Master Thesis
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Kinematic reconstruction of diffractive processes
with tagged protons in the ALFA detector at $\sqrt{s} = 8$ TeV

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Abstract

This thesis presents a kinematic reconstruction of diffractively scattered protons at a centre-of-mass energy of 8 TeV and a $\beta$ function at the interaction point (IP) of 90 metres. Knowledge about the magnetic lattice of the LHC between the IP in the ATLAS detector and the measurement of the proton in the ALFA detector at $z = 240$ m is used to parametrize the proton trajectory as a function of its kinematics and the IP. Intensive studies lead to a minimization procedure of a $\chi^2$ function from where the reconstructed kinematics and IP are extracted. Detector resolution effects are simulated to give a relative energy resolution of $\sim 1\%$. Systematic effects from the positions of ATLAS and ALFA as well as the uncertainty on the optics are found to be smaller. The reconstructed kinematic distributions in data show a rough agreement with a toy model simulation. Only a vague sign of a momentum correlation between the proton and the excited system measured in ATLAS has been observed.
**Resumé på dansk**

Dette speciale præsenterer en kinematisk rekonstruktion af diffaktivt spredte protoner ved en massemidtpunktssenergi på 8 TeV og en $\beta$ funktion i kollisionspunktet på 90 meter. Viden om de LHC magneter, som er placeret mellem kollisionspunktet i ATLAS detektoren og målingen af protonen i ALFA detektoren ved $z = 240$ m, er udnyttet til at parametrere protonens bane som funktion af dens kinematik og kollisionspunktet. Intensive studier har resulteret i minimeringsproceduren af en $\chi^2$ funktion, hvorfra man kan udlede den rekonstruerede kinematik og kollisionspunkt. Effekter af detektorernes opløsning er simuleret og viser en relativ energiopløsning på omkring 1%. De systematiske effekter fra positionerne af ATLAS og ALFA samt usikkerheden på optikken viser sig at være mindre. De rekonstruerede kinematiske fordelinger i data udviser en grov overensstemmelse med en legetøjssimulering. Der er kun observeret et svagt tegn på en impulskorrelation mellem protonen og det exciterede system i ATLAS.
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1 | Introduction

The physical laws aim to describe Nature from the smallest scales of less than an atomic nuclei ($\sim 10^{-15}$ m [1]) in particle physics to the enormous scales of the universe ($> 10^{26}$ m ) in the Big Bang theory of cosmology. Amazingly, these two areas complement each other: Particle physics explains the early Big Bang with the formation of atoms, and cosmology predicts that new particles must exist in order to balance the evolution of the universe.

The LHC experiment at CERN, the European Organization for Nuclear Research, opens for the study of elementary particles at a new energy regime. Two of the main goals of the LHC was to find the Higgs particle to prove our understanding of the unification of the electroweak theory, and to test whether Nature has supersymmetry. A new particle has been found [2][3] and work is in progress to determine whether it has the properties of the Higgs particle predicted by the standard model of particle physics. Furthermore, no supersymmetric particles has been found but the lower limits on the masses of supersymmetric particles has been increased [4]. The belief that supersymmetry is a symmetry of Nature has thus been diminished.

Among other thing, also diffractive processes are studied at the LHC. A Pythia8 [5] simulation shows that the diffractive cross section constitutes about 25 % of the total cross section at a centre-of-mass energy of 8 TeV, but despite of the large cross section, diffractive processes are still not well understood. Regge theory is used as the theoretical approach to diffraction. However, the successful description of elastic scattering by Regge theory is not continued to the diffractive processes where issues remain unsolved. In addition, diffraction is believed to interact via the strong force, but Regge theory is still not combined with quantum chromodynamics. Since the momentum transfer in diffractive processes is small, the study of diffraction may provide insight to the soft regime of quantum chromodynamics.

