(p_h) | S | \gamma(p_e, \alpha), \gamma(p_e, \beta) \rangle_{BSM} = (1 + \frac{\delta R_h}{2})(1 + \delta R_A)(1 + \delta R_e)^2 \sum_{x=a..u} A_x, \quad (6.71)$$

where $\delta v$ has been absorbed into $A_x$, the amplitude of the tree-level interaction, represented by diagram (u) in figure 6.4.

The Higgs self-energy is calculated in appendix F. The diagrams contributing to the self-energy are seen in figure 6.6. The $R$ factor, $R_h = 1 + \delta R_h$ can be found using the renormalization condition, (5.30) [62, 114]. In Feynman gauge, using

$$\mathcal{I}[m^2] \equiv \int_0^1 dx \log \left( \frac{m^2 - m_h^2 x (1-x)}{m_h^2} \right), \quad \mathcal{I}_{xx}[m^2] \equiv \int_0^1 dx \frac{m^2}{m^2 - m_h^2 x (1-x)}, \quad (6.72)$$

it is given by

$$\begin{align*}
\delta R_h &= - \left( g_1^2 + g_2^2 \right) \mathcal{I}[m_\psi^2] + \frac{1}{2} \mathcal{I}_{xx}[m_\psi^2] \left( \left( g_1^2 + g_2^2 \right) \left( 1 - \frac{3 m_\psi^2}{m_h^2} \right) - 2 \lambda \right) \\
&\quad + \frac{1}{2} \left( g_1^2 + g_2^2 \right) \left( \frac{3 m_\psi^2}{m_h^2} - 1 \right) + \left( g_1^2 - 3 g_2^2 + Y_\psi^2 N_c \right) \log \left( \frac{m_h^2}{\mu^2} \right) + Y_\psi^2 N_c \\
&\quad - 2 g_2^2 \mathcal{I}[m_w^2] + \mathcal{I}_{xx}[m_w^2] \left( g_2^2 \left( 1 - \frac{3 m_w^2}{m_h^2} \right) - 2 \lambda \right) + g_2^2 \left( \frac{3 m_w^2}{m_h^2} - 1 \right) \\
&\quad + Y_\psi^2 N_c \mathcal{I} \left( \frac{2 m_\psi^2}{m_h^2} + 1 \right) - 2 \sqrt{3} \pi \lambda + 12 \lambda - 2 \frac{m_\psi^2}{m_h^2} Y_\psi^2 N_c \log \left( \frac{m_\psi^2}{m_h^2} \right), \quad (6.73)
\end{align*}$$

where $m_\psi^2 = \frac{1}{2} Y_\psi^2 v^2$. The tree level VEV of the Higgs is defined through minimizing the potential

$$\mathcal{L}_V = - \lambda \left( H^\dagger H - \frac{v^2}{2} \right)^2. \quad (6.74)$$

Radiative corrections also affect the Higgs potential, so the minimum gets shifted. Counterterms are therefore needed to correct for this shift. They are determined through the requirement that the renormalized VEV is the actual minimum of the effective Higgs potential [114]. The one loop correction ($\delta v$) to the VEV is as a consequence fixed by the condition that the one point function of the Higgs field vanishes to one loop order, including $\delta v$ in the definition of $H$ in equation (2.3). Including one loop corrections in the BFM, shown in figure 6.7, the finite terms linear in the Higgs field are modified to

$$T = m_h^2 \ h \ v \ i \ \frac{1}{16 \pi^2} \left[ -16 \pi^2 \ \frac{\delta v}{v} + 3 \lambda \left( 1 + \log \left( \frac{\mu^2}{m_h^2} \right) \right) + \frac{1}{4} g_2^2 \xi \left( 1 + \log \left( \frac{\mu^2}{\xi \ m_\psi^2} \right) \right) \right] \quad (6.75)$$

$$\begin{align*}
&\quad + \frac{1}{8} \left( g_1^2 + g_2^2 \right) \log \left( \frac{\mu^2}{\xi \ m_\psi^2} \right) + 2 Y_\psi^2 N_c \frac{m_\psi^2}{m_h^2} \left( 1 + \log \left( \frac{\mu^2}{m_\psi^2} \right) \right) \\
&\quad + \frac{g_2^2}{2} \frac{m_w^2}{m_h^2} \left( 1 + 3 \log \left( \frac{\mu^2}{m_w^2} \right) \right) + \frac{1}{4} \left( g_1^2 + g_2^2 \right) \frac{m_\psi^2}{m_h^2} \left( 1 + 3 \log \left( \frac{\mu^2}{m_\psi^2} \right) \right) \left( 1 + 3 \log \left( \frac{\mu^2}{m_\psi^2} \right) \right),
\end{align*}$$
6.7. The Finite Renormalization Contributions to the $h \to \gamma \gamma$ Decay

Figure 6.6: 1PI diagrams contributing to the Higgs self energy. $h$ is a Higgs, $\phi_0$ a neutral Goldstone, $\phi_\pm$ are charged Goldstones, $u_0$ are the neutral ghosts, $u_\pm$ the charged ghosts, $\psi$ are fermions and $W, Z$ are gauge bosons.
leading to the following renormalization of the VEV at finite level,

\[
\frac{\delta v}{v} = \frac{1}{16 \pi^2} \left[ 3 \lambda \left( 1 + \log \left( \frac{\mu^2}{m_h^2} \right) \right) + \frac{1}{4} g_2^2 \xi \left( 1 + \log \left( \frac{\mu^2}{\xi m_\sigma^2} \right) \right) \right. \\
+ \frac{1}{8} \left( g_1^2 + g_2^2 \right) \xi \left( 1 + \log \left( \frac{\mu^2}{\xi m_\phi^2} \right) \right) - 2 Y_\psi^2 N_e \frac{m_\psi^2}{m_h^2} \left( 1 + \log \left( \frac{\mu^2}{m_\psi^2} \right) \right) \\
+ \frac{g_2^2}{2} \frac{m_\psi^2}{m_h^2} \left( 1 + 3 \log \left( \frac{\mu^2}{m_\psi^2} \right) \right) + \frac{1}{4} \left( g_1^2 + g_2^2 \right) \frac{m_\sigma^2}{m_h^2} \left( 1 + 3 \log \left( \frac{\mu^2}{m_\sigma^2} \right) \right). 
\]  

(6.76)

Setting $T = 0$ defines the VEV at one loop and fixes the finite terms of $\delta v$. The remaining contributions to $\Gamma(h \rightarrow \gamma \gamma)$ from the sum of all other tadpole diagrams and insertions of $\delta v$ vanish due to this chosen renormalization condition.

The R factors of the photon and the electric coupling cancel in the BFM, due to the Ward identities of the theory, as established in section 5.1, setting

\[
\delta R_e = -\delta R_A. 
\]  

(6.77)

In the calculation in question, only the Higgs and the VEV R-factors as seen in equations (5.30) and (6.76), contribute.

### 6.8 Calculations

The complete effective Lagrangian combines the ICs and DCs,

\[
\mathcal{L}_{\text{eff}}^{h \gamma \gamma} = \mathcal{L}_{\text{eff}}^{\text{Exp}} + \mathcal{L}_{\text{eff}}^{\text{GF}} + \mathcal{L}_{\text{eff}}^{\text{YM}} + \mathcal{L}_{\text{eff}}^{H} + \mathcal{L}_{\text{eff}}^{\text{gh}} + \mathcal{L}_{\text{eff}}^{Y}. 
\]  

(6.78)

On account of this effective Lagrangian, the Feynman rules for the effective vertices can be derived and are given in appendix H. The SM Feynman rules in the BFM are correspondingly given in appendix C. These rules allow for the calculation of the diagrams, which originate from operators contributing directly as shown in figure 6.2 and the ones from operators contributing indirectly as in figure 6.3. The amplitudes for these diagrams are written up in detail in appendices I and J, respectively. The integrals can thereafter be calculated using a combination of FORM and Mathematica, as described in appendix B.

The combination of Wilson coefficients given by

\[
C_{\gamma \gamma} = C_{HB} + C_{HW} - C_{HWB} 
\]  

(6.79)
6.8. Calculations

corresponds to the effective tree-level Wilson coefficient for \( h \to \gamma \gamma \). Using the following definitions,

\[
A_{\alpha \beta}^{h\gamma\gamma} = \langle h \mid h A_{\mu\nu} A_{\mu\nu} \mid \gamma(p_{\alpha}), \gamma(p_{\beta}) \rangle = -4 \left( p_{\alpha} \cdot p_{\beta} g^{\alpha \beta} - p_{\alpha}^{\beta} p_{\beta}^{\alpha} \right),
\]

\[
C_e = \frac{i c^2 v}{16 \pi^2 \epsilon},
\]

(6.80)

the divergent part of the diagrams with insertion of effective vertices from direct operators are calculated to be,

\[
iA_a = C_e C_{\gamma\gamma} (-4 \lambda - \frac{1}{4} (g_1^2 + g_2^2) \xi) A_{\alpha \beta}^{h\gamma\gamma},
\]

\[
iA_b = C_e (C_{\gamma\gamma} + 2 C_{HWB}) (-2 \lambda - \frac{1}{2} g_2^2 \xi) A_{\alpha \beta}^{h\gamma\gamma},
\]

\[
iA_c = C_e g_2^2 C_{HW} \left[ \left( 1 - \frac{1}{\xi} \right) \frac{4}{3} p_{\alpha}^{\beta} p_{\beta}^{\alpha} - 12 m_w^2 (3 + \frac{1}{\xi}) g^{\alpha \beta} + \left( 13 + \frac{5}{\xi} \right) \frac{4 p_{\alpha} \cdot p_{\beta}}{3} g^{\alpha \beta} \right],
\]

\[
iA_d = \frac{3}{2} C_e C_W g_2 \left( 1 + \xi \right) A_{\alpha \beta}^{h\gamma\gamma},
\]

\[
iA_e = 6 C_e g_2^2 C_{HW} m_w^2 \left[ 3 + \xi^2 \right] g^{\alpha \beta},
\]

\[
iA_f = C_e g_2^2 C_{HWB} \frac{3 + \xi}{2} A_{\alpha \beta}^{h\gamma\gamma},
\]

\[
iA_g = 4 g_2^2 C_e \left[ -3 m_w^2 (\xi^2 + 3) C_{HW} \frac{g^{\alpha \beta} - (\xi + 3)}{4} (2 C_{HW} - C_{HWB}) A_{\alpha \beta}^{h\gamma\gamma} \right],
\]

\[
iA_h = C_e g_2^2 C_{HWB} (-1) A_{\alpha \beta}^{h\gamma\gamma},
\]

\[
iA_i = C_e g_2^2 C_{HWB} (-1) \frac{1 + \xi}{2} A_{\alpha \beta}^{h\gamma\gamma},
\]

\[
iA_j = \frac{4}{3} C_e g_2^2 C_{HW} \left[ \left( 6 \xi + 17 \frac{1}{\xi} \right) p_{\alpha}^{\beta} p_{\beta}^{\alpha} + \frac{9}{2} m_w^2 (\xi^2 + 9 + \frac{2}{\xi}) g^{\alpha \beta} - \left( 6 \xi + 31 \frac{5}{\xi} \right) \frac{g^{\alpha \beta}}{3} p_{\alpha} \cdot p_{\beta} \right],
\]

\[
iA_k = \frac{3}{2} C_e C_W g_2 \left( 5 - \xi \right) A_{\alpha \beta}^{h\gamma\gamma},
\]

\[
iA_l = C_e \left( C_{\psi B} - C_{\psi W} \right) [Y_\psi]_{rs} N_c Q_\psi A_{\alpha \beta}^{h\gamma\gamma},
\]

\[
iA_m = C_e \left( C_{\psi B} - C_{\psi W} \right) [Y_\psi]_{sr} N_c Q_\psi A_{\alpha \beta}^{h\gamma\gamma}.
\]

(6.81)

The indirect contributions from the operators \( O_{Hd}, O_{HD} \) and \( O_{\psi H} \) give the SM result times an overall shift proportional to these indirect operators. Therefore, the gauge dependence and divergences act as in the SM and cancel likewise. Not including these operators, the divergences of the diagrams with indirect operators entering the effective vertices are given by,
6.9. Complete Finite Contribution to the $h \to \gamma \gamma$ Decay

The finite terms in the calculation come about from expanding the results of the diagrams in figures 6.2 and 6.3 to $O(\epsilon^0)$, as well as from finite one loop terms defined via renormalization conditions in the $\overline{\text{MS}}$ scheme for the Higgs and the VEV entering at tree level in the calculation, as described in section 6.7.

In the following, the complete finite contribution from dimension six operators to the $h \to \gamma \gamma$ decay is listed. The result for general $\xi$ is given in appendix K and the $R$ factors for the Higgs and the VEV are listed in equations (6.73) and (6.77). Setting $\xi = 1$ and using equation (6.72) as well as the following replacement,

$$I_y[m^2] \equiv \int_0^{1-x} dy \int_0^1 dx \frac{m^2}{m^2 - m_h^2 x (1 - x - y)},$$

the complete result is given by

$$iA_{t} = \frac{C_c}{2} \left[ C_{HWB} (6 g_2^2 + 4 \lambda) - 12 \lambda (C_{HB} + C_{HW}) \right] A_{a\beta}^{h\gamma\gamma}$$

$$+ C_c \left( 9 g_2^4 C_W - 2 (C_{eB} - C_{eW}) [Y_e]_{sr} + \frac{4}{3} N_c (C_{\mu B} + C_{\mu W}) [Y_{\mu}]_{sr} - \frac{2}{3} N_c (C_{d B} - C_{d W}) [Y_d]_{sr} \right) A_{a\beta}^{h\gamma\gamma},$$

(6.83)

which can subsequently be found to cancel against the divergences of the RGE contributions, also seen in equation (6.69), as expected. The cancellation of the $\xi$ dependence as well as of the divergences provide checks of the calculation. On account of this, the contributions can therefore safely be expanded to next order in $\epsilon$, to obtain the finite contributions.

6.9 Complete Finite Contribution to the $h \to \gamma \gamma$ Decay

$$iA_b = C_c C_{HWB} e^2 v^2 (8 \lambda + 2 g_2^2 \xi) g^{\alpha\beta},$$

$$iA_d = \frac{1}{2} C_c C_{HWB} e^2 v^2 g_2^2 \left( 9 + \frac{3}{\xi} + 3 \xi^2 \right) g^{\alpha\beta},$$

$$iA_k = -\frac{1}{2} C_c C_{HWB} e^2 v^2 g_2^2 \left( 9 + \frac{3}{\xi} + 3 \xi^2 \right) g^{\alpha\beta},$$

$$iA_o = -C_c C_{HWB} e^2 v^2 (8 \lambda + 2 g_2^2 \xi) g^{\alpha\beta},$$

$$iA_i = 4 C_c C_{HWB} e^2 g_2^2 v^2 g^{\alpha\beta},$$

$$iA_t = -4 C_c C_{HWB} e^2 g_2^2 v^2 \xi g^{\alpha\beta}.$$  

(6.82)

The divergences from ICs cancel amongst themselves, as expected. In this calculation, they all come in shifting the overall SM amplitudes, which themselves add up to be finite.
6.9. Complete Finite Contribution to the $h \rightarrow \gamma \gamma$ Decay

\[ \frac{i A_{NP}^{\text{total}}}{i v e^2 A_{\alpha \beta}^{h \gamma \gamma}} = C_{\gamma \gamma} \left( 1 + \frac{\delta R_h}{2} + \frac{\delta v}{v} \right) \]
\[ + \frac{1}{16 \pi^2} \left( C_{\gamma \gamma} \left( \frac{g_1^2}{4} + \frac{3 g_2^2}{4} + 6 \lambda \right) + C_{\text{HWB}} \left( -3 g_2^2 + 4 \lambda \right) \right) \]
\[ - 9 C_W g_2^4 + 2 \left( C_{\psi W} - C_{\psi B} \right) \left[ Y_{\psi} |_{sr} Q_\psi \right] \log \left( \frac{m_h^2}{A^2} \right) \]
\[ + \frac{C_{\gamma \gamma}}{16 \pi^2} \left( 2 e^2 \left( 1 + 6 \frac{m_w^2}{m_h^2} \right) - 2 g_2^2 \left( 1 + \log \frac{m_w^2}{m_h^2} \right) \right) \left( 4 \lambda - g_2^2 \right) I[m_w^2] \]
\[ + 4 \left( 3 e^2 - g_2^2 - 6 e^2 \frac{m_w^2}{m_h^2} \right) I_y[m_w^2] \]
\[ - \frac{g_2^2 C_{\text{HW}}}{4 \pi^2} \left( 3 \frac{m_w^2}{m_h^2} + 4 - \frac{m_w^2}{m_h^2} - 6 \frac{m_w^2}{m_h^2} \right) \]
\[ - \frac{3 g_4^2 C_{W}}{16 \pi^2} \left( 4 + 3 I[m_w^2] + 2 I_y[m_w^2] - 2 I_x[m_w^2] \left( 1 - \frac{m_w^2}{4 m_h^2} \right) \right) \]
\[ + \frac{2 [Y_{\psi}]_{sr} Q_\psi}{16 \pi^2} \left( C_{\psi B} - C_{\psi W} \right) \left( -2 - 4 I_y[m_\psi^2] - 2 I[m_\psi^2] + \log \left( \frac{m_\psi^2}{m_h^2} \right) \right) \]
\[ - \frac{1}{16 \pi^2} \left( C_{H \square} - \frac{1}{4} C_{\text{HD}} \right) \left( 1 + 6 \frac{m_w^2}{m_h^2} + 6 I_y[m_w^2] \left( 1 - 2 \frac{m_w^2}{m_h^2} \right) \right) \]
\[ + \frac{Q_\psi^2}{16 \pi^2} \left( - C_{\psi H} + C_{H \square} - \frac{1}{4} C_{\text{HD}} \right) \left( 4 \frac{m_\psi^2}{m_h^2} + 2 I_y[m_\psi^2] \left( 1 - 4 \frac{m_\psi^2}{m_h^2} \right) \right). \quad (6.85) \]

### 6.9.1 Verification of the Result

Besides having checked the finiteness of the result, finding complete cancellation of divergences as shown in sections 6.8, gauge invariance has also been verified numerically for the finite result in $R_\xi$ gauge listed in appendix K. This has been done using the CUBA library [128].

Another check of the result has been carried out performing the calculations explicitly in the unitary gauge. Structures are relocated drastically as a result of this. In the $R_\xi$ gauge, terms proportional to $\lambda$ appear only in diagrams (a) and (b) of figure 6.2. In the unitary gauge, diagram (b) of figure 6.2 does not exist. Therefore, the $\lambda$ terms are expected to be produced elsewhere. The finite contributions in the $R_\xi$ gauge proportional to $C_{\text{HWB}}$ are given by
\[
\frac{A^{Rc}_{\alpha \beta}}{e^2 v A^{h, \gamma \gamma}_{\alpha \beta}} = 4 C_{HWB}(\Lambda) \left( \log \left( \frac{\Lambda^2}{\xi m_w^2} \right) + 2 \right) + \frac{2}{\sqrt{-1 + \frac{4 m^2_{\xi}}{m_t}} \arctan \left( \frac{1}{\sqrt{-1 + \frac{4 m^2_{\xi}}{m_t}}} \right)}.
\] (6.86)

Due to the structure of the gauge field propagator in the unitary gauge, the \( \lambda \) dependence of diagram (b) gets shifted to diagram (k). This happens due to the propagator in the unitary gauge containing a term \( \sim \frac{d \cdot q}{m^2_{\xi}} \), which produces an extra \( \lambda \) term, cancelling a \( m^2_{w} \) factor and producing a \( p^2 = m^2_{R} \).

From diagrams (a) and (k), in the unitary gauge, the part proportional to \( C_{HWB} \) given above is recovered, leading to the result,

\[
\frac{A^{uni}_{\alpha \beta}}{e^2 v A^{h, \gamma \gamma}_{\alpha \beta}} = 4 C_{HWB}(\Lambda) \left( \log \left( \frac{\Lambda^2}{m^2_{w}} \right) + 2 \right) + \frac{2}{\sqrt{-1 + \frac{4 m^2_{\xi}}{m_t}} \arctan \left( \frac{1}{\sqrt{-1 + \frac{4 m^2_{\xi}}{m_t}}} \right)}.
\] (6.87)

which corresponds to equation (6.86), as can be seen immediately, setting \( \xi = 1 \).

### 6.10 Numerical Results and Discussion

The ratios of the decay rates for the isolated decay channel \( h \rightarrow \gamma \gamma \) including new physics compared to the SM decay rate is given by \( \kappa_{\gamma} \) as addressed in section 3.1,

\[
\kappa_{\gamma}^2 \equiv \left( \frac{\Gamma(h \rightarrow \gamma \gamma)}{\Gamma_{SM}(h \rightarrow \gamma \gamma)} \right) \simeq 1 + \left| \frac{A_{NP_{total}}}{A_{SM}} \right|^2,
\] (6.88)

where \( A_{SM} \) was given in section 6.1 and can be rewritten here using the notation compatible with the rest of the thesis,

\[
i A_{SM} = \frac{i e^2}{16 \pi^2 v} \left( \left( -1 - 6 \frac{m^2_{\psi}}{m^2_{h}} - 6 \mathcal{I}_y [m^2_{\psi}] \left( 1 - 2 \frac{m^2_{w}}{m^2_{h}} \right) \right) \right)
+ 2 Q^2_{\psi} \left( \frac{m^2_{\psi}}{m^2_{h}} + \mathcal{I}_y [m^2_{\psi}] \left( 1 - 4 \frac{m^2_{w}}{m^2_{h}} \right) \right) A^{h, \gamma \gamma}_{\alpha \beta}.
\]

This \( \kappa \) parameter represents the possible deviation of the decay, including new physics from effective operators in the SMEFT, from the decay expected in the SM.
The one-loop calculations carried out in the previous section can be included in the amplitude $A_{\text{NP}}^{\text{total}}$, which is therefore taken to contain both tree-level and one-loop level contributions. This reduces some theoretical errors, as addressed in section 3.6. The actual reduction can be seen studying the impact of these one-loop contributions on the interpretation of experimental constraints for the process $\Gamma(h \to \gamma \gamma)$.

The complete result of the SMEFT amplitude for the $h \to \gamma \gamma$ decay, as given in equation (6.85) contains contributions from log enhanced RG terms as already calculated in [6, 9–11], as well as the pure finite terms, which have been the focus of this thesis and presented also in [119, 120, 129]. The calculation of the RG contributions is carried out in the unbroken theory, making this part of the calculation comparably simpler than the calculation of the finite terms. On top of this, initial calculations in the one-loop sector of the SMEFT concerning these RG contributions have been well motivated. They are log enhanced as $\log \left( \frac{\Lambda^2}{m_h^2} \right)$. This means that for a large cutoff, these contributions will come in as more significant than the pure finite terms.

On the other hand, in order to see effects of new physics in the first place, the cutoff scale cannot be pushed up too high, which would eventually lead to a complete decoupling of the new physics. No traces will in this case be evident at the low energy level and the approach of implementing new physics through the SMEFT can no longer be supported. This Run II of the LHC will at most push the scale of new physics up to about $\Lambda \sim 2$ TeV. With no new physics in sight at this level, but still expecting new physics to not decouple completely, a SMEFT can still prove useful in the case where the cutoff is taken to be up to $\Lambda \sim 3$ TeV. In this case, the log enhanced contributions will be enhanced by a factor $\log \left( \frac{\Lambda^2}{m_h^2} \right) \sim 6$ compared to the pure finite contributions. However, there is also a possibility for this run of the LHC to discover new physics at a lower scale, say $\Lambda \sim 1$ TeV. In this case, the enhancement is bigger by a factor $\sim 4$. At the TeV scale, RG contributions are therefore only a factor of $\mathcal{O}(1)$ bigger, which does not support neglecting the pure finite terms.

With this motivation for calculating finite contributions, and having done so, as presented in this thesis, it is interesting to study the actual impact of these finite terms, compared to that of the RG contributions.

For the coefficient of $C_{HW,B}$, the ratio of the full one loop terms, to the RG contribution is given by

$$ R_{C_{HW,B}} \simeq 1 + 0.7 \log^{-1} \frac{m_h^2}{\Lambda^2}. \quad (6.89) $$

For a cutoff scale of $\Lambda \sim 1$ TeV, the RG log terms of this coefficient are seen to be larger by a factor of $\sim 6$, meaning a factor of $\mathcal{O}(1)$ bigger, as expected. With regards to $C_W$, the ratio of the full one loop terms to the RG induced part of the result is in the same way,

$$ R_{C_W} \simeq 1 - 0.5 \log^{-1} \left( \frac{m_h^2}{\Lambda^2} \right), \quad (6.90) $$

which shows an RG contribution only $\sim 8$ bigger than the pure finite terms. The situation is also interesting for the dipoles, where

$$ R_{\text{dip}} \simeq 1 + 1.1 \log^{-1} \left( \frac{m_h^2}{\Lambda^2} \right), \quad (6.91) $$
6.10. Numerical Results and Discussion

gives an RG induced contribution only a factor $\sim 4$ bigger, assuming a TeV cutoff scale.

The part of the result proportional to $C_{HW}$ which is not included in $C_{\gamma \gamma}$, as well as all of the indirect operator coefficients $C_{H\Box}$, $C_{HD}$ and $C_{\psi H}$ do not contain any RG terms. It is therefore interesting to compare the size of these pure finite terms with the full one loop contributions of other coefficients. The ratio between the $C_{HW}$ pure finite terms and the $C_{HWB}$ terms, which are not already included in $C_{\gamma \gamma}$, is

$$R_{C_{HWB}/C_{HW}} \simeq \frac{C_{HWB}}{C_{HW}} \left(0.5 + 0.7 \log \frac{m_h^2}{\Lambda^2}\right), \quad (6.92)$$

and the ratios between the complete contribution of the direct operator $O_W$ and the contributions of the indirect operators are found to be,

$$R_{C_{\ell}/C_{H\Box}} \simeq \frac{C_{\ell}}{C_{H\Box}} \left(-0.8 + 1.5 \log \left(\frac{m_h^2}{\Lambda^2}\right)\right),$$

$$R_{C_{\ell}/C_{HD}} \simeq \frac{C_{\ell}}{C_{HD}} \left(0.2 - 0.4 \log \left(\frac{m_h^2}{\Lambda^2}\right)\right),$$

$$R_{C_{\ell}/C_{\psi H}} \simeq \frac{C_{\ell}}{C_{\psi H}} \left(4.5 - 8.5 \log \left(\frac{m_h^2}{\Lambda^2}\right)\right). \quad (6.93)$$

These operators contribute significantly. The pure finite terms are not negligible in favour of an RG analysis for cutoff scales in the TeV range. The enhancement of the RG contributions compared to the finite contribution is as expected only bigger by a factor of $O(1)$. Furthermore, contributions enter at the finite level, which do not have corresponding RG contributions. This interesting revelation further increases the importance of these finite contributions compared to the RG contributions.

The inclusion of the finite terms, including the RG contributions will reduce the theoretical errors needed for a global fit with experiments, as outlined in section 3.6. It is possible to study this numerical impact of the calculated one-loop contributions to see the exact effect. Such a study can be carried out for the relevant and interesting values of the cutoff scale, namely $\Lambda = 800$ GeV and $\Lambda = 3$ TeV, seeing that $v^2/(0.8 \text{ TeV})^2 \sim 0.1$.

The numerical analysis will be carried out, looking at the percentage deviations on account of these one-loop new physics contributions on the upper and lower bounds of the one $\sigma$ range for $\kappa_\gamma$. These have been measured by the two experiments ATLAS and CMS, and were listed in section 3.1 to be $\kappa_\gamma = 0.93^{+0.36}_{-0.17}$ and $\kappa_\gamma = 0.98^{+0.17}_{-0.16}$, respectively, see figures 3.8 and 3.9. Inferring from equation (6.88),

$$A_{\text{total}}^{\text{NP}} \sim A_{\text{SM}}(\kappa_\gamma - 1), \quad (6.94)$$

such that the upper and lower bounds on the NP amplitude in the one sigma range can be found from

$$\text{upper/lower} = \frac{I_7}{8\pi^2}(\kappa_{\text{upper}}/\kappa_{\text{lower}} - 1), \quad (6.95)$$
the ATLAS constraint gives the one sigma range,

\[ -0.020 \leq \hat{C}^\text{NP}_{\gamma\gamma} \frac{v^2}{\Lambda^2} \leq 0.024, \Rightarrow \]

\[ -0.020 \leq \left( \hat{C}^\text{NP}_{\gamma\gamma} + \frac{\hat{C}^\text{NP}_i f_i}{16 \pi^2} \right) \frac{v^2}{\Lambda^2} \leq 0.024, \quad (6.96) \]

whereas the range for CMS is

\[ -0.015 \leq \left( \hat{C}^\text{NP}_{\gamma\gamma} + \frac{\hat{C}^\text{NP}_i f_i}{16 \pi^2} \right) \frac{v^2}{\Lambda^2} \leq 0.012. \quad (6.97) \]

Here \( f_i \) are the coefficients of the Wilson coefficients, as seen in equation (6.85), and the hatted notation on the Wilson coefficients shows that \( \frac{1}{\Lambda^2} \) has been factored out. \( \hat{C}^\text{NP}_{\gamma\gamma} \) is the one-loop correction appearing in 6.85, including the tree-level contribution, \( C^0_{\gamma\gamma} \).

\[
C^4_{\gamma\gamma} = C^0_{\gamma\gamma} \left( 1 + \frac{f_{\gamma\gamma}}{16 \pi^2} \right) = C^0_{\gamma\gamma} \left( 1 + \frac{\delta R_h}{2} \right) + \frac{\delta v}{v} + \left( \sqrt{3} \pi - 6 \right) \frac{\lambda}{16 \pi^2} \\
+ \frac{C^0_{\gamma\gamma}}{16 \pi^2} \left( \frac{g_1^2}{4} + \frac{3 g_2^2}{4} + 6 \beta \right) \log \left( \frac{m_h^2}{\Lambda^2} \right) \\
+ \frac{C^0_{\gamma\gamma}}{16 \pi^2} \left( \frac{g_1^2}{4} I[m_e^2] + \left( \frac{g_2^2}{4} + \beta \right) (I[m_e^2] + 2 I[m_w^2]) \right). \quad (6.98)
\]

The VEV renormalization \( \delta v \) entering at the finite level also contains log enhanced terms, which can only be captured in these calculations carried out in the broken phase of the theory. It therefore deviates from what has been found in the unbroken phase, which does not capture effects introduced by the VEV. The log dependence of \( \delta v \) shows a running of the VEV, which is important to take into account. The scale dependent term of \( \delta v \) given by

\[
2 Y^2 N_c \frac{m_e^2}{m_h^2} \log \left[ \frac{\mu^2}{m_h^2} \right], \quad (6.99)
\]

will show the biggest effect from the top quark Yukawa. However, the interpretation of the numerical impact of this correction on \( \kappa_\gamma \) is more subtle. Naively using \( v + \delta v \) as a numerical value of the VEV extracted from a measured \( G_F \) in \( \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \) would assign a gauge dependent quantity a numerical value. Instead, the value for \( v + \delta v \) should be extracted from the \( \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu \) interaction, calculated at one loop in the SMEFT, where the one loop calculation has only been done in the SM [130]. This calculation would also induce a gauge dependent loop correction to the VEV, which will correspond to the gauge dependence of \( \delta v \) found in the \( h \rightarrow \gamma \gamma \) decay. The measurement of the muon decay can therefore be used as a direct input into the prediction of \( \kappa_\gamma \). Doing so, it is expected that the large log in equation (6.99) will be absorbed into the numerical value of the VEV in the SMEFT. From the remaining numerically small \( \mu \) independent contributions to \( \delta v \), as seen in equation (6.77), the scheme dependent effects defining the VEV in \( \delta v \), are expected to be below the few percent level for \( \Lambda \) in the range [0.8, 3] TeV.
The change in percentage on the upper and lower bounds of $\hat{C}_{\gamma\gamma}^0$ from the $f_i$ corrections are determined using equation (6.96), by shifting the quoted bounds by the SMEFT perturbative corrections. The individual effects of the $f_i$ are interesting to analyse initially. Therefore, the contribution proportional to $C_{\gamma\gamma}^0$ is factored out in the following way,

$$\begin{align*}
\text{lower} & \leq (\hat{C}_{\gamma\gamma}^0 (1 + \frac{f_i}{16\pi^2} + \frac{\hat{C}_i f_i}{16\pi^2}) \frac{v^2}{\Lambda^2} \leq \text{upper}, \Rightarrow \\
\frac{\text{lower}}{(1 + \frac{f_i}{16\pi^2})} & \leq (\hat{C}_{\gamma\gamma}^0 (1 + \frac{\hat{C}_i f_i}{16\pi^2}) \frac{v^2}{\Lambda^2} \leq \frac{\text{upper}}{(1 + \frac{f_i}{16\pi^2})}, \Rightarrow \\
1 + \Delta C_{\gamma\gamma}^{\text{lower}} & \equiv \frac{1}{\text{lower}} (\hat{C}_{\gamma\gamma}^0 \frac{v^2}{\Lambda^2} \geq (\text{lower} - \hat{C}_i f_i \frac{v^2}{\Lambda^2}) \frac{1}{(1 + \frac{f_i}{16\pi^2})} \\
1 + \Delta C_{\gamma\gamma}^{\text{upper}} & \equiv \frac{1}{\text{upper}} (\hat{C}_{\gamma\gamma}^0 \frac{v^2}{\Lambda^2} \leq (\text{upper} - \hat{C}_i f_i \frac{v^2}{\Lambda^2}) \frac{1}{(1 + \frac{f_i}{16\pi^2})}. 
\end{align*}$$

The envelope of the upper ($\Delta C_{\gamma\gamma}^{\text{upper}}$) and lower ($\Delta C_{\gamma\gamma}^{\text{lower}}$) bound percentage variations is quoted in the form $[\cdot, \cdot]$, for values of $\Lambda$ varying from $[0, 10]$ TeV including the values from ATLAS and CMS.

The DCs, including the ICs for operators varying both ways, show the following deviations, for positive values of the Wilson coefficients,

$$\begin{align*}
\left[\Delta \hat{C}_{\gamma\gamma}^{0,\text{NP}}\right]_{HWB}^{\text{ATLAS}} & \sim [10, 1] \times |\hat{C}_{HWB}|, \\
\left[\Delta \hat{C}_{\gamma\gamma}^{0,\text{NP}}\right]_{HWB}^{\text{CMS}} & \sim [13, 2] \times |\hat{C}_{HWB}|, \\
\left[\Delta \hat{C}_{\gamma\gamma}^{0,\text{NP}}\right]_{HW}^{\text{ATLAS}} & \sim [6, 1] \times |\hat{C}_H|, \\
\left[\Delta \hat{C}_{\gamma\gamma}^{0,\text{NP}}\right]_{HW}^{\text{CMS}} & \sim [9, 1] \times |\hat{C}_H|, \\
\left[\Delta \hat{C}_{\gamma\gamma}^{0,\text{NP}}\right]_{W}^{\text{ATLAS}} & \sim [25, 3] \times |\hat{C}_W|, \\
\left[\Delta \hat{C}_{\gamma\gamma}^{0,\text{NP}}\right]_{W}^{\text{CMS}} & \sim [32, 4] \times |\hat{C}_W|, \\
\left[\Delta \hat{C}_{\gamma\gamma}^{0,\text{NP}}\right]_{\text{dip}}^{\text{ATLAS}} & \sim [20, 3] \times \left(\text{Re}(|\hat{C}_{IB}|) + \text{Re}(|\hat{C}_{IW}|)\right), \\
\left[\Delta \hat{C}_{\gamma\gamma}^{0,\text{NP}}\right]_{\text{dip}}^{\text{CMS}} & \sim [26, 4] \times \left(\text{Re}(|\hat{C}_{IB}|) + \text{Re}(|\hat{C}_{IW}|)\right),
\end{align*}$$

and for negative values of the Wilson coefficients,
only considering the top quark coupling effects in $A_{SM}$ for simplicity. The effect of ICs yields for positive Wilson coefficients

\begin{align}
\left\{ \begin{array}{l}
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{ATLAS}}_{HWB} \sim [9, 1] \% \times |\hat{C}_{HWB}|, \\
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{CMS}}_{HWB} \sim [14, 2] \% \times |\hat{C}_{HWB}|,
\end{array} \right. & \quad (6.105) \\
\left\{ \begin{array}{l}
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{ATLAS}}_{HW} \sim [6, 1] \% \times |\hat{C}_{HW}|, \\
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{CMS}}_{HW} \sim [8, 1] \% \times |\hat{C}_{HW}|,
\end{array} \right. & \quad (6.106) \\
\left\{ \begin{array}{l}
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{ATLAS}}_{W} \sim [21, 2] \% \times |\hat{C}_{W}|, \\
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{CMS}}_{W} \sim [38, 4] \% \times |\hat{C}_{W}|,
\end{array} \right. & \quad (6.107) \\
\left\{ \begin{array}{l}
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{ATLAS}}_{\text{dip}} \sim [17, 3] \% \times \left(\text{Re}(|\hat{C}_{tB}|) + \text{Re}(|\hat{C}_{tW}|)\right), \\
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{CMS}}_{\text{dip}} \sim [31, 5] \% \times \left(\text{Re}(|\hat{C}_{tB}|) + \text{Re}(|\hat{C}_{tW}|)\right),
\end{array} \right. & \quad (6.108)
\end{align}

and for negative Wilson coefficients

\begin{align}
\left\{ \begin{array}{l}
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{ATLAS}}_{tH} \sim [5, 1] \% \times \text{Re}(\hat{C}_{tH}^{33}), \\
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{CMS}}_{tH} \sim [8, 1] \% \times \text{Re}(\hat{C}_{tH}^{33}),
\end{array} \right. & \quad (6.109) \\
\left\{ \begin{array}{l}
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{ATLAS}}_{H\Box} \sim [11, 1] \% \times \hat{C}_{H\Box}, \\
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{CMS}}_{H\Box} \sim [19, 1] \% \times \hat{C}_{H\Box},
\end{array} \right. & \quad (6.110) \\
\left\{ \begin{array}{l}
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{ATLAS}}_{HD} \sim [6, 1] \% \times \hat{C}_{HD}, \\
\Delta \hat{C}^{0,\text{NP}}_{\gamma\gamma}^{\text{CMS}}_{HD} \sim [6, 1] \% \times \hat{C}_{HD},
\end{array} \right. & \quad (6.111)
\end{align}
6.10. Numerical Results and Discussion

The sizes of the Wilson coefficients of the dimension six operators are UV dependent. Therefore, they have to be treated as free parameters, in this model independent approach, when aiming at constraining them through experiments. Only the naive power counting suppression $O(v^2/\Lambda^2)$ can be implemented in the linear parametrization used in the calculations of this thesis. Therefore, an adequate approach is to keep the coefficients undefined in a numerical analysis, as was done above or consider $\hat{C}_i \sim 1$. The latter can be assumed when adding all of the $f_i$ perturbative corrections in quadrature, that is

$$f_{\text{combined}} \frac{v^2}{16\pi^2} = \sqrt{\sum_i \left( \frac{f_i v^2}{16\pi^2 \Lambda^2} \right)^2}$$  \hspace{1cm} (6.115)$$

where $i = (HWB, HW, W, fB, tH, H\Box, HD)$. This combination leads to the net impact of one loop corrections due to higher dimensional operators on the bound of the tree level Wilson coefficient, fitting with ATLAS data, choosing positive values for the Wilson coefficients

$$\Delta_{\text{ATLAS}}^{C_{0, NP}} \hat{C}_{\gamma \gamma}^{0, NP} \sim [34, 4] \%,$$  \hspace{1cm} (6.116)$$

and choosing negative values,

$$\Delta_{\text{ATLAS}}^{C_{0, NP}} \hat{C}_{\gamma \gamma}^{0, NP} \sim [29, 4] \%.$$  \hspace{1cm} (6.117)$$

For CMS,

$$\Delta_{\text{CMS}}^{C_{0, NP}} \hat{C}_{\gamma \gamma}^{0, NP} \sim [46, 6] \%,$$  \hspace{1cm} (6.118)$$

and

$$\Delta_{\text{CMS}}^{C_{0, NP}} \hat{C}_{\gamma \gamma}^{0, NP} \sim [52, 7] \%.$$  \hspace{1cm} (6.119)$$
for positive and negative Wilson coefficients respectively. It is possible that these corrections could add in a manner that is not in quadrature, as this depends on the unknown $C_i$ values. In some cases of weakly coupled and renormalizable UV scenarios, the contributions from $C_{\gamma\gamma}$, $C_W$, $C_{HWB}$, $C_{HW}$, $C_{tB}$ and $C_{tW}$ are all expected to be suppressed by a further loop factor, as the arguments of [87] imply. The ICs from pure scalar operators as well as the scalar operators involving fermions are then expected to make up the largest effects. Allowing for underlying strongly coupled theories, or the presence of EFTs in the UV, such general statements are not possible [88].

EWPD will become more precise in the near future, as already insinuated. As was shown in section 3.1, the estimated uncertainties on the signal strengths will drastically decrease already in Run II with luminosity $\int \mathcal{L} dt = 300 \text{fb}^{-1}$ and even more with luminosity $\int \mathcal{L} dt = 3000 \text{fb}^{-1}$. The numerical impact of the one-loop calculations carried out in this thesis will therefore increase accordingly. For a luminosity of $300 \text{fb}^{-1}$, a value of $\kappa_{\gamma}^{\text{proj:RunII}} = 1 \pm 0.045$ has been estimated by CMS [3,131]. With this value, the deviations in the one $\sigma$ bounds can be found to be,

$$(\Delta_{\text{quad}} C_{\gamma\gamma})_{\text{proj:RunII}} \sim [167, 21] \%.$$ (6.120)

For a luminosity of $3000 \text{fb}^{-1}$ it has been estimated that the LHC will have a sensitivity between $2 - 5\%$ in $\kappa_\gamma$ [4]. Therefore choosing $\kappa_{\gamma}^{\text{proj:HILHC}} = 1 \pm 0.03$ one finds

$$(\Delta_{\text{quad}} C_{\gamma\gamma})_{\text{proj:HILHC}} \sim [250, 31] \%.$$ (6.121)

The LHC will undoubtedly do a great job in this second run, evidenced by these projected uncertainties. It has however been reported by the Snowmass Working Group [4], that a TLEP collider will be able to further improve the above listed sensitivities. Here, the potential future sensitivity is predicted to be $\kappa_{\gamma}^{\text{proj:TLEP}} = 1 \pm 0.0145$. Performing the same exercise with this projection

$$(\Delta_{\text{quad}} C_{\gamma\gamma})_{\text{proj:TLEP}} \sim [513, 64] \%.$$ (6.122)

Therefore, the calculation of the precise impact of the one-loop improved amplitude greatly reduces the theoretical error, as was already argued. In section 3.6, the theoretical errors due to these one-loop corrections,

$$\Delta_{\text{1-loop}}^{\Lambda_6} \simeq \frac{g^2}{16\pi^2} \left( a + b \log \left( \frac{\Lambda^2}{v^2} \right) \right) \frac{v^2}{\Lambda^2},$$ (6.123)

setting $a = b = 1$ and $g = 0.65$, were found to be,

$$\Delta_{\text{1-loop}}^{\Lambda_6} \simeq 0.02 \frac{v^2}{\Lambda^2} \simeq 10^{-4}, \quad \Lambda = 3 \text{TeV},$$

$$\Delta_{\text{1-loop}}^{\Lambda_6} \simeq 0.01 \frac{v^2}{\Lambda^2} \simeq 10^{-3}, \quad \Lambda = 1 \text{TeV}. $$ (6.124)
The effects of the above estimates will become important, when data increases its precision for the \( h \rightarrow \gamma \gamma \) decay. Already this Run II will induce a bound on this decay of \( \mathcal{O}(10^{-3}) \). Compared to this bound, the estimate on the theoretical errors certainly become relevant.