Diffractive processes are characterized by large gaps in pseudorapidity, and the differential cross section has been studied by the ATLAS experiment in ref. [6]. However, single diffraction and central diffraction are characterized by the survival of one and both of the colliding hadrons, respectively, which will then be scattered in the extreme forward direction along the beam. The ALFA experiment, which is a sub-detector to ATLAS, is placed 240 metres from the interaction point and is able to measure protons only few millimetres from the beam center. This opens for the possibility to study diffraction by tagging the proton(s). ALFA is build to measure the elastic cross section and is only turned on during runs with very low luminosity to avoid a large multiplicity in the detector, hence the signal in ATLAS from the excited proton in diffractive events will be almost free of pile-up.

This thesis has the main emphasis on the kinematics of the diffractively scattered proton measured by ALFA. Chapter 2 outlines some of the basic knowledge needed to understand the contents of this thesis. Chapter 3 gives a brief introduction to
the standard model of particle physics with special focus on the strong force which is 
believed to describe diffractive processes. The specific diffraction theory is given in 
chapter 4 where it is shown how diffraction is related to rapidity gaps. Furthermore, an 
overview of Regge theory is given and some of the problems by incorporating diffraction 
in this theory is mentioned.

Chapter 5 is devoted to a description of the LHC as well as a review of accelerator 
physics since the magnetic lattice of the LHC plays a crucial role in the reconstruction 
of the proton kinematics. The ATLAS detector, which is used to measure the fragments 
of the dissociated proton, is described in chapter 6, while the ALFA detector tracking 
the intact proton is described in chapter 7. The kinematic region of protons having 
a chance to hit the ALFA detector, and thereby provide information on the possible 
physics one can study with ALFA, is examined in chapter 8.

The main emphasis of this thesis is in chapter 9 where an algorithm to recon-
struct the kinematics of the diffractively scattered proton measured in ALFA is devel-
oped. LHC magnets are located between the interaction point and the ALFA detector, 
here the proton trajectory depends on the kinematics. The proton trajectory will be 
parametrised as a function of the kinematics and the location of the interaction point. 
From the proton impact in the ALFA detector it is possible to numerically minimize 
a $\chi^2$ function and thereby get the kinematics of the proton. The resolution of the 
reconstructed kinematics due to detector resolution will be studied as well as system-
atic effects from detector position. The algorithm will be tested on elastic events in 
a $\sqrt{s} = 7$ TeV, $\beta^* = 90$ m optics in order to estimate the uncertainties arising from 
the optics. Furthermore, the algorithm will be compared to an algorithm developed 
in Cracow.

The actual data from the special ALFA run at an $\sqrt{s} = 8$ TeV, $\beta^* = 90$ m optics is 
analysed in chapter 10. The kinematic distributions of the single diffractively scattered 
protons are studied and compared to a toy model Monte Carlo simulation. Also, 
the correlations between the proton kinematics and the measurement in ATLAS are 
investigate in order to determine whether the ATLAS measurement can be used to 
guide the kinematic reconstruction of the protons.
2 | Preliminaries

2.1 Relativity

The experiments at the LHC operate with particles of very high speed, therefore one needs to use special relativity. This section gives a brief review of the aspects needed in this thesis.

The energy of a particle is given by

\[ E = \gamma m c^2, \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}, \tag{2.1} \]

where \( c = 299.792.458 \) m/s is the speed of light in vacuum, \( v \) is the velocity of the particle, and \( m \) is the mass of the particle. Likewise, the momentum of the particle is given by

\[ p = \gamma m v. \tag{2.2} \]

Another useful expression for the energy is

\[ E^2 = m^2 c^4 + p^2 c^2, \tag{2.3} \]

hence in the large energy limit, the mass is negligible and one can make the approximation \( E = |p| c \).

The energy \( E' \) and momentum \( p' \) in a reference frame \( S' \) boosted with velocity \( v \) along the \( x \)-direction wrt. a frame \( S \) is given by the Lorentz transformation:

\[ E' = \gamma (E - vp_x), \tag{2.4} \]
\[ p'_x = \gamma (p_x - vE), \tag{2.5} \]
\[ p'_y = p_y, \tag{2.6} \]
\[ p'_z = p_z. \tag{2.7} \]

In experimental particle physics, it is common to choose a coordinate system where the beam particles move along the \( z \) axis and to change variables such that only one variable transforms under boosts along the beam direction. This gives three invariant variables which are chosen to be the mass \( m \), the momentum component transverse to the beam \( p_T \), and the azimuthal angle \( \phi \), where

\[ p_T = \sqrt{p_x^2 + p_y^2}, \tag{2.8} \]
\[ p_x = p_T \cdot \cos (\phi), \tag{2.9} \]
\[ p_y = p_T \cdot \sin(\phi). \tag{2.10} \]
CHAPTER 2. PRELIMINARIES

The last variable, which is not Lorentz invariant, is chosen to be the rapidity given by

\[ y = \frac{1}{2} \ln \frac{E + p_c}{E - p_c}, \quad (2.11) \]

Under a Lorentz boost along the \( z \) axis, the rapidity transforms as

\[ y \rightarrow y + \frac{1}{2} \ln(\gamma \sqrt{1 + \beta}), \quad (2.12) \]

hence the difference in rapidity between two particles boosted along the \( z \) axis is invariant.

In the massless limit, the rapidity reduces to

\[ y \approx -\ln \left( \tan \frac{\theta}{2} \right) \equiv \eta, \quad (2.13) \]

where \( \theta \) is the angle between the momentum of the particle and the \( z \) axis and \( \eta \) is called the pseudorapidity.

An important quantity for a system of many particles is the invariant mass \( M \) given by

\[ M^2 c^4 = E_{\text{system}}^2 - p_{\text{system}}^2 c^2, \quad \text{where} \quad E_{\text{system}} = \sum_{i=1}^{N} E_i, \quad p_{\text{system}} = \sum_{i=1}^{N} p_i, \quad (2.14) \]

and \( N \) is the number of particles. The mass of a decaying particle \( A \) is given by the invariant mass of the decay products:

\[ m_A c^4 = E_A^2 - p_A^2 c^2 = \left( \sum_{i=1}^{N} E_i \right)^2 - \left( \sum_{i=1}^{N} p_i \right)^2 = M^2 c^4, \quad (2.15) \]

where energy and momentum conservation has been invoked in the second step. In the centre-of-mass (CM) frame defined where \( p_{\text{total}} = 0 \), the invariant mass is equal to the total energy and for a single particle this is simply the mass.

2.2 Units

In elementary particle physics it is common to use units where \( c = 1 \) and \( \hbar = 1 \), thus everything can now be expressed in units of energy. The unit used for the energy chosen to be the electronvolt, eV. The result in normal units can be obtained by multiplying with powers of \( c \) and \( \hbar \) found by dimensional analysis. Throughout this thesis both momentum and energy is therefore given in eV (or keV, MeV, GeV, TeV).

2.3 Cross section

The cross section \( \sigma \) for a given process is a measure of how probable it is for the colliding particles to produce the final state. Classically, the probability is given by

\footnotesize
\[ ^1 \text{see appendix A for a full derivation.} \]
\[ ^2 \text{see appendix A for a full derivation.} \]

the areas of the colliding particles. In quantum mechanics, the particles are described by waves, hence the concept of size is somewhat meaningless, but the cross section can still be thought of as an area.

What is typically calculated in theory is not the cross section but the scattering amplitude $A$. The cross section is then found as the absolute square of the amplitude modulus some constants and summing over all allowed final states. For example the differential cross section for a $2 \rightarrow 2$ process is given by

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s} |p_1|^2 |A|^2,$$  \hspace{1cm} (2.16)

where $s$ is the CM energy squared, $t$ is the four-momentum transfer squared, $p_1$ is the CM momentum of one of the incoming particles, and $|A|^2$ is the amplitude summed over final state spins and colors and averaged over initial state spins and colors.