Having calculated the contributions explicitly from the loop contributions within the SMEFT, the corrections to the SM error have been found explicitly. For a cutoff of \( \Lambda = 800 \) GeV, the corrections to the error is found to be of the same order as the SM error, namely \( \sim 10^{-2} - 10^{-3} \). For a cutoff of \( \Lambda = 3 \) TeV, the error is approximately one order less, namely \( 10^{-3} - 10^{-4} \). The error therefore corresponds well with the estimated error.

With data coming in soon from the second run of the LHC, it is therefore helpful to replace these sizeable theoretical errors by actual calculated corrections. This puts theorists in a better position to analyze data more accurately, in the quest to reveal underlying new physics.
6.10. Numerical Results and Discussion
With the preciseness of present and future experiments, more accurate calculations are needed to keep up with measurements. The focus of this thesis has so far been on the calculation of one-loop finite contributions, completing the task of obtaining the full one-loop result for the $h \to \gamma \gamma$ decay, following already obtained calculations of RG contributions. At this stage, it is clear that finite contributions at one-loop need to be calculated to account for theoretical errors, existing when bounding new physics today using present experiments. The amount of work still needed in this area is therefore overwhelming.

With experiments becoming even more precise, it is also likely that two-loop calculations within the SMEFT will be needed in the future. Such calculations make up an even more formidable task. It will require many technical tools and skills, within the area of the SMEFT and the more technical areas concerning calculating amplitudes at the multiloop level, to overcome such a task.

The task of such a full two-loop calculation is beyond the scope of this thesis. The goal of the following section is therefore to describe a framework for carrying out such a calculation, merely initiating work in this area. Based on present data and experimental precision, the $h \to \gamma \gamma$ decay will again be addressed, since a two-loop calculation will more likely be relevant for this type of interaction in the nearer future. Ongoing work calculating a subset of the two-loop contributions within this decay will be addressed, and initial results will be presented.

### 7.1 $h \to \gamma \gamma$ Decay at Two Loops

The two-loop calculation of the $h \to \gamma \gamma$ decay in the SMEFT is a tedious task. In the SM, calculations were carried out at two loops for this decay in 2005 in [132] implementing an expansion technique in $\frac{m_h^2}{4m_Z^2}$ and $\frac{m_h^2}{4m_t^2}$. Only in 2007, the exact numerical result was given in [133]. The Feynman topologies amount to 40 for such an interaction within the SM, as shown in figure 7.1. With the insertion of particles, the amount climbs up to $\mathcal{O}(1000)$, only within the SM. Introducing new interactions as a result of including dimension six operators extends this amount even further.
The calculations needed can again be divided into direct and indirect contributions. At the two-loop level, however, this division becomes somewhat smeared. At a first look, the ICs seem easier to calculate, deducing this from the one-loop case, since these would require a shift in SM interactions and parameters. Therefore, one would expect such a calculation to require the recalculation of the two-loop SM calculations, merely shifting parameters already in the calculation, as was seen for the ICs in the one-loop calculation. However, this is not the case for two-loop interactions, where DCs from indirect operators are seen to enter diagrams, which are not in the SM, through new effective vertices.

Besides calculating such ICs and DCs showing up as effective vertices in two-loop diagrams, a renormalization has to be accounted for. Such a renormalization takes place both at tree and loop level, in which renormalizations of both SM and SMEFT parameters are needed.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure7.1.png}
\caption{Topologies contributing to the $h \rightarrow \gamma \gamma$ decay in the SM at two loops.}
\end{figure}

The calculation of DCs from higher dimensional operators, entering through an effective Lagrangian and effective vertices in diagrams at two loops makes up an impressive task. It amounts to identifying the dimension six operators contributing, as well as the diagrams, the effective Lagrangian of these operators will induce. The subsequent two-loop calculation can
then be performed. The task of calculating these integrals is tedious, however, the process at this stage is more trivial.

ICs first of all enter the two-loop calculation through shifting vertices already in the SM, as was seen in the one-loop case. For such contributions, only SM diagrams need calculation. Therefore, it seems like the calculation of such ICs is simplest to carry out. However, some operators that enter indirectly in this way might also contribute directly to interactions involving more fields as was shown in section 6.4. At one-loop, this did not have an impact in the \( h \rightarrow \gamma \gamma \) case, since these DCs from indirect operators only show up at the level of four-point effective interactions. However, such interactions will be well represented at two loops. For instance, the operators \( \mathcal{O}_{H\Gamma} \), \( \mathcal{O}_{HD} \) and \( \mathcal{O}_{\psi H} \) contribute indirectly by shifting the Higgs field and the Yukawa couplings. However, they also contribute directly to the four-point Goldstone vertices, the five-point vertices containing four Goldstones and a Higgs field, as well as four and five point interactions amongst two fermion fields and a combination of Higgs or Goldstone fields. When calculating in the \( R_g \) gauge, these DCs manifest themselves through effective vertices, extending the amount of diagrams to be calculated. Therefore, ICs not only amount to an overall shift in SM amplitudes, but also to new diagrams with effective vertices to be calculated, as is the case for direct contributions.

### 7.3 Renormalization of the Two-Loop Calculation

In this section, an outline for carrying out the two-loop renormalization of the \( h \rightarrow \gamma \gamma \) decay in the SMEFT will be presented. The outline will involve two stages of calculations. Namely the tree-level stage and the one-loop level stage, in which SM and SMEFT renormalizations enter at two-loop and one-loop, respectively.

#### 7.3.1 Tree-Level Stage

At this stage, renormalization is carried out involving two-loop calculations. Mixing of operators cause two-loop contributions to enter already at tree-level through RGEs of the operators \( \mathcal{O}_{HW} \), \( \mathcal{O}_{HB} \) and \( \mathcal{O}_{HWB} \). These two-loop contributions enter the \( h \rightarrow \gamma \gamma \) decay through the effective Lagrangian,

\[
\mathcal{L}_{\text{eff}}^{\text{tree}} = h v \left( N_{HB} + N_{HW} - N_{HWB} \right) \epsilon^2 A_{\mu} \epsilon_{\nu} A^{\mu \nu},
\]

where the renormalized couplings now have the form

\[
N_j = \frac{1}{(16 \pi^2)^2} \left( \frac{C^1_i}{\epsilon} + \frac{C^2_i}{\epsilon^2} \right) Z_{i,j}.
\]

The anomalous dimensions can be calculated in the unbroken theory, as argued. Subtleties are however involved with identifying the anomalous dimensions for this subset of operators, as was already outlined in [9–11]. Besides taking into account all possible two-loop diagrams with two external gauge fields and two external Higgs doublets, one needs to consider also the effects of the EOMs and other relations amongst operators. As was shown in [11], the anomalous dimension contribution from an operator involving three gauge fields is found
7.3. Renormalization of the Two-Loop Calculation

Figure 7.2: Graphs taken from [11], showing the insertions of the three-gauge field operators, contributing to operators related to $\mathcal{O}_{HWB}$ and $\mathcal{O}_{HW}$.

through calculating the diagrams shown in figure 7.2. None of these seem to contribute to the anomalous dimension of the above mentioned three operators entering in the $h \to \gamma\gamma$ decay at tree-level. However, as is subsequently shown in the paper, contributions from these diagrams do lead to contributions to the anomalous dimensions of the operators considered here through basic rewritings. To include the complete RGE contributions from the tree-level operators, manipulations of these operators as well as implementations of EOMs are needed for a complete result.

The rewriting in the specific case, where the diagrams of figure 7.2 contribute to the anomalous dimensions of the operators entering the $h \to \gamma\gamma$ decay at tree-level, occurs as follows. Using integration by parts and the relation $D_\mu D_\nu = \frac{1}{2} ([D_\mu, D_\nu] + \{D_\mu, D_\nu\})$, where the symmetric part vanishes, due to the antisymmetry of $W^a_{\mu\nu}$ in its Lorentz indices, followed by $[D_\mu, D_\nu] = ig_2 W^a_{\mu\nu} T^a + ig_1 Y B_{\mu\nu}$, as well as using $W^a_{\mu\nu} W_{\mu\nu}^a = W^a_{\mu\nu} \tau_a W_{\mu\nu}^a \tau_a$, the relation becomes,

\begin{align*}
i g_2 D_\mu H^{\dagger} \tau^I D_\nu H W_{\mu\nu}^I &= -i g_2 H^{\dagger} \tau^I (D_\mu D_\nu H) W_{\mu\nu}^I - i g_2 H^{\dagger} \tau^I D_\nu H D_\mu W_{\mu\nu}^I \\
 &= -\frac{1}{2} i g_2 H^{\dagger} \tau^I ([D_\mu, D_\nu] H) W_{\mu\nu}^I - i g_2 H^{\dagger} \tau^I D_\nu H D_\mu W_{\mu\nu}^I \\
 &= -\frac{1}{2} i g_2 H^{\dagger} \tau^I ((i g_2 W^a_{\mu\nu} T^a + i g_1 Y B_{\mu\nu}) H) W_{\mu\nu}^I \\
 &= -i g_2 H^{\dagger} \tau^I D_\nu H D_\mu W_{\mu\nu}^I \\
 &= \frac{1}{4} \mathcal{O}_{HW} + \frac{1}{4} \mathcal{O}_{HWB} - i g_2 H^{\dagger} \tau^I D_\nu H D_\mu W_{\mu\nu}^I. \quad (7.3)
\end{align*}

The three-gauge field operators mix into operators with two gauge fields and a Higgs, which in turn can be rewritten as the operators entering the $h \to \gamma\gamma$ decay. This spelled out subtlety demonstrates the importance of taking relations amongst operators into account when calculating the anomalous dimensions. It is however not necessary to calculate all anomalous dimensions at two-loop to account for all of these subtleties. It is enough to carefully account for contributions that enter under various relations of the above nature or through EOMs. These subtleties have already been met in the one-loop case in [6, 9–11]. Since the relations will be equivalent at two loops, it is not necessary to rederive these in this case. The relations can simply be adapted from [11].

Considering the anomalous dimensions of the three operators, $\mathcal{O}_{HW}$, $\mathcal{O}_{HB}$ and $\mathcal{O}_{HWB}$, the diagrams needed for this two-loop calculation are therefore of the shapes in figure 7.3. Diagrams at two loops with effective vertices from dimension six operators fitting into these shapes need to be calculated to get contributions from all operators to the anomalous dimensions. With no exaggeration, this is still a highly involved task.
7.3. Renormalization of the Two-Loop Calculation

Besides RG contributions from mixing of dimension six operators, SM renormalizations also enter at tree-level, requiring two-loop calculations of $Z_h$ and $Z_v$ as well as of the R factors $\delta R_h$ and $\delta v$. Since this decay does not contain tree-level contributions in the SM, the mixing of dimension six operators into the SM renormalization is luckily avoided in this case, but should be accounted for in any other two-loop calculation involving interactions occurring at tree-level in the SM.

7.3.2 One-Loop Level Stage

Renormalization has to be accounted for in various ways at the one-loop level. The calculations themselves are simpler, since they only involve one-loop diagrams. However, the subtlety becomes to keep track of all possible combinations of renormalizations operating at this stage. The various steps to be taken will be listed in the following.

- More operators are seen to enter directly, namely the operators already met in the one-loop calculation, $\mathcal{O}_W$, $\mathcal{O}_{\psi W}$, $\mathcal{O}_{\psi B}$, $\mathcal{O}_{H^\pm}$, $\mathcal{O}_{HD}$ and $\mathcal{O}_{\psi H}$. All possible dimension six operators entering the RGE’s of these operators will appear at this stage. For each one-loop diagram already encountered in section 6, the one-loop RG contribution to the operator of this diagram is inserted, causing another two-loop contribution.

- Renormalization of SM parameters enter the one-loop effective diagrams.

- Renormalization of SM parameters with insertions of dimension six operator vertices show up in the one-loop SM diagrams.

- The one-loop SM R factors for the $S$ matrix element are needed, which were already calculated in section 6, namely $R_h$ and $\delta v$.

- On top of the SM R factor contributions, the SMEFT contributions, in which dimension six operators enter the R factors, need to be included.

- Redefinitions of SM parameters due to dimension six operators. For example, the one-loop contribution of $\mathcal{O}_{HWB}$ to the rotation of the gauge fields into their mass eigenstates at one-loop will enter the one-loop SM diagrams.

- RGEs of SM parameters also come in at this point, through which dimension six operators will contribute.
7.4 SecDec

Multiloop integrals are too complicated to carry out analytically. For this purpose, the program SecDec, [134–141] is implemented to carry out this part of the two-loop calculation. It uses the technique of sector decomposition, and singularities are regulated using DREG. Under sector decomposition, overlapping divergences are disentangled and the integrals carried out numerically. In the algebraic part of the program, the decomposition into sectors is performed and the singularities are subtracted, with a subsequent expansion in \( \epsilon \). In the numerical part, Fortran functions are integrated using Monte Carlo integration programs like BASES or routines from the CUBA library [128, 142, 143]. Monte Carlo programs are used for such multiloop integrals, since the factorizations of the singularities in \( \epsilon \) are too complicated to be integrated analytically. This however, limits the precision.

Carrying out the integrals over the Feynman parameters in all the separated sectors and summing them up, the poles can be extracted from the program, expanded in terms of \( \epsilon \) in a Laurent series,

\[
A = \sum_{n=-LP}^{r} C_n \epsilon^n + \mathcal{O}(\epsilon^{r+1}) \tag{7.4}
\]

where \( LP \) is the leading pole, which is at most 2L for L loops and \( r \) is the number of subsectors.

In the program, the files "graph.input", "graph.m" and "graph_kinem.input" are altered for each diagram, specifying the number of loops, propagators, internal and external momenta, propagator expressions, powers of the propagators and masses.

7.5 The Two-Loop Program, FORM to SecDec

In this section, a short outline of how the calculation of two-loop diagrams is carried out, implementing an interface between FORM and SecDec, through Mathematica, will be presented. The procedure follows many of the steps carried out in the one loop case, as described in appendix B. However, in general it is not possible to acquire analytical results in the two-loop case. Therefore, the last step, the actual evaluation of the integral, will be performed by SecDec.

To apply SecDec, it is necessary to remove the Lorentz index dependence from numerators. Therefore, projections should be implemented similar to the ones applied in the one-loop calculation of the \( h \rightarrow \gamma \gamma \) decay in this thesis. This extracts the form factors of the Passarino-Veltman decomposition, which emerge as scalar products of momenta. After the steps involving projections, contractions, traces and reductions carried out in FORM, as described in appendix B, the integrals are carried out by SecDec.

Mathematica provides the interface between FORM and SecDec and gathers the final results. Numerator outputs from FORM are separated into coefficients and momenta and translated into SecDec language, from which the ".input", ".m" and ".kinem.input" files are generated, defined in the previous section.

Shell scripts are constructed in Mathematica to run the generated SecDec files and gather the results. These are then reimplemented in Mathematica, where the total result is recovered. The full interface, which is at this stage only constructed for the \( h \rightarrow \gamma \gamma \) decay, will appear...
7.6 Initiating the Two-Loop Calculation of the $h \to \gamma \gamma$ Decay - the $\lambda$ Dependence

The two-loop integrals can be carried out using the combined program as described in the previous section. However, in such a calculation within the SMEFT, the main challenge is to identify the integrals to be calculated. As outlined throughout this thesis, besides calculating the diagrams through which operators contribute directly or indirectly, a vast renormalization procedure also has to be accounted for.

Due to the extent of calculating contributions from dimension six operators to the $h \to \gamma \gamma$ decay at two loops, it can prove useful to define an initial subset of the calculation. For instance, following the approach of [6, 9–11], the calculation can be limited to one coupling structure to begin with.

The simplest possible coupling structure for the $h \to \gamma \gamma$ two-loop decay is of the form $C_i e^2 \lambda^2$.

The operators to be included in this case, have been narrowed down to the following,

\[
\begin{align*}
\mathcal{O}_{HH}^{(0)} &= g_1^2 H^\dagger H B_{\mu\nu} B^{\mu\nu}, \\
\mathcal{O}_{HW}^{(0)} &= g_2^2 H^\dagger H W_{\mu}^{a} W^a_{\mu\nu}, \\
\mathcal{O}_{H_W B}^{(0)} &= g_1 g_2 H^\dagger T H W_{\mu}^{I} B^{\mu\nu}, \\
\mathcal{O}_{H_W}^{(0)} &= \lambda (H^\dagger H)^3, \\
\mathcal{O}_{H_D}^{(0)} &= \lambda (H^\dagger D_{\mu} H)^* (H^\dagger D^\mu H). 
\end{align*}
\] (7.5)

There is a subtlety involved with choosing coupling structures, when dimension 6 operators are involved, regarding how they are normalized. The advantages of the Ward identities in the BFM were exploited in the one-loop case, normalizing the gauge field operators to include the respective couplings associated with the gauge fields in the operator. Besides $\mathcal{O}_H$, the scalar operators have so far not been normalized with additional couplings, but will in the following be normalized pulling a coupling $\lambda$ outside the Wilson coefficients. This will match the coupling structures of SM and SMEFT interactions to more consistently extract the coupling structure $C_i e^2 \lambda^2$, including both areas. This is a choice, which can be made as one sees fit to simplify the calculation.

Scalar operators have in models, where the Higgs is assumed to be strongly coupled to the underlying UV theory, been advocated to be less suppressed than other EW operators. Since these are the operators contributing mainly in this calculation, considering only the $\lambda$ part, this could also favour the initiation of such a two-loop calculation in this sector.

The various steps already described, regarding this two-loop calculation will in the following be described in detail for this subset of the calculation. The various contributions through diagrams will be listed, as well as the results of divergences in some cases for the purpose of illustration. At this stage, the following outline should only be used as a demonstration. The results of this section are therefore not complete, and they are subject to possible corrections. In the eventual calculation, all the divergences are to be calculated to verify the convergence of the result. The calculations are thereafter expanded to include the finite contributions to obtain the full result. The outline of the calculation goes as follows.

- Only one diagram contributes to the RGE’s of the three operators entering at tree-level,
7.6. Initiating the Two-Loop Calculation of the $h \to \gamma \gamma$ Decay - the $\lambda$ Dependence

$O_{HW}$, $O_{HB}$ and $O_{HWB}$, which contains only internal scalar fields. It is given in figure 7.4. The operators contributing to the two-loop anomalous dimensions are the tree-level contributing operators themselves. The integrals for these diagrams are simple to carry out, since they can be split into two one-loop integrals. The amplitudes in the unbroken phase for these integrals can be evaluated as multiplying two one-loop integrals, leading to the following result, for $O_{HW}$,

$$A_{\text{RGE}}^{HW} = 36 i 7 \lambda^2 C_{HW} \left( -4 (p_a \cdot p_b g_{a\beta} - p_{a\beta} p_b) \right) \left( \mu^2 e^{\gamma_E} \left( \int \frac{d^d p}{(2\pi)^d} \frac{1}{q^2} \frac{1}{(q - p)^2} \right)^2 \right)$$

$$= 4 \cdot 36 i \lambda^2 (p_a \cdot p_b g_{a\beta} - p_{a\beta} p_b) \left( \frac{1}{\epsilon^2} + 2 \epsilon \left( \log \left( \frac{\mu^2}{-p^2} \right) + 2 \right) \right) . \quad (7.6)$$

The same can be done for $O_{HB}$ and $O_{HWB}$. The RGEs become

$$\dot{C}_{HW} = \frac{72 \lambda^2}{(16 \pi^2)^2} C_{HW} \left( \frac{1}{\epsilon^2} + 2 \epsilon \left( \log \left( \frac{\mu^2}{-p^2} \right) + 2 \right) \right) ,$$

$$\dot{C}_{HB} = \frac{72 \lambda^2}{(16 \pi^2)^2} C_{HB} \left( \frac{1}{\epsilon^2} + 2 \epsilon \left( \log \left( \frac{\mu^2}{-p^2} \right) + 2 \right) \right) ,$$

$$\dot{C}_{HWB} = \frac{8 \lambda^2}{(16 \pi^2)^2} C_{HB} \left( \frac{1}{\epsilon^2} + 2 \epsilon \left( \log \left( \frac{\mu^2}{-p^2} \right) + 2 \right) \right) . \quad (7.7)$$

These enter through $N_i = \frac{i}{32 \pi^2} \dot{C}_i$ in the S matrix element as follows,

$$\langle h | v \sqrt{Z_v} (v \gamma (N_{HB} + N_{HW} - N_{HWB}) e^2 A_{\mu\nu} A^{\mu\nu} | h \rangle = \sqrt{Z_h} \left( v \gamma (N_{HB} + N_{HW} - N_{HWB}) e^2 A_{\mu\nu} A^{\mu\nu} | h \right) . \quad (7.8)$$

- The SM renormalization at tree level requires the two-loop calculation of $Z_h$ and the renormalization of the VEV, $\left( \sqrt{Z_v} + \frac{\delta v}{v} \right)$. In the BFM, the two renormalizations are the same at the divergence level,

$$\left( \sqrt{Z_v} + \frac{\delta v}{v} \right)_{\text{div}} = \sqrt{Z_h} . \quad (7.9)$$

At $O(\lambda^2)$, the only diagram contributing to the Higgs renormalization constant is the sunset diagram, as shown in figure 7.5. The divergent part of this amplitude can be calculated using SecDec as described in section 7.4, and yields
7.6. Initiating the Two-Loop Calculation of the $h \to \gamma \gamma$ Decay - the $\lambda$ Dependence

\[
\frac{h/\phi_0}{\phi_{\pm}}
\]

\[
\frac{h}{h} \quad \frac{h}{h}
\]

**Figure 7.5:** Two-loop diagram contributing to the self-energy of the Higgs.

\[
\frac{h}{\phi_0/\phi_{\pm}}
\]

\[
\frac{h}{h} \quad \frac{h}{h}
\]

**Figure 7.6:** Two-loop diagrams contributing to the VEV renormalization.

\[
\Sigma_{\text{div}}^h(p^2) = \frac{\lambda^2}{(16\pi^2)^2} \left( \frac{1}{\epsilon^2} \frac{3}{2} m_h^2 + \frac{1}{\epsilon} \left( \frac{9}{2} m_h^2 - 3 m_H^2 \log \left( \frac{m_H^2}{\mu^2} \right) - \frac{1}{2} p^2 \right) \right).
\]

(7.10)

From this amplitude, the renormalization constant at two loops, is found from

\[-i\Sigma^{r,h}(p^2) = i \Sigma^h(p^2) + i p^2 \delta Z_h, \]

(7.11)

and given by,

\[
Z_h = 1 + \delta Z_h = 1 + \frac{1}{4} \frac{\lambda^2}{(16\pi^2)^2} \frac{1}{\epsilon}.
\]

(7.12)

- The Higgs R factor contributing at two loops can also be extracted from the sunset diagram, expanding the above amplitude to the finite level. The VEV R factor is calculated from twelve tadpole diagrams at two loops, given in the form of two topologies as shown in figure 7.6.