The luminosity $L$ is the number of particles per unit area per unit time. For a storage ring with two colliding beams we have

$$L = f n_1 n_2 \frac{N_1 N_2}{A},$$  \hspace{1cm} (2.17)

where $f$ is the revolution frequency, $n_i$ is the number of bunches in beam $i$, $N_i$ is the number of particles in each bunch, and $A$ is the transverse area of the bunches. The luminosity can be determined by van der Meer scans \cite{7}. The rate $R$ at which the process occur is therefore given by

$$R = L \cdot \sigma.$$  \hspace{1cm} (2.18)
To this day, only four fundamental forces of Nature has been observed:

- The electromagnetic force,
- The strong force,
- The weak force,
- Gravity.

Particle physics aims to describe Nature by its smallest constituents, the elementary particles, and their interactions through the four fundamental forces. The Standard Model (SM) is the modern description based on Quantum Field Theory and describes the electromagnetic, the strong and the weak interactions by gauge symmetries. Gravity is not yet included, and the lack of particle candidates for Dark Matter, an explanation for the observed neutrino masses, and other challenges show that the SM is still only an approximation. Nevertheless, this chapter gives an introduction to the SM since it has given very successful predictions. It will lead to a discussion about the strong force and why diffractive processes are not well understood.

The information in this chapter is based on ref. [8], [9] and [10].

### 3.1 The elementary particles

The particles of the SM are shown in figure 3.1 and are divided into three groups.

![Figure 3.1: The elementary particles of the Standard Model [11].](image)

---
The matter particles are spin-$\frac{1}{2}$ fermions. The fermions are further divided into leptons and quarks, and each of these groups have three generations. The masses of the particles are different in the three generations, but apart from that they have similar interactions. All fermions have a corresponding anti-particle carrying opposite color, weak, and electric charge.

The electron is the lightest of the charged leptons and was the first elementary particle to be discovered. The electrically neutral leptons are called neutrinos and are treated as massless in the SM. The upper bounds in their masses are experimental limits. Only left-handed leptons, i.e. leptons where the projection of the spin component on the momentum is negative, carry weak charge. None of the leptons have color charge.

All the quarks have electric and weak charge, and in addition they have also color charge. The quarks cannot exist freely but combines in colorless hadrons consisting of either three quarks (baryons) or a quark and an anti-quark (mesons). Protons and neutrons, which make up the atomic nucleus, are bound states of up-up-down quarks and up-down-down quarks, respectively.

The fundamental forces of the SM are described as exchanges of the force carrying spin-1 bosons. The photon mediates the electromagnetic force, the $Z^0$ and $W^\pm$ the weak force, and the gluon the strong force.

The spin-0 Higgs boson was introduced in the SM to give masses to $Z^0$ and $W^\pm$, but it gives masses to the other particles as well.

The interactions between the particles of the SM are described below.

### 3.2 Quantum Field Theory

The SM must be put in a framework where quantum mechanics and special relativity is combined since by the Heisenberg uncertainty principle it is seen that small length scales correspond to large momentum scales. Furthermore, the framework must incorporate particle production and annihilation. This is obtained in Quantum Field Theory (QFT) where the elementary particles are described by different kinds of fields. This section gives a brief introduction to QFT leading to a discussion of the *running coupling constants*. This running will play an important role in the description of the strong interactions which are the subject of this thesis. The theory is presented for a scalar field unless otherwise stated since this is the most simple form.

#### 3.2.1 The path integral

The transition between two states is described by the amplitude $A$. In the Feynman path integral formulation it is given by

$$A = \sum_{\text{all paths}} e^{\frac{i}{\hbar}S(\text{path})},$$

(3.1)

where *all paths* means all possible interactions fulfilling certain boundary condition for the specific amplitude, e.g. properties of the initial state such as mass and momentum. The action $S$ is given by

$$S(\text{path}) = \int_{\text{path}} d^4x \mathcal{L} (\phi_r(x), \partial_\mu \phi_r(x)),$$

(3.2)
where $\phi_r$ is the field of particle $r$ (summation over $r$ is implied) and

$$ (x) = (x^\mu) = (x^0, x^1, x^2, x^3) = (t, \mathbf{x}), \quad \partial_\mu \equiv \frac{\partial}{\partial x^\mu}. $$

The integrant $\mathcal{L}$ is the Lagrangian density but will throughout this thesis be called simply the Langrangian.

The classical path in the path integral (3.1) is the one which minimizes the action. Doing a variation of the action for fixed boundary conditions and setting it to zero one finds the Euler-Lagrange equations of motion:

$$ \frac{\partial \mathcal{L}}{\partial \phi_r} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi_r)} = 0. $$

The Lagrangian for a free scalar field $\phi$ is

$$ \mathcal{L} = \frac{1}{2} \partial_\mu \phi_r \partial^\mu \phi_r - \frac{1}{2} m^2 \phi_i \phi_i, $$

whereas for a free massive spin-$\frac{1}{2}$ field $\psi$ it is given by

$$ \mathcal{L} = i \bar{\psi} \gamma^\mu \partial_\mu \psi - m \bar{\psi} \psi, $$

where $\bar{\psi} \equiv \psi^\dagger \gamma^0$. Using the Euler-Lagrange equations on these Lagrangians gives the Klein-Gordon\(^1\) equation for the scalar field and the Dirac equation for the spin-$\frac{1}{2}$ field.

In the classical limit where $S \gg \hbar$, the sum of the paths in equation (3.1) cancels out due to the large difference between $S/\hbar$ for neighbouring paths, except in the neighbourhood of the classical path where the action is stationary and the amplitude reduces to the classical one.

### 3.2.2 Feynman diagrams and rules

Feynman diagrams and the associated rules are very useful for perturbation theory in QFT. The diagrams and rules can be extracted directly from the Lagrangian describing the theory. Consider a theory with the interaction

$$ V(\phi) = g \phi^n, $$

where $n$ is a positive integer larger than 2 (for $n = 2$ it is simply a mass term, see equation (3.5)). The procedure for drawing a Feynman diagram in $N$th order of perturbation theory in $g$ is as follows:

1) Write $N$ vertices on a piece of paper, each with $n$ legs.

2) Write a dot for each incoming and outgoing particle.

3) Connect all dots and legs with lines.

\(^1\)The Klein-Gordon equation states that $\partial_\mu \partial^\mu \phi + m^2 \phi = 0$ which can easily be obtained from the energy-momentum relation, $E^2 = p^2 + m^2$, by the replacement $E \rightarrow i \frac{\partial}{\partial t}$ and $p \rightarrow -i \nabla$. 

Typically, the dots of the incoming and outgoing particles are removed afterwards. An example of a Feynman diagram is shown in figure 3.2. Time flows from left to right and the arrows indicate the momentum direction.

Each element of the diagram represents a contribution to the path integral. A vertex gives a factor \(-g\), and because of 4-momentum conservation there is also a \(\delta\)-function at each vertex. The lines represent particles and each gives a propagator

\[
\frac{1}{p_i^2 + m_i^2},
\]

where \(p_i\) and \(m_i\) is the four momentum and mass of the scalar particle \(i\), respectively. The momenta of the incoming and outgoing particles are fixed by the boundary conditions, but the momenta of the internal particles are not. In the end, one has to integrate over all the internal four momenta. Different combinations of connecting the dots and vertices correspond to different paths in equation (3.1).

Different line styles are used to visualize the different kind of particles, i.e. scalars, fermions, gluons, and vector bosons. They all have different propagators. The vertex factor is found from the Lagrangian.