- The one-loop diagram contributing in the limit considered are shown in figure 7.7. Other dimension six operators enter these one-loop diagrams through RGE mixing into the operators of the effective vertices. The operators contributing to the vertices of these diagrams are $O_{HW}, O_{HB}, O_{HWB}, O_{H\Box}, O_{HD}$, listed in the beginning of this section. The RGEs for these operators contributing at $O(\lambda)$ are the following

\[
\begin{align*}
\dot{C}_{HW} &= 12 \lambda C_{HW}, \\
\dot{C}_{HB} &= 12 \lambda C_{HB}, \\
\dot{C}_{HWB} &= 4 \lambda C_{HWB}, \\
\dot{C}_{H\Box} &= 24 \lambda C_{H\Box}, \\
\dot{C}_{HD} &= 12 \lambda C_{HD}.
\end{align*}
\]

(7.13)

These replace the coefficients of the already calculated one-loop amplitudes, extracting only the $O(\lambda)$ part,
7.6. Initiating the Two-Loop Calculation of the $h \to \gamma\gamma$ Decay - the $\lambda$ Dependence

Figure 7.7: One-loop diagram contributing to $h \to \gamma\gamma$ decay at $O(\lambda^2)$.

\[ iA_{1\text{loop}}^{\text{tot}} = \frac{i e^2 \lambda v}{16 \pi^2} \left( \frac{1}{\epsilon} \left( 6 C_{\gamma\gamma} + 4 C_{\text{HWB}} \right) \right. \]
\[ \left. + C_{\gamma\gamma} \left( I[\xi m^2] + 2 I[\xi m^2_w] + \left( \sqrt{3} \pi - 6 \right) + 6 \log \left( \frac{m^2_h}{\mu^2} \right) \right) \right) \]
\[ + 4 C_{\text{HWB}} \left( I[\xi m^2_w] + \log \left( \frac{m^2_h}{\mu^2} \right) \right) \right) A_{\alpha\beta}^{h\gamma\gamma}. \] (7.14)

As can be seen, only operators involving gauge fields contribute in this limit. The following divergences are found

\[ iA_{1\text{loop}+1\text{loop}}^{\text{RG}} = \frac{i e^2 \lambda^2 v}{16 \pi^2} \left( \frac{1}{\epsilon} \left( 36 C_{HB} + 36 C_{HW} - 4 C_{HWB} \right) \right. \]
\[ \left. + (6 C_{HB} + 6 C_{HW} - 2 C_{HWB}) \left( I[\xi m^2] + 2 I[\xi m^2_w] + \left( \sqrt{3} \pi - 6 \right) \right) \right) \]
\[ + 6 \log \left( \frac{m^2_h}{\mu^2} \right) + 8 C_{HWB} \left( I[\xi m^2_w] + \log \left( \frac{m^2_h}{\mu^2} \right) \right) \right) A_{\alpha\beta}^{h\gamma\gamma}. \] (7.15)

- A SM one-loop renormalization enters the various vertices of the one-loop effective diagrams. Due to the simplifications of working in the BFM, taking advantage of the Ward identities, only the 3-point vertex involving scalar fields in the one-loop diagrams in figure 7.7 will acquire SM renormalization contributions. This scalar vertex involves a renormalization of the $\lambda$ coupling, the VEV and the scalar fields, $h$, $\phi_0$ and $\phi_{\pm}$. In the BFM, the renormalizations of the scalar fields are equivalent and do not show $\lambda$ dependence, therefore giving no contribution in this calculation. Only the $\lambda$ renormalization constant is included, which enters as an overall multiplication factor of the amplitude $A_{1\text{loop}}^{\text{tot}}$. 

108/171
7.6. Initiating the Two-Loop Calculation of the $h \to \gamma \gamma$ Decay - the $\lambda$ Dependence

The renormalization of the coupling $\lambda$ is found through calculation of the one-loop contribution to the Higgs four-point vertex, shown in figure 7.8, giving the following result,

$$Z_\lambda = 1 + \frac{5 \lambda}{4 \pi^2 \epsilon}.$$  \hspace{1cm} (7.16)

The divergent contribution from the SM renormalization of the one-loop effective diagrams is therefore

$$i \delta Z_\lambda A_{\text{tot}}^{\text{1loop}} = \frac{5 i \lambda}{4 \pi^2 \epsilon} A_{\text{tot}}^{\text{1loop}}.$$  \hspace{1cm} (7.17)

Figure 7.8: SM $\lambda$ renormalizations contributing to $h \to \gamma \gamma$ decay at $O(\lambda^2)$.

- A renormalization at one-loop, where the SM one-loop diagrams get renormalized by renormalization constants, which themselves involve the dimension 6 operators, $O_H$, $O_{H\Box}$ and $O_{HD}$ also has to be accounted for. The SM diagrams to be renormalized are diagrams (c) and (d) of figure 6.1. The $h \phi_+ \phi_-$ vertex of these diagrams also involve the renormalization of the $\lambda$, VEV and scalar fields. Again, the convenience of calculating only the $\lambda$ contribution means that only $\lambda$ needs to be renormalized. The Feynman rules for the effective vertices involved in the diagram of figure 7.8 can be found from the Lagrangian of equation 6.33 and are given in Appendix H. The operators contribute both indirectly and directly in this case, leading to the following contribution to $Z_\lambda$,

$$i A_\lambda^6 = i \frac{\lambda^2 v^2}{16 \pi^2 \epsilon} \left( \frac{-87}{8} C_H + \frac{55}{2} C_{H\Box} - \frac{43}{8} C_{HD} \right).$$  \hspace{1cm} (7.18)

The renormalization constant becomes,

$$Z_\lambda = 1 + \frac{\lambda v^2}{16 \pi^2 \epsilon} \left( \frac{87}{2} C_H - 110 C_{H\Box} + \frac{43}{2} C_{HD} \right).$$  \hspace{1cm} (7.19)

The divergent part of the SM diagrams (c) and (d) of figure 6.1 cancels. The finite contribution from these diagrams is given by,

$$i A_\lambda^{\text{SM}} = \frac{i e^2}{16 \pi^2 v} \left( 1 - 6 I_y |m_w|^2 \right) A_{\alpha \beta}^{h \gamma \gamma}.$$
7.6. Initiating the Two-Loop Calculation of the $h \to \gamma \gamma$ Decay - the $\lambda$ Dependence

such that the two-loop contribution in this case, combining the above amplitude and renormalization constant, yields

$$i \delta Z_\lambda A^3_{\text{SM}} = i \frac{\lambda v^2}{16 \pi^2 \epsilon} \left( \frac{87}{2} C_H - 110 C_{H\Box} + \frac{43}{2} C_{HD} \right) A^3_{\text{SM}}$$  \hspace{1cm} (7.20)

- At one-loop, the R factors come into the picture in two ways, as do the renormalization constants. The SM renormalization of one-loop effective diagrams, concerns the R factors $R_h$ and $\delta v$, which have already been calculated for the purpose of the one-loop calculation.

The scalar dimension six operators $O_H$, $O_{H\Box}$ and $O_{HD}$ enter the renormalization of the $\lambda$ vertex as well as the Higgs self-energy and the Higgs one-point tadpoles. In the case of the Higgs self-energy, they will however not induce a renormalization of the Higgs field, as was also the case in the SM for this particular coupling study. The VEV renormalization will however get a contribution from these operators, from the tadpoles shown in figure 7.9.

- The operator $O_{HWB}$, which has been seen to contribute in the one-loop calculation through a rotation of gauge fields into their mass eigenstates, also contributes at two loops. In this case, however, also one-loop effects to the redefinition of the mixing should be accounted for. The operator enters with a $\lambda$ contribution at one-loop into the $W_\mu^3 - B_\mu$ mixing, as seen in figure 7.10.

The amplitude of this loop is given by

$$i A_{\text{loop}}^{C_{HWB}} = -\frac{1}{2} \int \frac{d^4q}{(2\pi)^4} \frac{i^2}{q^2 - m_h^2} \langle W_\mu^3, B_\mu^\nu \rangle$$  \hspace{1cm} (7.21)

where the factor $-\frac{1}{2}$ comes from the Higgs doublets expanded, $H^\dagger \tau^3 H$, leading to the following divergent loop contribution to the $W_\mu^3 - B_\mu$ mixing,

$$\frac{1}{16 \pi^2} \lambda v^2 g_1 g_2 C_{HWB} C \langle W_\mu^3, B_\mu^\nu \rangle$$  \hspace{1cm} (7.22)
where \( C = \left( \frac{1}{\epsilon} + 1 + \log \left( \frac{\Lambda^2}{m^2} \right) \right) \). The tree-level contribution to this mixing is given by

\[
-\frac{1}{2} v^2 g_1 g_2 C_{\text{HWB}} \langle W_{\mu \nu}^3 B^{\mu \nu} \rangle
\]

(7.23)
such that the mixing becomes

\[
\begin{pmatrix}
W_{\mu}^3 \\
B_{\mu}
\end{pmatrix} = \begin{pmatrix}
1 & -\frac{1}{2} v^2 g_1 g_2 C_{\text{HWB}} \left( 1 - \frac{1}{8\pi^2} \lambda C \right) \\
-\frac{1}{2} v^2 g_1 g_2 C_{\text{HWB}} \left( 1 - \frac{1}{8\pi^2} \lambda C \right) & 1
\end{pmatrix}
\begin{pmatrix}
Z_{\mu} \\
A_{\mu}
\end{pmatrix}
\times \begin{pmatrix}
c_w & s_w \\
-s_w & c_w
\end{pmatrix}
\]

(7.24)

The one-loop contribution of \( \mathcal{O}_{\text{HWB}} \) to the mixing of gauge fields merely shifts its own tree-level contribution. This one-loop contribution therefore enters wherever \( C_{\text{HWB}} \) enters indirectly at one-loop. However the \( C_{\text{HWB}} \) contributions entering \( A_{\text{tot}}^{\text{1-loop}} \) of equation (7.14), only arise from direct contributions, and therefore no contribution from this one-loop contribution to the gauge field rotation will occur in this calculation.

- The diagrams contributing directly to the \( h \to \gamma \gamma \) decay with the above mentioned couplings are the ones shown in figure 7.11. The operators entering the effective vertices shown in the figure are \( \mathcal{O}_{\text{HW}}, \mathcal{O}_{\text{HB}}, \mathcal{O}_{\text{HWB}}, \mathcal{O}_{\text{H}}, \mathcal{O}_{\text{H}^2} \) and \( \mathcal{O}_{\text{HD}} \).

The divergent parts of these diagrams should cancel off the divergences appearing at tree-level and at one-loop. The calculation of these diagrams after insertion of fields amount to 100, which is beyond the scope of this thesis, but will be addressed in the future.
7.6. Initiating the Two-Loop Calculation of the $h \rightarrow \gamma \gamma$ Decay - the $\lambda$ Dependence

Figure 7.11: Two-loop diagrams contributing to $h \rightarrow \gamma \gamma$ at $\mathcal{O}(\lambda^2)$, where an effective scalar vertex is normalized to contribute with a $\lambda$ coupling.
The discovery of a Higgs like field has provided the answer to many previously open questions within particle physics. An important part of the SM has been verified through its discovery. However, new findings bring about new questions. Therefore, the hunt for new physics continues. There are still many unanswered questions within and outside the SM, demanding attention and hinting towards the existence of new physics.

The areas to look for new physics are vastly being explored both experimentally and theoretically. The LHC has started up again in Run II and will expectedly provide interesting data in the near future. The question is in which form these interesting data will show up. Will it be in the shape of new particles, or in the shape of very precise data revealing underlying new physics through deviations in interactions within the SM?

Until new physics shows up in the shape of new particles, particle physicists can only hope to see glimpses of such existence through deviations. To properly account for the precise measurements expected to reveal such findings, theoretical calculations are necessary at the same precision level.

To account for possible new physics, a general approach implemented by theorists is the assumption of having a SMEFT. The extension of the SM Lagrangian, including 59 new dimension six operators plus Hermitian conjugates, without taking into account the various fermion families, could provide a peak hole for theorists to the underlying new physics, without possessing specific knowledge on the theory. Exploiting this connection to the underlying new physics, however, makes up a formidable task. It consists of accounting for all possible effects of dimension six operators on all interactions of the SM, as well as providing a solid understanding of the deviations, if these show up in experiments. However, it might be our only guide towards discovering and understanding the structure of existing new physics.

In this thesis, the aim has been to proceed along this pathline, calculating the one-loop contributions to the $h \rightarrow \gamma\gamma$ decay within the SMEFT. The full one-loop result has been presented, showing a deviation on the bounds of the signal strength of this decay at the percentage order. Therefore, the deviations from such SMEFT corrections to the SM decay will become within the measurable range of this second run of the LHC and are therefore highly relevant for future global fit studies.
The log enhanced RG contributions to the $h \to \gamma \gamma$ decay have already been calculated. In this thesis, the finite contributions calculated in the broken phase were found, supplementing these results to give the full one-loop contribution to the decay in the SMEFT. Even though the RG contributions are log enhanced, it was found that at the level, where new physics is expected to show up to account for the use of this EFT approach, these log enhanced contributions are only bigger by a factor of $\mathcal{O}(1)$. Furthermore, it was found that some operators contribute with finite contributions, which do not have corresponding contributions in the RG calculation. Therefore, the calculation of RG contributions is simply insufficient.

This is however only a small step towards a general understanding of possible deviations in future experiments. The interpolation between the SMEFT and data still requires plenty of theoretical calculations. A global fit, comparing data with the SMEFT is at this point not possible, without introducing vast theoretical errors. Such errors can be overcome through explicit calculation. Therefore, there are plenty of calculations to be done for theorists within this general framework of the SMEFT. The full one-loop calculation of a SM interaction has so far only been carried out for the $h \to \gamma \gamma$ decay, as addressed in this thesis, as well as for the muon decay, as calculated in [145]. Even though measurements are becoming exceptionally precise for these particular interactions, many other SM interactions can also be seen to require loop level calculations at this stage to account for experimental precision. As an example, the $Z$ width has been measured at an incredible preciseness by experiments as shown in section 3.5. However, one-loop calculations are yet to be completed in this area.

The introduction of dimension six operators, extending the SM Lagrangian provides an interesting approach to reveal possible new physics, but also a big task ahead of theorists. Furthermore, the dimension eight operators have recently all been found in [146], contributing with the extraordinary amount of 535 new operators. It is very likely that the inclusion of these operators will become necessary in the future, greatly enhancing the task of covering loop calculations within the SMEFT. The effect of these dimension eight operators will especially become important, if the cutoff scale is found to be at a lower level, say around or below 1 TeV. This can be seen in equation (3.32) of section 3.6, showing the expected theoretical error brought about by these operators.

Another step is to proceed to the two-loop level, as has also been discussed in this thesis. Such calculations are not yet a priority within the area of the SMEFT but, as for the dimension eight operators, could become relevant at some point in the future. Especially for interactions occurring already at the loop level in the SM, such as the $h \to \gamma \gamma$ decay, as well as interactions which are being measured at the most precise level by experiments. The decay studied in this thesis also falls under this category. Therefore, a two-loop calculation, will it become relevant in the future, seems adequate to explore for this particular interaction. Certain models of the underlying UV theory moreover provide power counting tools, advocating for some operators being more suppressed than others. In this case, operators coming in at two-loop level might contribute at the same level as other operators entering at one-loop level. In such situations, where certain models hint towards a specific power counting, the two-loop calculations could become more relevant to carry out for a full global analysis. For these reasons, the two-loop calculation of the $h \to \gamma \gamma$ decay would be interesting to explore. As outlined in this thesis, this great task can be overcome through intermediate steps following couplings. Starting out with the contributions from $\lambda$ couplings in this decay could be an adequate approach. To manage this overwhelming task, the implementation of programs such as FORM and SecDec could provide valuable tools to support such a calculation.

In this thesis, the interface between FORM and SecDec was discussed and calculations were
carried out at one-loop using FORM, evaluating the integrals in Mathematica, whereas the two-loop calculations were evaluated using SecDec, before the results were gathered in Mathematica. At this point, the amplitudes within the BFM and the SMEFT are written up by hand in the FORM file, making the full two-loop calculation a vast task in terms of manual work. A way forward would here be to generate a program, such as FeynArts and FeynCalc [147–149], which could account for the BFM and the SMEFT to generate the diagrams and amplitudes automatically. This would be a natural extension of the Form2SecDec program encountered in this thesis.
Bibliography


Bibliography


[71] Georges Aad et al. Measurements of the Higgs boson production and decay rates and coupling strengths using pp collision data at $\sqrt{s} = 7$ and 8 TeV in the ATLAS experiment. 2015.


[99] Brian Henning, Xiaochuan Lu, and Hitoshi Murayama. How to use the Standard Model effective field theory. 2014.


[120] Christine Hartmann and Michael Trott. Higgs decay to two photons at one-loop in the SMEFT. 2015.


Erratum

This revised version has been printed for distribution at the Ph.D. defence on Friday the 13th of November, 2015. Compared to the submitted thesis on the 19th of October, 2015, the following corrections have been made in this version:

- Section 3.5: Listed operators contributing to the $S$ and $T$ parameters corrected; $O_{DW}$, $O_{DB}$, $O_{WWW}$ and $O_B$ removed.
- Section 5.4, equation (5.27) and section 6.7, equation (6.77): Factor $\frac{1}{2}$ removed to obtain consistency in convention.
- Section 6: Removed $h \to gg$ in listing of more noticeable interactions.
- Section 6.3, figure 6.2: Caption corrected to include operators $O_W$, $O_{\psi W}$ and $O_{\psi B}$, as already described in the text and shown diagrammatically.
- Section 7.6, equations (7.6), (7.7), (7.16) and (7.17): Contraction factors corrected.
- Appendix C: Feynman rule for $\bar{\psi} \psi Z_\alpha$ corrected
- Appendix G: Fermion tadpole diagram added in figure and listed amplitude. Already included in result, which remains unchanged.
List of Abbreviations

In this Appendix, the abbreviations used throughout the thesis are defined.

- LHC: Large Hadron Collider
- UV: Ultraviolet
- IR: Infrared
- DREG: Dimensional regularization
- RGE: Renormalization group equation
- 1PI: One particle irreducible
- EFT: Effective field theory
- SM: Standard model
- SMEFT: Standard model effective field theory
- BSM: Beyond the standard model
- χPT: Chiral perturbation theory
- χSB: Chiral symmetry breaking
- SSB: Spontaneous symmetry breaking
- VEV: Vacuum expectation value
- SUSY: Supersymmetry
- EW: Electroweak
- EWSB: Electroweak symmetry breaking
- EWPT: Electroweak precision tests
List of Abbreviations

- EWPD: Electroweak precision data
- NDA: Naive dimensional analysis
- NLO: Next-to-leading order
- MFV: Minimal flavour violation
- $\overline{\text{MS}}$: Modified minimal subtraction
Appendix B

Calculational Methods

B.1 FORM

FORM is a calculational tool [150], used in part throughout the calculations of this thesis. It performs the contractions of the indices, the calculation of traces, the implementation of spinor technology and Passarino-Veltman reductions. All of this is carried out on the amplitudes, written up within the SMEFT using the BFM and $R_\xi$ gauge. The FORM code is inspired by the tool FormCalc [147], from which the code has been changed to accommodate working in the SMEFT and in an $R_\xi$ gauge in the BFM. The amplitudes are written up in the C++ FORM files manually and the FORM file is run in a shell script. The interface with Mathematica is made using FormGet.

The FORM file is prepared as follows. Parameters to be used are defined and amplitudes are written up. Next, the traces and contractions are carried out. Finally, the identities are given, which involve on-shell conditions, spinor technology and Passarino-Veltman reductions.

B.2 Passarino-Veltman Decomposition

In order to calculate the amplitude of any interaction, it is useful to implement a Passarino-Veltman decomposition [109]. Such a decomposition writes up the amplitude in terms of its most general Lorentz invariant structure with respective form factors. A triangle diagram with a massive incoming and two massless outgoing particles, where $p = p_1 + p_2$, has the amplitude

$$ I^\mu = \int \frac{d^d k}{(2 \pi)^d} \frac{k^\mu}{k^2 (k + p_1)^2 (k + p_1 + p_2)^2} $$

(B.1)

This amplitude can be decomposed into the following structure with Lorentz invariant form factors, $A$ and $B$,

$$ I^\mu = A p_1^\mu + B (p_1 + p_2)^\mu $$

(B.2)
The form factors can be recovered by contracting the amplitude with \( p_1^\mu \) and \((p_1 + p_2)^\mu\) respectively,

\[
p_1^\mu I_\mu = \frac{1}{2} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 (k + p_1)^2 (k + p_1 + p_2)^2} - \frac{1}{2} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k + p_1)^2 (k + p_1 + p_2)^2}
\]

\[
= \frac{1}{2} B_0 ((p_1 + p_2)^2) - \frac{1}{2} B_0 (p_2^2)
\]

\[
(p_1 + p_2)^\mu I_\mu = \frac{1}{2} \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 (k + p_1)^2 (k + p_1 + p_2)^2} - \frac{1}{2} \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k + p_1)^2 (k + p_1 + p_2)^2}
\]

\[
- \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 (k + p_1)^2 (k + p_1 + p_2)^2}
\]

\[
= \frac{1}{2} B_0 (p_2^2) - \frac{1}{2} B_0 (p_2^2)
\]

The form factors can now be expressed in terms of these \( B \) functions,

\[
\frac{1}{2} (B_0 ((p_1 + p_2)^2) - B_0 (p_2^2)) = p_{1\mu} I_\mu = p_{1\mu} (A p_1^\mu + B (p_1 + p_2)^\mu)
\]

\[
= A p_1^2 + B (p_1^2 + p_1 \cdot p_2)
\]

\[
B = \frac{B_0 ((p_1 + p_2)^2) - B_0 (p_2^2)}{m^2}
\]

\[
\frac{1}{2} (B_0 (p_1^2) - B_0 (p_2^2)) = (A + B) p_1 \cdot p_2
\]

\[
A + B = \frac{B_0 (p_1^2) - B_0 (p_2^2)}{m^2}
\]

where \( m \) is the mass of the incoming particle.

The form factors of the \( h \to \gamma \gamma \) decay can in the same way be projected out. The amplitudes for this decay are associated with polarization vectors of the photons. The general structure of the amplitude is given by

\[
T^{\mu\nu} \epsilon_\mu (p_1) \epsilon_\nu (p_2) = (p_1^\mu p_1^\nu T_1 + p_2^\mu p_2^\nu T_2 + p_1^\mu p_2^\nu T_3 + p_1^\mu p_2^\nu T_4 + p_1 \cdot p_2 g^{\mu\nu} T_5
\]

\[
+ \epsilon^{\mu\nu\rho\sigma} p_1^\rho p_2^\sigma T_6)
\]

\[
\epsilon_\mu (p_1) \epsilon_\nu (p_2)
\]

Assuming the decay to be CP-even, the \( T_6 \) term can be neglected. From Abelian gauge invariance, it follows that \( T_1 = T_2 = 0 \) and \( T_4 = -T_5 \). Finally, \( T_3 \) does not contribute for onshell photons, since in this case, \( p_1^\mu \epsilon_{1\mu} = p_2^\mu \epsilon_{2\mu} = 0 \). The only form factors needed for the \( h \to \gamma \gamma \) decay are therefore \( T_4 \) and \( T_5 \). Using the following projector,

\[
P_{\mu\nu} = g_{\mu\nu} - \frac{2}{s_{12}} (p_{1\mu} p_{2\nu} + p_{2\mu} p_{1\nu})
\]

\[
T^{\mu\nu} \epsilon_\mu (p_1) \epsilon_\nu (p_2)
\]
the $T_5$ part of the Passarino-Veltman decomposition can be extracted,

$$p_{\mu \nu} T_{\mu \nu} = \frac{2}{(d-2) p^2} \left( p_2 \cdot p_3 T_4 + d p_2 \cdot p_3 T_5 - \frac{2}{s_{12}} ((p_2 \cdot p_3)^2 T_4 + 2 (p_2 \cdot p_3)^2 T_5) \right)$$

$$= \frac{2}{(d-2) p^2} \left( p_2 \cdot p_3 (T_4 + d T_5 - T_4 - 2 T_5) \right) = T_5. \quad (B.7)$$

To project out $T_4$, the following projector can be used

$$Q_{\mu \nu} = \frac{4}{p^4} \left( \frac{d - 1}{d - 2} p_2 \mu p_3 \nu + \frac{1}{d - 2} p_2 \nu p_3 \mu \right) - \frac{2}{(d-2) p^2} g_{\mu \nu}. \quad (B.8)$$

These projections are carried out in FORM.

### B.3 Passarino-Veltman Reduction

When performing Passarino-Veltman reductions on loop integrals, tensor integrals are reduced to scalar integrals, or so-called master integrals [109], simplifying the integrals.

The following identities are implemented in the FORM code in the case where $p = p_2 + p_3$ and $p_2$ and $p_3$ are the external momenta, $q$ is the integral momentum, and where $i, j = 2, 3$, $i \neq j$. Double indices are not to be summed over,

\begin{align*}
\frac{(q \cdot q)^2}{(q^2 - m_1^2)((q-p)^2 - m_2^2)} &= \left( \frac{q^2 + m_1^2}{(q-p)^2 - m_2^2} + \frac{m_1^2}{(q^2 - m_1^2)((q-p)^2 - m_2^2)} \right), \quad (B.8) \\
\frac{q \cdot q}{(q^2 - m_1^2)} &= \left( 1 + \frac{m_1^2}{(q^2 - m_1^2)} \right), \\
\frac{(q \cdot p)}{(q^2 - m_1^2)((q-p)^2 - m_2^2)} &= -\frac{1}{2} \left( \frac{1}{(q^2 - m_1^2)} - \frac{1}{(q-p)^2 - m_2^2} - \frac{p \cdot p + m_1^2 - m_2^2}{(q^2 - m_1^2)((q-p)^2 - m_2^2)} \right), \\
\frac{(q \cdot p_i)}{(q^2 - m_1^2)((q-p_i)^2 - m_2^2)} &= -\frac{1}{2} \left( \frac{1}{(q^2 - m_1^2)} - \frac{1}{(q-p_i)^2 - m_2^2} - \frac{p_i \cdot p_i + m_1^2 - m_2^2}{(q^2 - m_1^2)((q-p_i)^2 - m_2^2)} \right), \\
\frac{(q \cdot p_i)}{(q^2 - m_1^2)((q-p)^2 - m_2^2)} &= -\frac{1}{2} \left( \frac{1}{(q^2 - m_1^2)} - \frac{1}{(q-p)^2 - m_2^2} - \frac{p_i \cdot p_i + m_1^2 - m_2^2}{(q^2 - m_1^2)((q-p)^2 - m_2^2)} \right), \\
\frac{1}{((q-p)^2 - m_1^2)} &= \frac{1}{((q-p)^2 - m_1^2)} - \frac{1}{((q-p)^2 - m_2^2)} + \frac{p_i \cdot p_i + 2p_i \cdot p_j - m_1^2 + m_2^2}{((q-p)^2 - m_1^2)((q-p)^2 - m_2^2)}, \quad (B.9)
\end{align*}
B.4 Feynman Parametrization

The loop integrals are written in such a way as to combine the product of denominators into a single denominator, in a Feynman parametrization [151, 152],

\[
\frac{N}{D_1^{\nu_1} D_2^{\nu_2} \ldots D_N^{\nu_N}} = \frac{\Gamma(\sum_{i=1}^{N} \nu_i)}{\prod_{i=1}^{N} \Gamma(\nu_i)} \frac{\delta(1 - \sum_{j=1}^{N} z_j) N}{[z_1 D_1 + z_2 D_2 + \ldots + z_N D_N]^{\sum_{i=1}^{N} \nu_i}}, \tag{B.10}
\]

with \( z_i \) the Feynman parameters and \( D_i = ((q - p_i)^2 - m_i^2) \). When carrying out the integral over one of the Feynman parameters, \( z_N \), the delta functions set \( z_N = 1 - (\sum_{i=1}^{N-1} z_i) \). In the case of having a one-loop integral, expanding the denominator and completing the square for the part of the denominator containing the integral momentum, the integral is shifted, \( q \rightarrow l + \ldots \), both in the numerator and the denominator, such that the form of the integral becomes,

\[
\int \frac{d^dl}{(2\pi)^d} \frac{(l^2)^\alpha}{(l^2 - \Delta)^n}. \tag{B.11}
\]

B.5 Integrals in Dimensional Regularization

Using DREG, the solution can be found for the following group of integrals

\[
\int \frac{d^dl}{(2\pi)^d} \frac{(l^2)^\alpha}{(l^2 - \Delta + i\delta)^n}, \tag{B.12}
\]

where \( l^2 = \sum_{i=0}^{3} l_i^2 \). In dimensional regularisation, the integration can be carried out over \( l_0 \) first [153]. The propagators are expressed as

\[
\frac{1}{(l_0^2 - \Delta + i\delta)^n}, \tag{B.13}
\]

and contain poles at \( l_0^2 = \Delta - i\delta \). The function of the \( i\delta \) term is therefore to shift the poles into the imaginary plane. On the real \( l_0 \) axis, see figure B.1, the poles are shifted above and below the axis by \( i\delta \) respectively. At this point, it is useful to shift to Euclidian space, where \( l_E^2 = \sum_{i=1}^{4} l_i^2 \). This results in the following redefinition, \( l_0 \rightarrow il_4 \) leading to \( l^2 \rightarrow -l_E^2 = l_4^2 + l^2 \). The integration contour in the complex \( l_0 \) plane is therefore Wick rotated by 90 degrees. After this rotation, the contour no longer encloses any poles. Applying Cauchy’s integral formula

\[
f(z_0) = \frac{1}{2\pi i} \oint_{\gamma} f(z) \frac{dz}{z - z_0}, \tag{B.14}
\]

to the contour, results in the integrand vanishing as \( |l_0| \rightarrow \infty \) and the integration over the real \( l_0 \) can be expressed as the integral over the imaginary axis.

\[
\int_{-\infty}^{\infty} \frac{dl_0}{2\pi} \frac{(l_0^2)^\alpha}{(l_0^2 - \Delta + i\delta)^n} = i(-1)^{n+\alpha} \int_{-\infty}^{\infty} \frac{dl_E^2}{2\pi} \frac{(l_E^2)^\alpha}{((l_E^2)^2 + \Delta)^n}. \tag{B.15}
\]

The \( i\delta \) part in the original integral vanishes for \( \Delta \neq 0 \) and the integral becomes
B.5. Integrals in Dimensional Regularization

\[ i(-1)^{n+\alpha} \int \frac{d^dl_E}{(2\pi)^d} \frac{(l_E^2)^\alpha}{(l_E^2 + \Delta)^n}, \]

where \( l_E^2 = (l_E^0)^2 + \Delta^2 \). The integral over a d-dimensional Euclidean space can be carried out rewriting the integral in terms of spherical coordinates [153],

\[ i(-1)^{n+\alpha} \int \frac{d^dl_E}{(2\pi)^d} \frac{(l_E^2)^\alpha}{(l_E^2 + \Delta)^n} = i(-1)^{n+\alpha} \int \frac{d\Omega_d}{(2\pi)^d} \int_0^\infty dl_E \frac{l_E^{d-1+2\alpha}}{(l_E^2 + \Delta)^n}. \]

Using that \( \int d\Omega_d = \frac{2\pi^{d/2}}{\Gamma[d/2]} \) and \( dl_E = \frac{d(l_E^2)}{2l_E} \) and changing the definition \( l_E \rightarrow l \),

\[ i(-1)^{n+\alpha} \frac{2}{(4\pi)^{d/2}} \frac{1}{\Gamma[d/2]} \int_0^\infty dl^2 \frac{1}{2} \frac{(l^2)^{d/2+\alpha-1}}{(l^2 + \Delta)^n}. \]

Substituting \( x = \frac{\Delta}{l^2 + \Delta} \),

\[ i(-1)^{n+\alpha} \frac{1}{(4\pi)^{d/2}} \frac{1}{\Gamma[d/2]} \left( \frac{1}{\Delta} \right)^{n-d/2-\alpha} \int_0^1 dx x^{n-d/2-\alpha-1} (1-x)^{d/2+\alpha-1}. \]

The definition of the Beta function

\[ \int_0^1 dx x^\alpha (1-x)^\beta = B(\alpha + 1, \beta + 1) = \frac{\Gamma[\alpha + 1] \Gamma[\beta + 1]}{\Gamma[\alpha + \beta + 2]}, \]

gives the solution to the integral [32],

\[ \int \frac{d^dl}{(2\pi)^d} \frac{(l^2)^\alpha}{(l^2 - \Delta)^n} = i \frac{(-1)^{n+d/2}}{(4\pi)^{d/2}} \frac{\Gamma[\alpha + \beta + 2]}{\Gamma[n]}. \]

\[ \frac{\Gamma[d/2] \Gamma[n]}{\Gamma[\alpha + d/2] \Gamma[n - \alpha - d/2]}. \]
B.5. Integrals in Dimensional Regularization
The following convention is used for the propagators in the $R_\xi$ gauge,

\begin{align}
W : & \quad -i \frac{1}{q^2 - m_W^2} (g_{\mu \nu} - (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi m_W^2}) \\
Z : & \quad -i \frac{1}{q^2 - m_Z^2} (g_{\mu \nu} - (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi m_Z^2}) \\
\psi : & \quad \frac{i (q^2 - m_\psi)}{q^2 - m_\psi^2} \\
\phi_{\pm} : & \quad \frac{i}{q^2 - \xi m_\phi^2} \\
\phi_0 : & \quad \frac{i}{q^2 - \xi m_\phi^2} \\
h : & \quad \frac{i}{q^2 - m_h^2} \tag{C.1}
\end{align}

The Feynman rules are listed in the following in $R_\xi$ gauge, using the BFM in the SM. They are given for the various interactions relevant for calculations in this thesis. All fields are taken to be outgoing.
\[ \phi^+ \phi^- \hat{A}_\alpha \hat{A}_\beta: \]
\[ = e^2 g_{\alpha \beta} \]

\[ h W^+_{\mu} W^-_{\nu}: \]
\[ = g_2 m_D g_{\mu \nu} \]

\[ h \hat{Z}_\mu Z_\nu: \]
\[ = 2 \frac{g_2^2}{v_D^2} g_{\mu \nu} \]

\[ h \hat{Z}_\alpha \hat{Z}_\beta: \]
\[ = \frac{m_D^2}{v_D^2} g_{\alpha \beta} \]

\[ \hat{Z}_\alpha W^\pm_{\mu} \phi^\mp: \]
\[ = \pm i \frac{m_D}{v_D} g_2 g_{\mu \alpha} \]

\[ W^+_{\mu} W^-_{\nu} \hat{A}_\alpha: \]
\[ = -e \left( (p_b - p_c)_\alpha g_{\mu \nu} + \left( \frac{1}{2} p_b + p_c - p_a \right)_\mu g_{\nu \alpha} \right. \]
\[ \left. + (p_a - p_b - \frac{1}{2} p_c)_\nu g_{\mu \alpha} \right) \]
Feynman Rules in the Background Field Method

\[ W^+ W^- \hat{Z}_\alpha : \]

\[ W^+ W^- \hat{A}_\alpha \hat{A}_\beta : \]

\[ \phi^+ W^\pm \hat{h} : \]

\[ \phi^+ \phi^- \hat{A}_\alpha : \]

\[ h \phi_0 \hat{Z}_\alpha : \]

\[ \phi^+ \phi^- \hat{Z}_\alpha : \]

\[ W^+ W^- \hat{Z}_\alpha : \]

\[ W^+ W^- \hat{A}_\alpha \hat{A}_\beta : \]

\[ \phi^+ W^\pm \hat{h} : \]

\[ \phi^+ \phi^- \hat{A}_\alpha : \]

\[ h \phi_0 \hat{Z}_\alpha : \]

\[ \phi^+ \phi^- \hat{Z}_\alpha : \]

\[ = -c_w g_2 \left( (p_b - p_c)\alpha g_{\mu,\nu} + \left( \frac{1}{\xi} p_b + p_c - p_a \right)_{\mu,\nu} \right) \]

\[ + (p_a - p_b - \frac{1}{\xi} p_c)_{\nu,\nu} g_{\mu,\alpha} \]

\[ = -e^2 \left( 2 g_{\alpha,\beta\nu} - \frac{1}{2} \left( g_{\mu,\alpha} g_{\nu,\beta} + g_{\mu,\beta} g_{\nu,\alpha} \right) \right) \]

\[ = -i g_2 p_{a,\mu} \]

\[ = e (p_a - p_b)_{\alpha} \]

\[ = i \frac{m_\phi}{\nu} (p_a - p_b)_{\alpha} \]

\[ = \frac{m_\phi}{\nu} \left( c_w^2 - s_w^2 \right) (p_a - p_b)_{\alpha} \]
\[ \bar{u}_\pm u_\pm \hat{A}_\alpha : \]
\[ = \pm e (p_a - p_b)_\alpha \]

\[ \bar{u}_\pm u_\pm \hat{A}_\alpha \hat{A}_\beta : \]
\[ = 2 e^2 g_{\alpha\beta} \]

\[ \bar{\psi} \psi Z_\alpha : \]
\[ = -2 \frac{m_z}{v} \gamma_\alpha \left( \pm \frac{1}{4} (1 - \gamma_5) + Q_\psi s^2 \right) \]

\[ \bar{\psi} \psi A_\alpha : \]
\[ = -e Q_\psi \gamma_\alpha \]

\[ u_0 u_0 \hat{h} : \]
\[ = -2 \xi \frac{m^2}{v} \]

\[ \bar{u}_\pm u_\pm \hat{h} : \]
\[ = -2 \xi \frac{m^2_{\bar{u}u}}{v} \]
Figure C.1: Feynman rules in the SM using the BFM.
Identities for Gamma Matrices

\[ \gamma^\mu \gamma_\mu = d\mathbf{I} \]
\[ \gamma^\mu \gamma^\alpha \gamma_\mu = (2 - d) \gamma^\alpha \]
\[ \gamma^\mu \gamma^\alpha \gamma^\beta \gamma_\mu = 4g^{\alpha\beta} \mathbf{I} + (d - 4) \gamma^\alpha \gamma^\beta \]
\[ \gamma^\mu \gamma^\alpha \gamma^\beta \gamma^\rho \gamma_\mu = -2\gamma^\rho \gamma^\beta \gamma^\alpha + (4 - d) \gamma^\alpha \gamma^\beta \gamma^\rho \]
\[ \text{Tr} \mathbf{I} = 4 \]
\[ \text{Tr} \gamma^\mu \gamma^\nu = 4g^{\mu\nu} \]
\[ \text{Tr} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma = 4(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} + g^{\nu\rho} g^{\mu\sigma}) \quad (D.1) \]
In the following, the amplitudes are written up for the one-loop $h \rightarrow \gamma \gamma$ decay in the SM, as seen in figure 6.1, using $d = 4 - 2\varepsilon$ and the BFM. The incoming momentum of the Higgs fields is given by the sum of the outgoing momenta of the photon fields $p = p_2 + p_3$. The on-shell conditions, $p^2 = m_h^2$ and $p_2^2 = p_3^2 = 0$ are taken to hold for the calculation of the S matrix element.

\[
i A_a = g_2^2 e^2 v g_{\mu \omega} \left( \frac{\mu^2 e^{\gamma_E}}{4\pi} \right)^\varepsilon \int \frac{d^d q}{(2\pi)^d} e^\alpha(p_2) e^\beta(p_3) \\
\times \left[ (2q - p - p_2)\beta g_{\sigma \tau} + \frac{1}{\xi} (q - p - q + p_2 - p_3)\tau g_{\beta \sigma} + (\frac{1}{\xi} q - p_2 - q + p + p_3)\sigma g_{\beta \tau} \right] \\
\times \left[ (2q - p_2)\alpha g_{\nu \rho} + \frac{1}{\xi} (q - p_2 - q - p_2)\rho g_{\alpha \nu} + (\frac{1}{\xi} q - q + 2 p_2)\nu g_{\alpha \rho} \right] \\
\times \left( g_{\mu \nu} - (1 - \xi) \frac{g_{\mu \nu}}{q^2 - \xi m_w^2} \right) \left( g_{\rho \sigma} - (1 - \xi) \frac{(q - p_2)\rho (q - p_2)\sigma}{(q - p_2)^2 - \xi m_w^2} \right) \\
\times \frac{1}{q^2 - m_w^2} \frac{1}{(q - p_2)^2 - m_w^2} \frac{1}{(q - p)^2 - m_w^2}
\]

\[
i A_b = \frac{1}{2} g_2^2 e^2 v g_{\mu \sigma} \left( \frac{\mu^2 e^{\gamma_E}}{4\pi} \right)^\varepsilon \int \frac{d^d q}{(2\pi)^d} e^\alpha(p_2) e^\beta(p_3) \\
\times \left[ (1 - \xi) g_{\beta \nu} g_{\alpha \rho} + (1 - \xi) g_{\beta \rho} g_{\alpha \nu} - 2 g_{\alpha \beta} g_{\rho \sigma} \right] \\
\times \left( g_{\mu \nu} - (1 - \xi) \frac{g_{\mu \nu}}{q^2 - \xi m_w^2} \right) \left( g_{\rho \sigma} - (1 - \xi) \frac{(q - p)\rho (q - p)\sigma}{(q - p)^2 - \xi m_w^2} \right)
\]

(E.1)
The Standard Model One-Loop $h \to \gamma \gamma$ Decay Amplitude

\[ i A_c = 4 e^2 v (\lambda + \frac{1}{4} \xi g_2^2) \left( \frac{\mu^2 e^{\gamma_E}}{4 \pi} \right)^\epsilon \int \frac{d^d q}{(2\pi)^d} e^\alpha(p_2) e^\beta(p_3) \frac{1}{q^2 - \xi m_w^2} \]

\[ \times \frac{1}{(q - p_2)^2 - \xi m_w^2} \frac{1}{(q - p)^2 - \xi m_w^2} (2q - p_2)_\alpha (2q - p - p_2)_\beta \]

\[ i A_d = -4 e^2 v \left( \lambda + \frac{1}{4} \xi g_2^2 \right) \left( \frac{\mu^2 e^{\gamma_E}}{4 \pi} \right)^\epsilon \int \frac{d^d q}{(2\pi)^d} e^\alpha(p_2) e^\beta(p_3) g_\alpha g_\beta \]

\[ \times \frac{1}{q^2 - \xi m_w^2} \frac{1}{(q - p)^2 - \xi m_w^2} \]

\[ i A_e = 2 m_\psi e^2 Q_\psi^2 N_c \left( \frac{\mu^2 e^{\gamma_E}}{4 \pi} \right)^\epsilon \int \frac{d^d q}{(2\pi)^d} e^\alpha(p_2) e^\beta(p_3) \frac{1}{q^2 - m_\psi^2} \frac{1}{(q - p_2)^2 - m_\psi^2} \]

\[ \times \frac{1}{(q - p)^2 - m_\psi^2} \text{Tr} \left[ (q - m_\psi) \gamma_\alpha (q - p_2 - m_\psi) \gamma_\beta (q - \psi - m_\psi) \right] \]

\[ i A_f = -2 e^2 v \xi g_2^2 \left( \frac{\mu^2 e^{\gamma_E}}{4 \pi} \right)^\epsilon \int \frac{d^d q}{(2\pi)^d} e^\alpha(p_2) e^\beta(p_3) (2q - p_2)_\alpha (2q - p - p_2)_\beta \]

\[ \times \frac{1}{q^2 - \xi m_w^2} \frac{1}{(q - p_2)^2 - \xi m_w^2} \frac{1}{(q - p)^2 - \xi m_w^2} \]

\[ i A_g = 2 e^2 v \xi g_2^2 \left( \frac{\mu^2 e^{\gamma_E}}{4 \pi} \right)^\epsilon \int \frac{d^d q}{(2\pi)^d} e^\alpha(p_2) e^\beta(p_3) g_\alpha g_\beta \frac{1}{q^2 - \xi m_w^2} \frac{1}{(q - p)^2 - \xi m_w^2} \]
The amplitudes in the BFM for the Higgs self-energy are written up as follows in appendix C. The integrals are calculated using the program as explained in appendix B. The divergent parts of the amplitudes for the Higgs self-energy will contribute to the Higgs mass and field renormalizations, $Z_h$ and $Z_m$. Only the field renormalization will enter the renormalization of the $h \to \gamma \gamma$ decay in the SMEFT and the BFM. The diagrams contributing to the Higgs self-energy can be seen in figure 6.6. In the BFM, the Feynman rules are given in appendix C. The integrals are calculated using the program as explained in appendix B.

The amplitudes in the BFM for the Higgs self energy are written up as follows

\[
\begin{align*}
i_A &= 3\lambda \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_h^2} + \left(\lambda + \frac{1}{4}(g_1^2 + g_2^2)\xi\right) \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - \xi m_w^2}, \\
i_B &= (2\lambda + \frac{1}{2}g_2^2\xi) \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_w^2}, \\
i_C &= \frac{1}{4}(g_1^2 + g^2)g_{\mu\nu}\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_w^2} (g_{\mu\nu} - (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi m_w^2}), \\
i_D &= \frac{1}{2}g_2^2 g_{\mu\nu}\left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_w^2} (g_{\mu\nu} - (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi m_w^2}), \\
i_E &= \frac{1}{2} g^2 m_w^2 \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_w^2} \text{Tr}((g + m_\psi)(g - p + m_\psi) \\
&+ 2(\lambda v + \frac{1}{4}(g_1^2 + g_2^2)\xi v)^2 \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - \xi m_Z^2}((q - p)^2 - \xi m_Z^2)), \\
i_F &= \frac{18}{2} g^2 v^2 \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_h^2} \frac{1}{(q - p)^2 - m_h^2} \\
&+ 2(\lambda v + \frac{1}{4}(g_1^2 + g_2^2)\xi v)^2 \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - \xi m_Z^2}((q - p)^2 - \xi m_Z^2)), \\
i_G &= (2\lambda v + \frac{1}{2} g_2^2 v)^2 \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - \xi m_Z^2}((q - p)^2 - \xi m_Z^2)), \\
i_H &= -(g_1^2 + g^2)\xi^2 m_Z^2 \left(\frac{\mu^2 e^{\gamma_E}}{4\pi}\right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - \xi m_Z^2}((q - p)^2 - \xi m_Z^2)),
\end{align*}
\]
The divergent part of these amplitudes are given by

\[ i A_i = -2g_2^2\xi^2 m_w^2 \left( \frac{\mu^2 e^{7\xi}}{4\pi} \right)^\epsilon \int \frac{d^dq}{(2\pi)^d} \frac{1}{q^2 - \xi m_w^2} \frac{1}{(q - p)^2 - \xi m_w^2}, \]

\[ i A_j = g_2^2 m_w^2 g_{\mu\rho} g_{\nu\sigma} \left( \frac{\mu^2 e^{7\xi}}{4\pi} \right)^\epsilon \int \frac{d^dq}{(2\pi)^d} \frac{1}{q^2 - m_w^2} \frac{1}{(q - p)^2 - m_w^2} \times \left( g_{\mu\nu} - (1 - \xi) \frac{g_{\mu\nu}}{q^2 - \xi m_w^2} \right) \left( g_{\rho\sigma} - (1 - \xi) \frac{(q - p)_\rho (q - p)_\sigma}{(q - p)^2 - \xi m_w^2} \right), \]

\[ i A_k = \frac{1}{2}(g_1^2 + g_2^2) m_w^2 g_{\mu\rho} g_{\nu\sigma} \int \frac{d^dq}{(2\pi)^d} \frac{1}{q^2 - m_w^2} \frac{1}{(q - p)^2 - m_w^2} \times \left( g_{\mu\nu} - (1 - \xi) \frac{g_{\mu\nu}}{q^2 - \xi m_w^2} \right) \left( g_{\rho\sigma} - (1 - \xi) \frac{(q - p)_\rho (q - p)_\sigma}{(q - p)^2 - \xi m_w^2} \right), \]

\[ i A_l = -g_2^2 \left( \frac{\mu^2 e^{7\xi}}{4\pi} \right)^\epsilon \int \frac{d^dq}{(2\pi)^d} p_{\mu\rho} p_{\nu\sigma} \frac{1}{q^2 - m_w^2} \frac{1}{(q - p)^2 - \xi m_w^2} \times \left( g_{\mu\rho} - (1 - \xi) \frac{g_{\mu\rho}}{q^2 - \xi m_w^2} \right) \left( g_{\nu\sigma} - (1 - \xi) \frac{(q - p)_\nu (q - p)_\sigma}{(q - p)^2 - \xi m_w^2} \right), \]

\[ i A_m = -(g_1^2 + g_2^2) \left( \frac{\mu^2 e^{7\xi}}{4\pi} \right)^\epsilon \int \frac{d^dq}{(2\pi)^d} p_{\mu\rho} p_{\nu\sigma} \frac{1}{q^2 - m_w^2} \frac{1}{(q - p)^2 - \xi m_w^2} \times \left( g_{\mu\nu} - (1 - \xi) \frac{g_{\mu\nu}}{q^2 - \xi m_w^2} \right) \left( g_{\rho\sigma} - (1 - \xi) \frac{(q - p)_\rho (q - p)_\sigma}{(q - p)^2 - \xi m_w^2} \right), \]

\[ i A_n = -\frac{1}{2}(g_1^2 + g_2^2)\xi \left( \frac{\mu^2 e^{7\xi}}{4\pi} \right)^\epsilon \int \frac{d^dq}{(2\pi)^d} \frac{1}{q^2 - \xi m_w^2}, \]

\[ i A_o = -g_2^2 \xi \int \frac{d^dq}{(2\pi)^d} \frac{1}{q^2 - \xi m_w^2}. \]
Higgs Field Renormalization

\[ \mathcal{A}^{\text{div}}_i = -\frac{1}{16\pi^2\epsilon} 2g_2^2m_w^2\xi^2, \]
\[ \mathcal{A}^{\text{div}}_j = \frac{1}{16\pi^2\epsilon} g_2^2m_w^2 (\xi^2 + 3), \]
\[ \mathcal{A}^{\text{div}}_k = \frac{1}{16\pi^2\epsilon} \frac{1}{2} m_z^2 (\xi^2 + 3) (g_1^2 + g_2^2), \]
\[ \mathcal{A}^{\text{div}}_l = -\frac{1}{16\pi^2\epsilon} \frac{1}{4} g_2^2 (\xi + 3) p^2, \]
\[ \mathcal{A}^{\text{div}}_m = -\frac{1}{16\pi^2\epsilon} \frac{1}{8} (\xi + 3) p^2 (g_1^2 + g_2^2), \]
\[ \mathcal{A}^{\text{div}}_n = -\frac{1}{32\pi^2\epsilon} \frac{g_2^2 m_w^2 \xi^2}{g_w^2}, \]
\[ \mathcal{A}^{\text{div}}_o = -\frac{1}{16\pi^2\epsilon} \frac{g_2^2 m_w^2 \xi^2}{\epsilon}. \]  

(F.2)

The divergent part of the Higgs self-energy sums up to,

\[ \mathcal{A}_{\text{tot}} = \frac{1}{16\pi^2\epsilon} \frac{1}{2} \left( 15\lambda m_h^2 + \frac{3g_2^2}{4c_w^2} (3 + \xi^2) m_z^2 + \frac{3g_2^2}{2} (3 + \xi^2) m_w^2 \right. \]
\[ \left. - \frac{m_\psi^2}{v^2} (9m_\psi^2 - 2p^2) - \frac{1}{4} (g_1^2 + 3g_2^2)(3 + \xi) p^2 - \frac{3}{8} \xi (g_1^2 + 3g_2^2) m_h^2 \right). \]  

(F.3)

The renormalization constant \( Z_h \) of the Higgs wavefunction is constructed to cancel the divergence in the above result proportional to \( p^2 \). Therefore, in the BFM,

\[ Z_h = 1 + \frac{3 + \xi (g_1^2 + 3g_2^2)}{64\pi^2\epsilon} - \frac{Y}{16\pi^2\epsilon}, \]  

(F.4)

where

\[ Y = \text{Tr} \left[ 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d + Y_e^\dagger Y_e \right]. \]  

(F.5)

When extracting the finite part of the amplitude, the following definitions will be implemented,
\[ I[m^2] \equiv \int_0^1 dx \log \left( \frac{m^2 - m_H^2 x (1-x)}{m_H^2} \right), \]
\[ I^\xi[m^2] \equiv \int_0^1 dx \log \left( \frac{m^2(1-x(1-\xi)) - m_H^2 x (1-x)}{m_H^2} \right), \]
\[ I^{\xi\xi}[m^2] \equiv \int_0^1 dx \log \left( \frac{m^2(\xi - \xi(\xi - 1)) - m_H^2 x (1-x)}{m_H^2} \right), \]
\[ I_{xx}[m^2] \equiv \int_0^1 dx \frac{m^2}{m^2 - m_H^2 x (1-x)}, \]
\[ I^\xi_{xx}[m^2] \equiv \int_0^1 dx \frac{m^2}{m^2(1-x(1-\xi)) - m_H^2 x (1-x)}, \]
\[ I^{\xi\xi}_{xx}[m^2] \equiv \int_0^1 dx \frac{m^2}{m^2(\xi - \xi(\xi - 1)) - m_H^2 x (1-x)}, \]
\[ I^{\xi\xi}_{xx}[m^2] \equiv \int_0^1 dx \frac{x^2 m^2}{m^2(\xi - \xi(\xi - 1)) - m_H^2 x (1-x)}, \]
\[ I^{\xi\xi}_{xx2}[m^2] \equiv \int_0^1 dx \frac{x^2 m^2}{m^2(\xi - \xi(\xi - 1)) - m_H^2 x (1-x)}. \] (F.6)

As can be seen, setting \( \xi = 1 \) leads to \( I^{\xi\xi}[m^2] = I^\xi[m^2] = I[m^2] \), \( I^{\xi\xi}_{xx}[m^2] = I^{\xi\xi}_{xx}[m^2] = I_{xx}[m^2] = I_{xx}[m^2] \) and \( I^{\xi\xi}_{xx2}[m^2] = I^{\xi\xi}_{xx2}[m^2] \). This encourages the choice of the Feynman gauge, resulting in simpler looking expressions.

In Feynman gauge, setting \( \xi = 1 \), the finite part of the amplitude for the Higgs self energy is given by
Higgs Field Renormalization

\( A_{\text{fin}}^a = m_z^2 \left( \frac{1}{4} (g_1^2 + g_2^2) \left( 1 - \log \left( \frac{m_h^2}{\mu^2} \right) \right) + \lambda \left( 1 - \log \left( \frac{m_h^2}{\mu^2} \right) \right) \right) + 3 \lambda m_h^2 \left( 1 - \log \left( \frac{m_h^2}{\mu^2} \right) \right), \)

\( A_{\text{fin}}^b = m_w^2 \left( \frac{1}{2} g_2^2 \left( 1 - \log \left( \frac{m_w^2}{\mu^2} \right) \right) + 2 \lambda \left( 1 - \log \left( \frac{m_w^2}{\mu^2} \right) \right) \right), \)

\( A_{\text{fin}}^c = \frac{1}{2} m_z^2 (g_1^2 + g_2^2) \left( 1 - 2 \log \left( \frac{m_z^2}{\mu^2} \right) \right), \)

\( A_{\text{fin}}^d = g_2^2 m_w^2 \left( 1 - 2 \log \left( \frac{m_w^2}{\mu^2} \right) \right), \)

\( A_{\text{fin}}^e = 2 m^2 \mu^2 Y^2 \left( 1 - \log \left( \frac{m^2_w}{\mu^2} \right) - 2 I[m_\psi^2] \right) - p^2 Y^2 I[m_\psi^2], \)

\( A_{\text{fin}}^f = -\frac{1}{2} \left( m_h^2 \left( g_1^2 + g_2^2 + 2\lambda \right) + m_w^2 \left( g_1^2 + g_2^2 \right) + g_t^2 m_z^2 \right) I[m_z^2] \)

\( - 9 \lambda m_h^2 I[m_\psi^2], \)

\( A_{\text{fin}}^g = -\left( g_2^2 m_w^2 + 2 \lambda \left( m_h^2 + 4 m_w^2 \right) \right) I[m_w^2], \)

\( A_{\text{fin}}^h = m_z^2 \left( g_1^2 + g_2^2 \right) I[m_z^2], \)

\( A_{\text{fin}}^i = 2 g_2^2 m_w^2 I[m_w^2], \)

\( A_{\text{fin}}^j = -2 g_2^2 m_w^2 \left( 1 + 2 I[m_w^2] \right), \)

\( A_{\text{fin}}^k = -m_z^2 \left( g_1^2 + g_2^2 \right) \left( 1 + 2 I[m_z^2] \right), \)

\( A_{\text{fin}}^l = 2 g_2^2 p^2 I[m_w^2], \)

\( A_{\text{fin}}^m = p^2 \left( g_1^2 + g_2^2 \right) I[m_z^2], \)

\( A_{\text{fin}}^n = -\frac{1}{2} m_z^2 \left( g_1^2 + g_2^2 \right) \left( 1 - I[m_z^2] \right), \)

\( A_{\text{fin}}^o = -g_2^2 m_w^2 \left( 1 - I[m_w^2] \right). \)

(F.7)

Summing up the above contributions to the Higgs self energy and using equation (5.30), the R factor in Feynman gauge is given in equation (6.73). The R factor in the \( R_\xi \) gauge is given below.
\[ \delta R_h^\xi = \frac{1}{2} \left( 8 \lambda - g_2^2 (1 + \xi) \right) \left( \mathcal{I}_x^\xi \left[ m_w^2 \right] - \mathcal{I}_x^{\xi \xi} \left[ m_w^2 \right] \right) + \left( 2 \lambda + g_2 \frac{3m_w^2}{m_h^2} \right) \left( 1 - \mathcal{I}_{xx} \left[ m_w^2 \right] \right) \]

\[ + 4 \mathcal{I}_{xx} \left[ m_w^2 \right] g_2^2 + \frac{1}{2} \left( \xi \log \left( \frac{m_w^2}{m_h^2} \right) - \log \left( \frac{m_w^2}{m_h^2} \right) \right) \]

\[ + \frac{1}{4} \left( 1 - \xi \right)^2 - \frac{2m_h^2(1 + \xi)}{m_w^2} \right) \left( \mathcal{I}_x^{\xi \xi} \left[ m_w^2 \right] - \mathcal{I}_{xx} \left[ m_w^2 \right] \right) \]

\[ + \lambda \left( \frac{1}{2} \left( g_1^2 + g_2^2 \right) \xi \right) \left( \mathcal{I}_x^{\xi \xi} \left[ m_w^2 \right] - \mathcal{I}_{xx} \left[ m_w^2 \right] \right) \]

\[ + \frac{1}{2} \left( g_1^2 + g_2^2 \right) \mathcal{I}_{xx} \left[ m_w^2 \right] + \left( \frac{g_2^2}{8m_h^3 m_w^2} \left( \frac{3m_h^2}{m_h^2} \right) \left( 1 - \xi^2 \right) - 2\lambda(1 + \xi) \right) \left( \mathcal{I}_x^{\xi \xi} \left[ m_w^2 \right] - \mathcal{I}_{xx} \left[ m_w^2 \right] \right) \]

\[ + \left( m_h^2 + m_w^2(1 - \xi) \right) \mathcal{I}_x^{\xi \xi} \left[ m_w^2 \right] - \left( m_h^2 - m_w^2(1 - \xi) \right) \mathcal{I}_{xx} \left[ m_w^2 \right] \]

\[ + \left( \frac{g_1^2 + g_2^2}{16m_h^2 m_w^2} \left( 2m_h^2(1 + \xi) - m_h^2(1 - \xi)^2 \right) - \lambda \right) \]

\[ + \left( m_h^2 + m_w^2(1 - \xi) \right) \mathcal{I}_x^{\xi \xi} \left[ m_w^2 \right] - \left( m_h^2 - m_w^2(1 - \xi) \right) \mathcal{I}_{xx} \left[ m_w^2 \right] \]

\[ + \frac{1}{4} \left( g_1^2 + 3g_2^2 \right) (3 + \xi) - Y \right) \log \left( \frac{m_h^2}{m_h^2} \right) + \frac{1}{4} \left( g_1^2 + g_2^2 \right) \xi \log \left( \frac{m_h^2}{m_h^2} \right) - \log \left( \frac{m_h^2}{m_h^2} \right) \]

\[ + \left( \frac{1}{8m_h^2} \left( g_1^2 + g_2^2 \right) m_h^2(1 - \xi)^2 - 2m_h^2(1 + \xi) \right) \log \left( \frac{m_h^2}{m_h^2} \right) - \log \left( \frac{m_h^2}{m_h^2} \right) \]

\[ + \left( 2 \lambda + \frac{g_2^2}{4m_h^2} \left( m_w^2(1 - \xi)^2 - 2m_h^2(1 + \xi) \right) \right) \left( \log \left( \frac{m_h^2}{m_h^2} \right) - \log \left( \frac{m_h^2}{m_h^2} \right) \right). \quad (F.8) \]
Appendix \( G \)

Vacuum Expectation Value

Renormalization

\( \delta v \) is constructed to cancel the linear term in the Lagrangian involving \( h \), since such terms are considered to be field excitations about the wrong VEV. It can therefore be found through the condition

\[
\frac{\delta v}{v} = \frac{1}{m_h^2} \frac{\Pi^{h(1)}}{v} \tag{G.1}
\]

where \( \Pi^{h(1)} \) is the amplitude of the one-loop contributions to the one-point functions shown in figure G.1.

The amplitudes of the separate diagrams are as follows,

\[
i A_a = 3\lambda v \left( \frac{\mu^2 e^{\gamma_E}}{4\pi} \right) \epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_h^2} + (\lambda + \frac{1}{4}(g_1^2 + g_2^2)\xi) v \left( \frac{\mu^2 e^{\gamma_E}}{4\pi} \right) \epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - \xi m_w^2},
\]

\[
i A_b = 2(\lambda + \frac{1}{4}g_2^2\xi) v \left( \frac{\mu^2 e^{\gamma_E}}{4\pi} \right) \epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - \xi m_w^2},
\]

\[
i A_c = -g_2(m_w^2)\xi \left( \frac{\mu^2 e^{\gamma_E}}{4\pi} \right) \epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - \xi m_w^2},
\]

\[
i A_d = -2g_2m_w\xi \left( \frac{\mu^2 e^{\gamma_E}}{4\pi} \right) \epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - \xi m_w^2},
\]

\[
i A_e = \frac{1}{2}g_2m_z\mu \left( \frac{\mu^2 e^{\gamma_E}}{4\pi} \right) \epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_z^2} \left( g^{\mu
u} - (1 - \xi) g^{\mu\nu} \right),
\]

\[
i A_f = g_2m_w\mu \left( \frac{\mu^2 e^{\gamma_E}}{4\pi} \right) \epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_w^2} \left( g^{\mu\nu} - (1 - \xi) g^{\mu\nu} \right),
\]

\[
i A_g = -\frac{m_\psi}{v} \left( \frac{\mu^2 e^{\gamma_E}}{4\pi} \right) \epsilon \int \frac{d^d q}{(2\pi)^d} \frac{\text{Tr}(g - m_\psi)}{q^2 - m_\psi^2}.
\]
Vacuum Expectation Value Renormalization

\[
\begin{align*}
\Pi^{(1)} &= \frac{i}{16\pi^2} m_h^2 \nu \left( \frac{1}{\epsilon} \left( 3\lambda + \frac{1}{8} \xi (g_1^2 + 3g_2^2) - 2 Y_{\psi}^2 N_c \frac{m_\psi^2}{m_h^2} + \frac{3}{2} (g_1^2 + g_2^2) \frac{m_w^2}{m_h^2} + 3g_2^2 \frac{m_w^2}{m_h^2} \right) 
+ 3 \lambda \left( 1 + \log \left[ \frac{\mu^2}{m_h^2} \right] \right) + \frac{m_w^2}{\nu^2} \xi \left( 1 + \log \left[ \frac{\mu^2}{\xi m_w^2} \right] \right) 
+ \frac{1}{2} \frac{m_z^2 m_\psi^2}{m_h^2} \left( 1 + 3 \log \left[ \frac{\mu^2}{m_z^2} \right] \right) + \frac{1}{4} (g_1^2 + g_2^2) \frac{m_z^2}{m_h^2} \left( 1 + 3 \log \left[ \frac{\mu^2}{m_z^2} \right] \right) \right) \right), 
\end{align*}
\]

so that

\[
\frac{\delta v}{v} = \frac{i}{16\pi^2} \left( \frac{1}{\epsilon} \left( 3\lambda + \frac{1}{8} \xi (g_1^2 + 3g_2^2) - 2 Y_{\psi}^2 N_c \frac{m_\psi^2}{m_h^2} + \frac{3}{2} (g_1^2 + g_2^2) \frac{m_w^2}{m_h^2} + 3g_2^2 \frac{m_w^2}{m_h^2} \right) 
+ 3 \lambda \left( 1 + \log \left[ \frac{\mu^2}{m_h^2} \right] \right) + \frac{m_w^2}{\nu^2} \xi \left( 1 + \log \left[ \frac{\mu^2}{\xi m_w^2} \right] \right) 
+ \frac{1}{2} \frac{m_z^2 m_\psi^2}{m_h^2} \left( 1 + 3 \log \left[ \frac{\mu^2}{m_z^2} \right] \right) + \frac{1}{4} (g_1^2 + g_2^2) \frac{m_z^2}{m_h^2} \left( 1 + 3 \log \left[ \frac{\mu^2}{m_z^2} \right] \right) \right) \right). 
\]

Figure G.1: Higgs tadpoles.

Added together, the finite contribution in $R_\xi$ gauge becomes
The following are the Feynman rules of the new interactions involving SM fields connected through vertices of the dimension six operators.
Feynman Rules for Effective Operators

\[ h h A_\alpha A_\beta / \phi_0 \phi_0 A_\alpha A_\beta: \]

\[ = -2 e^2 C_{\gamma\gamma} (p_a \cdot p_b g_{\alpha\beta} - p_{a,\beta} p_{b,\alpha}) \]

\[ \phi_+ \phi_0 A_\alpha A_\beta: \]

\[ = -4 e^2 C_{\gamma\gamma} (p_a \cdot p_b g_{\alpha\beta} - p_{a,\beta} p_{b,\alpha}) - 4e^4 v^2 C_{HWB} g_{\alpha\beta} \]

\[ h W_\mu W_\nu A_\alpha: \]

\[ = 4 e g_2^2 v \left( C_{HW} (g_{\mu\nu}(p_b - p_c)_\alpha - g_{\mu\alpha} p_b,\nu + g_{\nu\alpha} p_c,\mu) \right) + \left( C_{HW} - \frac{1}{2} C_{HWB} \right) (p_{a,\nu} g_{\mu\alpha} - p_{a,\mu} g_{\nu\alpha}) \]

\[ h W_\mu W_\nu: \]

\[ = g_2 m_w \left( 8 C_{HW} (p_{a,\nu} p_{b,\mu} - g_{\mu\nu} p_a \cdot p_b) + v^2 g_{\mu\nu} (C_H - \frac{1}{4} C_{HD}) \right) \]

\[ W_\mu W_\nu A_\alpha: \]

\[ = e g_2^2 v^2 C_{HWB} \left( - (p_{a,\nu} g_{\mu\alpha} - p_{a,\mu} g_{\nu\alpha}) + \frac{1}{4} e^2 g_2^2 (g_{\mu\alpha} p_b,\mu - g_{\mu\alpha} p_c,\nu) \right) + \frac{e^2 g_2}{g_2} (g_{\mu\alpha}(p_a - p_b)_\nu + g_{\nu\alpha} (p_c - p_a)_\mu + g_{\mu\nu}(p_b - p_c)_\alpha) \]

\[ + 6 s_w C_W \left( p_a \cdot p_b (p_{c,\mu} g_{\nu\alpha} - p_{c,\alpha} g_{\nu\mu}) + p_b \cdot p_c (p_{a,\nu} g_{\mu\alpha} - p_{a,\mu} g_{\nu\alpha}) \right) + p_a \cdot p_c (p_{b,\alpha} g_{\mu\nu} - p_{b,\nu} g_{\alpha\mu}) + p_{a,\mu} p_{b,\nu} p_{c,\alpha} - p_{a,\nu} p_{b,\alpha} p_{c,\nu} \]

**Figure H.1**: Feynman rules of dimension six operators.
$W_\mu W_\nu A_\alpha A_\beta$: 

$$
= 2 e^4 C_{HWB} v^2 \left( 2g_{\alpha\beta}g_{\mu\nu} - \left(1 - \frac{1}{4} \right) (g_{\mu\alpha}g_{\nu\beta} + g_{\mu\beta}g_{\nu\alpha}) \right)
- 6 C_{W} e^2 g_2^2 \left[ (p_a \cdot p_d + p_b \cdot p_c) (g_{\beta\nu} g_{\mu\alpha} - g_{\beta\alpha} g_{\mu\nu}) 
+ (p_b \cdot p_d + p_a \cdot p_c) (g_{\alpha\nu} g_{\beta\mu} - g_{\beta\alpha} g_{\mu\nu}) 
+ g_{\mu\nu} (p_{\alpha,\beta} (p_c + p_d)_{\alpha} + p_{\beta,\alpha} (p_c + p_d)_{\beta}) 
+ g_{\beta\alpha} (p_{\alpha,\nu} (p_d + p_a)_{\alpha} + p_{\beta,\nu} (p_d + p_a)_{\beta}) 
+ g_{\mu\nu} (p_{\alpha,\mu} (p_d - p_c)_{\beta} - p_{\beta,\mu} (p_d - p_a)_{\beta}) 
+ g_{\alpha\nu} (p_{\alpha,\nu} (p_d - p_c)_{\beta} - p_{\beta,\nu} (p_d - p_a)_{\beta}) 
+ g_{\mu\nu} (p_{\alpha,\mu} (p_d - p_c)_{\beta} - p_{\beta,\nu} (p_d - p_a)_{\beta}) 
+ g_{\alpha\nu} (p_{\alpha,\nu} (p_d - p_c)_{\beta} - p_{\beta,\nu} (p_d - p_a)_{\beta}) 
+ g_{\mu\nu} (p_{\alpha,\mu} (p_d - p_c)_{\beta} - p_{\beta,\nu} (p_d - p_a)_{\beta}) 
+ g_{\alpha\nu} (p_{\alpha,\nu} (p_d - p_c)_{\beta} - p_{\beta,\nu} (p_d - p_a)_{\beta}) \right] 
$$

$h W_\mu W_\mu A_\alpha A_\beta$:

$$
= 4 g_2^2 e^2 C_{HW} v (2g_{\alpha\beta}g_{\mu\nu} - g_{\mu\alpha}g_{\nu\beta} - g_{\mu\beta}g_{\nu\alpha})
$$

$h\phi^\pm W_\mu A_\alpha$:

$$
= \pm 2 i e g_2 C_{HWB} (p_{\alpha,\mu} p_{\beta,\alpha} - g_{\alpha\mu} p_{\alpha} \cdot p_{\beta})
$$

$\phi^\pm W_\mu A_\alpha$:

$$
= \pm 2 i e g_2 C_{HWB} (p_{\alpha,\mu} p_{\beta,\alpha} - g_{\alpha\mu} p_{\alpha} \cdot p_{\beta})
$$

$\phi^\pm W_\mu A_\alpha A_\beta$:

$$
= 2 i e^2 g_2 v C_{HWB} (g_{\mu\beta} p_{\beta,\alpha} + g_{\mu\alpha} p_{\alpha,\beta} - g_{\alpha\beta} (p_a + p_b)_{\mu})
$$

**Figure H.2:** Feynman rules of dimension six operators.
Feynman Rules for Effective Operators

\( \phi^+ \phi^- A_\alpha : \)

\[ = i e^3 v^2 C_{HWB} (p_b - p_a)_\alpha \]

\( \tilde{u}^\pm u^\pm A_\alpha : \)

\[ = \pm e^3 v^2 C_{HWB} (p_b - p_a)_\alpha \]

\( \tilde{u}^\pm u^\pm A_\alpha A_\beta : \)

\[ = -4 e^4 v^2 C_{HWB} g_{\alpha\beta} \]

\( \bar{\psi} \psi A_\alpha : \)

\[ = i \frac{1}{\sqrt{2}} e v \left( \pm C_{uW} + C_{uB} \right) \sigma^{\mu\nu} (p_{a\mu} g_{\alpha\nu} - p_{a\nu} g_{\alpha\mu}) \]

\( \bar{\psi} \psi h A_\alpha : \)

\[ = i \frac{1}{\sqrt{2}} e \left( \pm C_{uW} + C_{uB} \right) \sigma^{\mu\nu} (p_{a\mu} g_{\alpha\nu} - p_{a\nu} g_{\alpha\mu}) \]

\( \tilde{u}^\pm u^\pm h : \)

\[ = -\frac{1}{2} g_3^2 \xi v^3 \left( C_{HD} - \frac{1}{4} C_{HDD} \right) \]

Figure H.3: Feynman rules of dimension six operators.
\[ \phi^+ \phi^- h: \]
\[ = 2v \left( C_{H\Box} - \frac{1}{4} C_{HD} \right) \left( \lambda v^2 + p_a \cdot (p_b + p_c) + \frac{1}{4} \xi g_2^2 v^2 \right) \]

\[ \bar{\psi} \psi h: \]
\[ = -m_{\bar{\psi}} v \left( C_{H\Box} - \frac{1}{4} C_{HD} - C_{fH} \right) \]

\[ h h h h: \]
\[ = \frac{1}{4} \lambda v^2 \left( 6 C_H - \frac{8}{3} C_{H\Box} + \frac{2}{3} C_{HD} \right) \]

\[ h h \phi^+ \phi^-: \]
\[ = 2 \left( C_{H\Box} - \frac{1}{4} C_{HD} \right) \left( p_a + p_b \right) \left( p_c + p_d \right) + \lambda v^2 \left( \frac{9}{4} C_H - 2 C_{H\Box} + \frac{1}{2} C_{HD} \right) \]

\[ h h \phi_0 \phi_0: \]
\[ = 2 C_{H\Box} \left( p_a + p_b \right) \left( p_c + p_d \right) + \lambda v^2 \left( C_H - C_{H\Box} + \frac{1}{4} C_{HD} \right) \]

**Figure H.4:** Feynman rules of dimension six operators.
The amplitudes are written up in detail in the following for the diagrams in figure 6.2. These amplitudes arise from dimension six operators entering directly into the effective Lagrangian, namely the operators $O_{HW}, O_{HB}, O_{HWB}, O_{\psi W}$ and $O_{\psi B}$. 

\[
\begin{align*}
&\ i\ A_a = \alpha e^2 \gamma^4 \left( \frac{\lambda^2}{4} \right) \int \frac{d^4q}{(2\pi)^d} \left( \frac{1}{q^2 - m_h^2} - \frac{1}{m_h^2} \right) \\
&\quad \quad + 4(\lambda + \frac{1}{4}(g_1^2 + g_2^2)\xi) \left( \frac{1}{q^2 - \xi m_2^2} - \frac{1}{m_2^2} \right) (p_2 \cdot p_3 g^\alpha - p_2^\alpha p_3^\beta) \epsilon_\alpha (p_2) \epsilon_\beta (p_3), \\
&\ i\ A_b = 8(\lambda + \frac{1}{4}g_2^2\xi) e^2 (C_{\gamma^4} + 2 C_{HWB}) \left( \frac{\lambda^2}{4} \right) \int \frac{d^4q}{(2\pi)^d} \left( \frac{1}{q^2 - \xi m_2^2} - \frac{1}{m_2^2} \right) \\
&\quad \quad \times (p_2 \cdot p_3 g^\alpha - p_2^\alpha p_3^\beta) \epsilon_\alpha (p_2) \epsilon_\beta (p_3), \\
&\ i\ A_c = 4C_{HW} e^2 g_2^2 v \left( \frac{\lambda^2}{4} \right) \int \frac{d^4q}{(2\pi)^d} \left( \frac{2g^{\alpha\beta} g^{\alpha\beta} - (1 - \frac{1}{\xi})g^{\alpha\beta} g^{\sigma\alpha} - (1 - \frac{1}{\xi})g^{\rho\alpha} g^{\sigma\beta}}{q^2 - m^2} \right) \\
&\quad \quad \times \left( g^\nu (q - p)^\mu - g^\mu q \cdot (q - p) \right) \left( q^\sigma (q - p)^\rho - g^\rho q \cdot (q - p) \right) \epsilon_\alpha (p_2) \epsilon_\beta (p_3), \quad (I.1)
\end{align*}
\]
\[ i A_d = -3 e^2 g_2^4 v C_W g_{\mu\nu} \left( \frac{\mu^2 e^{\gamma_{1E}}}{4 \pi} \right) \epsilon \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_W^2} \frac{1}{(q - p)^2 - m_W^2} \]
\[ \times \left( -p_3 \cdot q + q \cdot p_2 - p_2 \cdot p_3 \right) \left( g_{\sigma\beta} g_{\alpha\nu} - g_{\nu\sigma} g_{\alpha\beta} \right) \]
\[ + \left( -q \cdot p_2 + q \cdot p_3 - p_3 \cdot p_2 \right) \left( g_{\sigma\beta} g_{\alpha\nu} - g_{\nu\sigma} g_{\alpha\beta} \right) + g_{\sigma\nu} \left( -p_3,\alpha \cdot p_\beta - p_2,\beta p_\alpha \right) \]
\[ + g_{\alpha\beta} \left( -p_3,\nu q_\sigma + p_3,\sigma (q - p)_\nu - p_2,\nu q_\sigma + p_2,\sigma (q - p)_\nu \right) \]
\[ + g_{\nu\beta} \left( -p_3,\sigma (2q - p)_\alpha + p_2,\sigma q_\alpha + p_3,\alpha q_\nu \right) \]
\[ + g_{\alpha\nu} \left( -p_2,\sigma (2q - p)_\beta + p_3,\sigma q_\beta + p_2,\beta q_\sigma \right) \]
\[ + g_{\beta\sigma} \left( p_3,\nu (2q - p)_\alpha - p_2,\nu (q - p)_\alpha - p_3,\alpha (q - p)_\nu \right) \]
\[ + g_{\alpha\sigma} \left( p_2,\nu (2q - p)_\beta - p_3,\nu (q - p)_\beta - p_2,\beta (q - p)_\nu \right) \]
\[ \times \left( g_{\mu\nu} - (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi m_W^2} \right) \left( g_{\nu\sigma} - (1 - \xi) \frac{(q - p)_\mu (q - p)_\sigma}{(q - p)_2 - \xi m_W^2} \right) \epsilon_\alpha(p_2) \epsilon_\beta(p_3), \]
\[ i A_c = 4 C_W e^2 g_2^2 v \left( \frac{\mu^2 e^{\gamma_{1E}}}{4 \pi} \right) \epsilon \int \frac{d^4 q}{(2\pi)^4} \left( -g_{\nu\alpha} g_{\mu\beta} - g_{\mu\beta} g_{\nu\alpha} + 2g_{\nu\beta} g_{\mu\nu} \right) \]
\[ \times \left( g_{\mu\nu} - (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi m_W^2} \right) \frac{1}{q^2 - m_W^2} \epsilon_\alpha(p_2) \epsilon_\beta(p_3), \]
\[ i A_f = -2 C_W e^2 g_2^2 v \left( \frac{\mu^2 e^{\gamma_{1E}}}{4 \pi} \right) \epsilon \int \frac{d^4 q}{(2\pi)^4} q_\mu \left( 2g_{\nu\beta} p_3^\alpha + 2g_{\nu\alpha} p_2^\beta - 2g_{\alpha\beta} p_2^\nu - 2g_{\alpha\beta} p_2^\nu \right) \]
\[ \times \left( g_{\mu\nu} - (1 - \xi) \frac{q_\mu q_\nu}{q^2 - \xi m_W^2} \right) \frac{1}{(q - p)^2 - m_W^2} \epsilon_\alpha(p_2) \epsilon_\beta(p_3), \]
\[ i A_g = -8 C_W e^2 g_2^2 v \left( \frac{\mu^2 e^{\gamma_{1E}}}{4 \pi} \right) \epsilon \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(q^2 - m_W^2)((q - p)^2 - m_W^2)} \]
\[ \times \left( (p_2 - p_3 - q)_\mu g_{\nu\beta} + (2q - p_2)_\beta g_{\mu\nu} - (q - p_3)_\nu g_{\mu\beta} \right) \]
\[ \times \left( \frac{1}{\xi} (q - p_2) - q - p_2)^\sigma g^{\sigma\alpha} + (2q - p_2)^\alpha g^{\sigma\alpha} + \frac{1}{\xi} q - q + 2p_2)^\rho g^{\sigma\alpha} \right) \]
\[ \times \left( g_{\mu\rho} - (1 - \xi) \frac{q_\mu q_\rho}{q^2 - \xi m_W^2} \right) \left( g_{\nu\sigma} - (1 - \xi) \frac{(q - p_2)_\mu (q - p_2)_\sigma}{(q - p_2)^2 - \xi m_W^2} \right) \epsilon_\alpha(p_2) \epsilon_\beta(p_3) \]
\[ - 4 g_2^2 C_W e^2 \left( \frac{\mu^2 e^{\gamma_{1E}}}{4 \pi} \right) \epsilon \int \frac{d^4 q}{(2\pi)^4} \left( p_3^\mu g_{\nu\beta} - p_2^\nu g_{\mu\beta} \right) \]
\[ \times \left( \frac{1}{\xi} (q - p_2) - q - p_2)^\sigma g^{\sigma\alpha} + (2q - p_2)^\alpha g^{\sigma\alpha} + \frac{1}{\xi} q - q + 2p_2)^\rho g^{\sigma\alpha} \right) \]
\[ \times \left( g_{\mu\rho} - (1 - \xi) \frac{q_\mu q_\rho}{q^2 - \xi m_W^2} \right) \frac{1}{(q^2 - m_W^2)((q - p_2)^2 - m_W^2)} \epsilon_\alpha(p_2) \epsilon_\beta(p_3), \] (1.2)
Direct Amplitudes

\[ i \mathcal{A}_h = 8e^2 g_2^2 v C_{HWB} \left( \frac{\mu^2 e^{\gamma_E}}{4 \pi} \right) \epsilon \int \frac{d^4q}{(2\pi)^3} \left( 2q - p_2 \right)^{\alpha} p^\mu \left( (q - p)^\beta p^\nu - p_3 \cdot (q - p) g^{\nu\beta} \right) \]

\[ \times \left( \frac{1}{(q^2 - \xi m_w^2)((q - p_2)^2 - \xi m_w^2)((q - p)^2 - m_w^2)} \right) \times \left( g_{\mu\nu} - (1 - \xi) \frac{(q - p_2)\mu(q - p_\nu)}{(q - p)^2 - \xi m_w^2} \right) \epsilon_\alpha(p_2) \epsilon_\beta(p_3), \]

\[ i \mathcal{A}_i = 4 \cdot 2e^2 g_2^2 v C_{HWB} \left( \frac{\mu^2 e^{\gamma_E}}{4 \pi} \right) \epsilon \int \frac{d^4q}{(2\pi)^3} p^\mu \left( (q - p_2)^\alpha p^\nu_2 - p_2 \cdot (q - p_2) g^{\nu\alpha} \right) \]

\[ \times \left( \frac{1}{(q^2 - \xi m_w^2)((q - p_2)^2 - m_w^2)((q - p)^2 - m_w^2)} \right) \times \left( g_{\mu\nu} - (1 - \xi) \frac{(q - p_2)\mu(q - p_\nu)}{(q - p)^2 - \xi m_w^2} \right) \epsilon_\alpha(p_2) \epsilon_\beta(p_3), \]

\[ i \mathcal{A}_j = 2 \cdot 4 C_{HWB} e^2 g_2^2 v \left( \frac{\mu^2 e^{\gamma_E}}{4 \pi} \right) \epsilon \int \frac{d^4q}{(2\pi)^3} \left( q_\nu(q - p_2 - p_3)_{\mu} - q \cdot (q - p_2 - p_3) g_{\mu\nu} \right) \]

\[ \times \left( 2p_2 - q + \frac{1}{\xi} q \right)^\rho g^{\rho\sigma} + (2q - p_2)^\alpha g^{\rho\sigma} - (q + p_2 - \frac{1}{\xi} (q - p_2))^\sigma g^{\rho\sigma} \]

\[ \times \left( -p - p_3 + q - \frac{1}{\xi}(q - p_2) \right)^\rho g^{\rho\sigma} + (2q + p_2 + p)^\beta g^{\rho\sigma} + (2p_2 + q + p_3) \]

\[ \left( \frac{1}{\xi}(q - p)^\rho g^{\rho\sigma} \right) \left( \frac{1}{(q^2 - m_w^2)((q - p_2)^2 - m_w^2)((q - p_2 - p_3)^2 - m_w^2)} \right) \times \left( g_{\mu\rho} - (1 - \xi) \frac{q_\rho q_\rho}{q^2 - \xi m_w^2} \left( g_{\sigma\omega} - (1 - \xi) \frac{(q - p_2)_\sigma(q - p_2)_\omega}{(q - p_2)^2 - \xi m_w^2} \right) \epsilon_\alpha(p_2) \epsilon_\beta(p_3) \right), \]

(1.3)
\[
\begin{align*}
    i \mathcal{A}_k &= 2 \cdot 2 \cdot \frac{1}{\Lambda^2} \varepsilon^2 g_2^2 v m_w^2 C_{HWB} \left( \frac{\mu^2 e^{\gamma E}}{4 \pi} \right)^\epsilon \int \frac{d^d q}{(2\pi)^d} g_{\mu\nu} \left( g^{\alpha\beta} p^\sigma q^\rho - g^{\alpha\rho} p^\sigma q^\beta \right) \\
    &\times \left( (-p - p_3 + q - \frac{1}{\xi}(q - p_2))^{\alpha\tau} + (-2q + p_2 + p)^\alpha g^{\tau\omega} \\
    &+ (q - p_2 + p_3 - \frac{1}{\xi}(q - p))^{\alpha\omega} \right) \frac{1}{(q^2 - m_w^2)((q - p_2)^2 - m_w^2)((q - p_2 - p_3)^2 - m_w^2)} \\
    &\times \left( g_{\nu\rho} - \frac{1}{\xi} \frac{g_\nu q_\rho}{q^2 - \xi m_w^2} \right) \left( g_{\sigma\omega} - \frac{1}{\xi} \frac{g_\sigma q_\omega}{q^2 - \xi m_w^2} \right) \\
    &\times \left( g_{\mu\rho} - \frac{1}{\xi} \frac{g_\mu q_\rho}{q^2 - \xi m_w^2} \right) \frac{1}{(q - p_2 - p_3)^2 - \xi m_w^2} \epsilon_\alpha(p_2)^\epsilon_\beta(p_3) \\
    &- 12 e^2 g_2^2 v g_{\mu\omega} \left( \frac{\mu^2 e^{\gamma E}}{4 \pi} \right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \frac{1}{q^2 - m_w^2} \frac{1}{(q - p_2)^2 - m_w^2} \frac{1}{(q - p)^2 - m_w^2} \\
    &\times \left( (q - p_2) - q_\rho q_\mu + \frac{1}{\xi} (q - q + 2p_2) \rho g_{\rho\mu} + (2q - p_2) g_{\rho\mu} \right) \\
    &\times \left( g_{\mu\rho} - \frac{1}{\xi} \frac{g_\mu q_\rho}{q^2 - \xi m_w^2} \right) \left( g_{\sigma\omega} - \frac{1}{\xi} \frac{g_\sigma q_\omega}{q^2 - \xi m_w^2} \right) \\
    &\times \left( g_{\nu\rho} - \frac{1}{\xi} \frac{g_\nu q_\rho}{q^2 - \xi m_w^2} \right) \frac{1}{(q - p_2 - p_3)^2 - \xi m_w^2} \epsilon_\alpha(p_2)^\epsilon_\beta(p_3) \\
\end{align*}
\]

Direct Amplitudes

\[
\begin{align*}
    i \mathcal{A}_l &= \frac{1}{2} e^2 Q_\psi |Y_\psi|_{sr} v \left( C_{\psi B} - C_{\psi W} \right) \left( \frac{\mu^2 e^{\gamma E}}{4 \pi} \right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \left( p_{3,\mu} g_{3\nu} - p_{3,\nu} g_{3\mu} \right) \\
    &\times \text{Tr} \left( (q + m_\psi)^\alpha (q - p_2 + m_\psi)(\gamma^\nu \gamma^\rho - \gamma^\nu \gamma^\mu)(q - p + m_\psi) \right) \\
    &\times \frac{1}{q^2 - m_\psi^2} \frac{1}{(q - p_2)^2 - m_\psi^2} \frac{1}{(q - p)^2 - m_\psi^2} \epsilon_\alpha(p_2)^\epsilon_\beta(p_3) \\
\end{align*}
\]

\[
\begin{align*}
    i \mathcal{A}_m &= \frac{1}{2 \sqrt{2}} e^2 |Y_\psi|_{sr} Q_\psi \left( C_{\psi B} - C_{\psi W} \right) \left( \frac{\mu^2 e^{\gamma E}}{4 \pi} \right)^\epsilon \int \frac{d^d q}{(2\pi)^d} \left( p_{3,\mu} g_{3\nu} - p_{3,\nu} g_{3\mu} \right) \\
    &\times \text{Tr} \left( (\gamma^\nu \gamma^\rho - \gamma^\nu \gamma^\mu)(q - p + m_\psi)^\gamma (q + m_\psi) \right) \frac{1}{q^2 - m_\psi^2} \frac{1}{(q - p_2)^2 - m_\psi^2} \epsilon_\alpha(p_2)^\epsilon_\beta(p_3) \\
\end{align*}
\]

(1.4)
The amplitudes in the following represent the diagrams in figure 6.3. These amplitudes arise from the contribution of dimension six operators entering indirectly into the effective Lagrangian, namely, $\mathcal{O}_{HWB}$, $\mathcal{O}_H$, $\mathcal{O}_H$, $\mathcal{O}_{HD}$ and $\mathcal{O}_H$.

\begin{align*}
i \mathcal{A}_b &= 8(\lambda + \frac{1}{4}g_2^2) v^3 e^4 C_{HWB} \left( \frac{\mu^2 e^{\gamma E}}{4 \pi} \right) \int \frac{d^dq}{(2\pi)^d} g_{\alpha \beta} \epsilon_\alpha(p_2) \epsilon_\beta(p_3) \\
&\quad \times \frac{1}{q^2 - \xi_m^2} \frac{1}{(q - p)^2 - \xi m^2} , \\
i \mathcal{A}_p &= -4 e^2 v(\lambda + \frac{1}{4}g_2^2) \left( 1 + (C_H - \frac{1}{4}C_{HD})v^2 \right) \left( \frac{\mu^2 e^{\gamma E}}{4 \pi} \right) \int \frac{d^dq}{(2\pi)^d} g_{\alpha \beta} \\
&\quad \times \frac{1}{q^2 - \xi m^2} \frac{1}{(q - p)^2 - \xi m^2} \epsilon_\alpha(p_2) \epsilon_\beta(p_3) , \\
i \mathcal{A}_c &= \frac{1}{2} g_2^2 v(1 + (C_H - \frac{1}{4}C_{HD})v^2) g_{\mu \nu} \left( \frac{\mu^2 e^{\gamma E}}{4 \pi} \right) \int \frac{d^dq}{(2\pi)^d} \frac{1}{q^2 - m^2} \frac{1}{(q - p)^2 - m^2} \\
&\quad \times \left( 1 - \frac{1}{\xi} \right) g_{\beta \nu} g_{\alpha \rho} + \left( 1 - \frac{1}{\xi} \right) g_{\beta \rho} g_{\alpha \sigma} - 2 g_{\alpha \beta} g_{\rho \sigma} \\
&\quad \times \left( g_{\mu \nu} - (1 - \xi) \frac{g_{\mu \nu}}{q^2 - \xi m^2} \right) \left( g_{\rho \sigma} - (1 - \xi) \frac{(q - p)_\rho (q - p)_\sigma}{(q - p)^2 - \xi m^2} \right) \epsilon_\alpha(p_2) \epsilon_\beta(p_3) , \\
i \mathcal{A}_d &= C_{HWB} e^3 v^3 g_2^2 \left( \frac{\mu^2 e^{\gamma E}}{4 \pi} \right) \int \frac{d^dq}{(2\pi)^d} \frac{1}{g_{\mu \nu}} \frac{1}{(q^2 - m^2)((q - p)^2 - m^2)} \\
&\quad \times \left( 2 g^{\sigma \beta} g^{\alpha \beta} - (1 - \frac{1}{\xi}) g^{\sigma \beta} g^{\rho \alpha} - (1 - \frac{1}{\xi}) g^{\rho \alpha} g^{\sigma \beta} \right) \\
&\quad \times \left( g_{\mu \rho} - (1 - \xi) \frac{(q - p)_\rho (q - p)_\rho}{(q - p)^2 - \xi m^2} \right) \left( g_{\nu \sigma} - (1 - \xi) \frac{q_\nu q_\sigma}{q^2 - \xi m^2} \right) \epsilon_\alpha(p_2) \epsilon_\beta(p_3) .
\end{align*}
Indirect Amplitudes

\[ i \mathcal{A}_j = g_2^2 e^2 v (1 + (C_{H \square} - \frac{1}{4} C_{HD}) v^2) g_{\mu \nu} \left( \frac{\mu^2 e^{\gamma} e}{4 \pi} \right)^e \int \frac{d^4 q}{(2\pi)^d} q^2 - \frac{1}{m_w^2} (q - p_2)^2 - m_w^2 \left( \frac{1}{q - p_2 - m_w^2} \right) g_{\sigma \tau} + \frac{1}{\xi} (q - p_2 - q + p_3) g_{\beta \tau} + \frac{1}{\xi} (q - p_2 - q - p_3) g_{\beta \tau} g_{\alpha \nu} + \frac{1}{\xi} (q - q + 2p_2 \nu g_{\alpha \nu}) (g_{\mu \nu} (1 - \xi) - \frac{q_{\mu} q_{\nu}}{q^2 - \xi m_w^2}) (g_{\mu \sigma} (1 - \xi) - \frac{q_{\mu} q_{\sigma}}{q^2 - \xi m_w^2}) \left( g_{\sigma \tau} - \frac{1}{\xi} (q - p_2 - q - p_2) \right) \epsilon_\alpha (p_2) \epsilon_\beta (p_3), \]

\[ i \mathcal{A}_k = 2 \cdot \frac{1}{A^2} e^4 v m_w^2 C_{H \square B} \left( \frac{\mu^2 e^{\gamma} e}{4 \pi} \right)^e \int \frac{d^4 q}{(2\pi)^d} g_{\mu \nu} \left( g_{\alpha \rho} \frac{1}{\xi} (q - p_2 - q + p_2) \right) + g_{\alpha \rho} \left( \frac{1}{\xi} (q - p_2 - q + p_2) \right) \left( \xi^\omega (q - p_2 + p) \right) + \frac{1}{\xi} (q - p_2 + p) (q - p_2 + p) \left( \xi^\omega (q - p_2 + p) \right) \left( \xi^\sigma (q - p_2 + p) \right) \left( \xi^\lambda (q - p_2 + p) \right) \epsilon_\alpha (p_2) \epsilon_\beta (p_3), \]

\[ i \mathcal{A}_n = 4 e^2 v (1 + \frac{1}{4} \xi g_2^2) \left( 1 + (C_{H \square} - \frac{1}{4} C_{HD}) v^2 \right) \left( \frac{\mu^2 e^{\gamma} e}{4 \pi} \right)^e \int \frac{d^4 q}{(2\pi)^d} (2q - p_2 - p_2). \]

\[ i \mathcal{A}_o = 4 e^4 v^3 C_{H \square B} \left( \frac{\mu^2 e^{\gamma} e}{4 \pi} \right)^e \int \frac{d^4 q}{(2\pi)^d} (2q - p_2 - p_2). \]

\[ i \mathcal{A}_q = 2 m_\psi^2 e Q_f N_c \left( 1 + (-C_f H + C_{H \square} - \frac{1}{4} C_{HD}) v^2 \right) \left( \frac{\mu^2 e^{\gamma} e}{4 \pi} \right)^e \int \frac{d^4 q}{(2\pi)^d} (2q - p_2 - p_2). \]

\[ i \mathcal{A}_s = -2 e^4 g_2^2 v^3 \xi C_{H \square B} \left( \frac{\mu^2 e^{\gamma} e}{4 \pi} \right)^e \int \frac{d^4 q}{(2\pi)^d} (2q - p_2 - p_2). \]

(J.1)
Indirect Amplitudes

\[ iA_r = -2e^2 v g_2^2 \left( 1 + \left( C_{H\Box} - \frac{1}{4} C_{HD}\right) v^2 \right) \left( \frac{\mu^2 e\gamma_E}{4\pi} \right)^\epsilon \int \frac{d^d q}{(2\pi)^d} (2q - p_2)_\alpha (2q - p - p_2)_\beta \]
\[
\times \frac{1}{q^2 - \xi m_w^2 (q - p_2)^2 - \xi m_w^2 (q - p)^2 - \xi m_w^2} \epsilon_\alpha (p_2) \epsilon_\beta (p_3),
\]

\[ iA_s = 2e^2 v g_2^2 \left( 1 + \left( C_{H\Box} - \frac{1}{4} C_{HD}\right) v^2 \right) \left( \frac{\mu^2 e\gamma_E}{4\pi} \right)^\epsilon \int \frac{d^d q}{(2\pi)^d} g^{\alpha\beta} \frac{1}{q^2 - \xi m_w^2} \]
\[
\times \frac{1}{(q - p)^2 - \xi m_w^2} \epsilon_\alpha (p_2) \epsilon_\beta (p_3),
\]

\[ iA_t = -4e^4 g_2^2 v^3 C_{HWB} \left( \frac{\mu^2 e\gamma_E}{4\pi} \right)^\epsilon \int \frac{d^d q}{(2\pi)^d} g^{\alpha\beta} \frac{1}{(q^2 - \xi m_w^2)((q - p)^2 - \xi m_w^2)} \]
\[
\times \frac{1}{q^2 - \xi m_w^2 (q - p_2)^2 - \xi m_w^2 (q - p)^2 - \xi m_w^2} \epsilon_\alpha (p_2) \epsilon_\beta (p_3). \quad (J.2)\]
In the following, the complete finite result for diagrams contributing both directly and indirectly as seen in figures 6.2 and 6.3, will be given for general $\xi$ in the $R_\xi$ gauge. The following definitions will be implemented,

\begin{align*}
\mathcal{I}[m^2] &= \int_0^1 dx \log \left( \frac{m^2 - m_h^2 x (1 - x)}{m_h^2} \right), \\
\mathcal{I}^\xi[m^2] &= \int_0^1 dx \log \left( \frac{m^2(1 - x(1 - \xi)) - m_h^2 x (1 - x)}{m_h^2} \right), \\
\mathcal{I}_y[m^2] &= \int_0^{1-x} dy \int_0^1 dx \frac{m^2}{m^2 - m_h^2 x (1 - x - y)}, \\
\mathcal{I}^\xi\xi[m^2] &= \int_0^1 dx \frac{\xi m^2}{m^2(\xi + x(1 - \xi)) - m_h^2 x (1 - x - y)}, \\
\mathcal{I}^{\xi\xi}[m^2] &= \int_0^1 dx \frac{\xi m^2}{m^2(1 - x(1 - \xi) - y(1 - \xi)) - m_h^2 x (1 - x - y)}, \\
\mathcal{I}^{\xi\xi}_y[m^2] &= \int_0^1 dx \frac{x \xi m^2}{m^2(1 - x(1 - \xi) - y(1 - \xi)) - m_h^2 x (1 - x - y)}, \\
\mathcal{I}^{\xi\xi}_x[m^2] &= \int_0^1 dx \frac{x^2 \xi m^2}{m^2(1 - x(1 - \xi) - y(1 - \xi)) - m_h^2 x (1 - x - y)}, \\
\mathcal{I}^{\xi\xi}_x[y][m^2] &= \int_0^1 dx \frac{m^2}{m^2(\xi + x(1 - \xi) + y(1 - \xi)) - m_h^2 x (1 - x - y)} - \frac{x m^2}{m^2(\xi + x(1 - \xi) + y(1 - \xi)) - m_h^2 x (1 - x - y)}, \\
\mathcal{I}^{\xi\xi}_x[y][m^2] &= \int_0^1 dx \frac{x^2 m^2}{m^2(\xi + x(1 - \xi) + y(1 - \xi)) - m_h^2 x (1 - x - y)}, \\
\mathcal{I}^{\xi\xi}_x[y][m^2] &= \int_0^1 dx \frac{x^2 m^2}{m^2(\xi + x(1 - \xi) + y(1 - \xi)) - m_h^2 x (1 - x - y)},
\end{align*}

(K.1)
Complete $R_\xi$ Result

\[
\frac{i A_{\text{BSM}}^{\gamma\gamma}}{v^2 e^2 A_{\alpha\beta}^{\gamma\gamma}} = \frac{C_{\gamma\gamma}}{16 \pi^2} \left( \frac{1}{2} (\xi g_2^2 + 4 \lambda) \mathcal{I}[\xi m_w^2] + \frac{1}{4} \left( \xi (g_1^2 + g_2^2) + 4 \lambda \right) \mathcal{I}[\xi m_t^2] \right)
\]

\[
\left( \sqrt{3} \pi - 6 \right) \lambda - \frac{1}{4} \left( \xi (g_1^2 + 3 g_2^2) + 24 \lambda \right) \log \left( \frac{\mu^2}{m_h^2} \right)
\]

\[
- \frac{g_2^2 C_{\text{HW}}}{4 \pi^2} \left( \frac{3 m_w^2}{m_h^2} + \left( 4 - \frac{m_h^2}{m_w^2} - 6 \frac{m_w^2}{m_h^2} \right) \mathcal{I}_y[m_w^2] \right)
\]

\[
+ \frac{C_{\text{HW}B}}{8 \pi^2} \left( e^2 \left( \frac{2 m_w^2}{m_h^2} \left( (\xi - 14) \xi + 1 \right) - 1 + 2 \left( 1 - \xi \right) \frac{m_w^2}{m_h^2} \right) \right)
\]

\[
\log \left( \frac{m_w^2}{m_h^2} \right) + \left( \frac{m_h^2}{m_w^2} - 2 \right) \mathcal{I}[\xi m_w^2] + 2 \left( 1 - \left( 1 + 7 \xi \right) \frac{m_h^2}{m_w^2} \right) \mathcal{I}[\xi m_w^2] \right)
\]

\[
- 2 \left( \frac{m_h^2}{m_w^2} (1 - \xi) + \frac{3}{\xi} \right) \mathcal{I}[\xi m_w^2] + 8 \left( \frac{4 m_w^2}{m_h^2} (1 - \xi) + 1 - \frac{1}{\xi} \right) \mathcal{I}[\xi m_w^2]
\]

\[
+ 8 \left( \frac{m_h^2}{m_w^2} - \frac{4}{\xi} \right) \mathcal{I}[\xi m_w^2] + 8 \left( 1 - \frac{1}{\xi} \right) \mathcal{I}[\xi m_w^2]
\]

\[
+ 8 \left( \frac{m_h^2}{m_w^2} + 1 - \frac{1}{\xi} \right) \mathcal{I}[\xi m_w^2] + 48 \left( \frac{2 m_w^2}{m_h^2} - 1 \right) \mathcal{I}[\xi m_w^2]
\]

\[
+ 4 \left( 8 - \frac{m_h^2}{m_w^2} \right) \mathcal{I}_y[m_w^2] - 4 (1 - \xi) \frac{m_w^2}{m_h^2} \log \left( \frac{m_w^2}{m_h^2} \right)
\]

\[
+ g_2^2 \left( - 2 - \left( 1 - \frac{m_h^2}{m_w^2} \right) \mathcal{I}[\xi m_w^2] - \frac{1}{2} \mathcal{I}[\xi m_w^2] \right)
\]

\[
+ \left( \frac{m_h^2}{2 m_w^2} \left( \xi^2 - 1 \right) - \xi + \frac{m_h^2}{2 m_w^2} \right) \left( \mathcal{I}[\xi m_w^2] - \mathcal{I}[\xi m_w^2] \right)
\]

\[
+ 2 \left( \mathcal{I}[\xi m_w^2] - \mathcal{I}[\xi m_w^2] \right) - 4 \mathcal{I}_y[m_w^2] - 2 \log \left( \frac{m_w^2}{m_h^2} \right)
\]

\[
+ 3 \log \left( \frac{\mu^2}{m_h^2} \right) + 4 \lambda \left( \mathcal{I}[\xi m_w^2] - \log \left( \frac{\mu^2}{m_h^2} \right) \right) \right)
\]

\[
+ \frac{g_3^2 C_{\text{W}}}{16 \pi^2} \left( -3 \left( 4 + 3 \log \left( \frac{m_h^2}{\mu^2} \right) + 3 \mathcal{I}[m_w^2] + 2 \mathcal{I}_y[m_w^2] + 2 \mathcal{I}_x[m_w^2] \left( 1 - \frac{m_h^2}{4 m_w^2} \right) \right) \right)
\]

\[
+ \frac{1}{4} \left( 1 - \xi \right) \frac{m_h^2}{m_w^2} - 2 (1 + \xi) \left( - \mathcal{I}[m_w^2] + \mathcal{I}[m_w^2] \right) \right)
\]

\[
+ \frac{2 |Y_{\psi}| \alpha_{\psi} Q_{\psi}}{16 \pi^2} \left( C_{\psi W} - C_{\psi B} \right) \left( \log \left( \frac{m_h^2}{\Lambda^2} \right) + 2 + 4 \mathcal{I}_y[m_w^2] + 2 \mathcal{I}[m_w^2] - \log \left( \frac{m_w^2}{m_h^2} \right) \right)
\]

\[
+ \frac{C_{\psi H} Q_{\psi}^2}{2 \pi^2} \left( 2 (\lambda - Y) \mathcal{I}_y[m_{\psi}^2] + Y \right) \right) \quad (K.2)
\]
$\frac{(C_H - \frac{C_{mu}}{4})}{16 \pi^2} \left( g_2 \left( \frac{1}{4} \mathcal{I}_{\xi} [m_w^2] \left( 1 - \frac{m_h^2}{m_w^2} + 7 \xi \right) + \frac{1}{4} \mathcal{I}_{\xi \xi} [m_w^2] \left( 1 + \frac{3 m_h^2}{m_w^2} - \frac{1}{\xi} \right) ight) + \mathcal{I}_{\xi \xi y} [m_w^2] \left( \frac{m_h^2}{2 m_w^2 \xi} - \frac{m_h^2}{2 m_w^2} \right) + \mathcal{I}_{\xi \xi y y} [m_w^2] \left( \frac{m_h^2}{2 m_w^2 \xi} + \frac{m_h^2}{2 m_w^2 \xi} - \frac{m_h^2}{2 m_w^2} \right) \
+ m_h^4 \mathcal{I}_{\xi \xi y y y} [m_w^2] - \frac{3 m_h^2 \mathcal{I}_{\xi y} [m_w^2]}{4 m_w^2 \xi} + \mathcal{I} [\xi m_w^2] \left( \frac{m_h^2}{4 m_w^2} - 2 \xi \right) \right) + \mathcal{I}_{\xi \xi y} [m_w^2] \left( \frac{m_h^2}{2 m_w^2 \xi} - \frac{m_h^2}{2 m_w^2} - 2(1 - \xi) \right) + \mathcal{I}_{\xi \xi y y} [m_w^2] \left( - \frac{m_h^4}{2 m_w^2 \xi} - \frac{m_h^2}{2 m_w^2 \xi} + \frac{5 m_h^2}{2 m_w^2} \right) + \mathcal{I}_{\xi \xi y y y} [m_w^2] + \frac{3 g_2^2 \mathcal{I}_{\xi \xi} [m_w^2]}{2 m_w^2 \xi} + \frac{2 \xi}{2 m_w^2 \xi} + \mathcal{I}_{\xi \xi} [\xi m_w^2] \left( \frac{m_h^4}{8 m_w^4 \xi} - \frac{m_h^2}{2 m_w^2 \xi} - \frac{m_h^2}{m_w^2} + 4 \right) \\+ \mathcal{I}_{\xi y} [\xi m_w^2] \left( \frac{m_h^2}{m_w^2} \right) + 3 \mathcal{I}_{\xi y} [m_w^2] \left( \frac{m_h^2}{m_w^2} - 2 \right) - \frac{3 g_2^2 m_h^2}{2 m_w^2 \xi} + \left( \frac{\xi}{4} - \frac{1}{4} \right) \log \left( \frac{m_h^2}{m_w^2} \right) \right) + \frac{m_h^2}{2 m_w^2} + \left( \frac{1}{4 \xi} - \frac{1}{4} \right) \log \left( m_w^2 \right) - \frac{17 \xi}{4} - \frac{1}{4 \xi} + \frac{7}{2} \right) \right) + 4 \lambda \left( 1 - 2 \mathcal{I}_{\xi y} [\xi m_w^2] - 2 \mathcal{I}_{\xi y} [m_w^2] \right) - 4 Q_Y^2 Y \left( 1 - 2 \mathcal{I}_{\xi y} [m_w^2] \right).$  \hspace{1cm} (K.3)