The Leading Order (LO) term in the path integral is the contribution from the diagrams with the lowest possible number of vertices which still describes the transition from initial to final state. The Next to Leading Order (NLO) is the contribution from diagrams with one more vertex, Next to Next to Leading Order (NNLO) is with two more vertices and so on. If the coupling constant \(g\) is less than one, a given precision can be reached within a finite order of perturbation theory.

### 3.2.3 Renormalization

The incoming and outgoing particles are on their mass shells, but this requirement does not apply for internal particles, i.e. particles connecting two vertices, where the momenta are undetermined. These momenta must be integrated out such that the amplitude is independent of intermediate and unmeasurable states. This can give rise to divergent diagrams like the one shown in figure 3.2. Momentum conservation at the first vertex gives the delta function \(\delta^4(p_1 + p_2 - q - q')\) such that the integral over \(q'\) simply gives

\[
q' = p_1 + p_2 - q.
\]

The integral over \(q\), on the other hand, diverges:

\[
\int \frac{d^4q_0 d^4q_1 d^4q_2 d^4q_3}{q_0^2 - q_1^2 - q_2^2 - q_3^2 + m^2} = \infty.
\]
These integrals can be renormalized and given a finite value in theories with four dimensions (which is the case for the SM), provided that no vertices with more than four legs are present. The details of the renormalization will not be given here, but one can write the contributions from the divergent integrals in the same analytic form as the terms in the Lagrangian, i.e. a mass term and coupling terms. The idea of renormalization is to add counter terms to the Lagrangian which cancels the divergent terms. The counter terms have the same structure as the divergent terms in the original Lagrangian, thus the renormalized Lagrangian can be written as

\[ \mathcal{L}_{\text{renormalised}}(\phi, m, \lambda) \equiv \mathcal{L}_{\text{original}}(\phi, m, \lambda) + \mathcal{L}_{\text{counter terms}}(\phi, m, \lambda) = \mathcal{L}_{\text{original}}(\phi_0, m_0, \lambda_0), \] (3.11)

which has the same functional form as the original Lagrangian but with the field \( \phi \), the mass \( m \), and the coupling constant \( \lambda \) replaced by the bare field, mass, and coupling constant. The divergences are now hidden in the bare parameters of the theory. The renormalized quantities \( \phi, m, \lambda \) are related to the bare quantities through a mass parameter \( \mu \) which has been added to the theory. At tree level, i.e. at LO where no loops are present in the Feynman diagrams, the renormalized and bare quantities are the same, but at a given order of perturbation theory they will differ.

The bare quantities may be interpreted as a short distance approximation where no interactions between the fields has taken place. As the distance increases, interactions occur and the renormalized quantities differ from the bare quantities.

The relation between the bare and the renormalized quantities depend on the mass scale (or inverse distance scale) \( \mu \). Using these relations one can obtain what happens if the momenta of the particles in the interaction are scaled with a factor \( t \),

\[ p_i \rightarrow tp_i. \] (3.12)

The coupling constant is then given by the following partial differential equation:

\[ t \frac{\partial \bar{\lambda}(t)}{\partial t} = \beta(\bar{\lambda}(t)) , \quad \bar{\lambda}(t = 1) = \lambda, \] (3.13)

and a similar equation can be written for the mass. The coupling \( \bar{\lambda} \) is not a constant with a scale of momentum and is called the running coupling constant, and \( \beta(t) \) can be calculated from the Lagrangian within perturbation theory.

The case of a negative \( \beta(\lambda) \) decreasing faster than \( \lambda \) will be important in the discussion of the strong force. Assuming that

\[ \beta(\lambda) = -\lambda^\alpha, \quad \alpha > 1, \] (3.14)

we get from equation (3.13) that

\[ t = \exp \int_\lambda^{\bar{\lambda}} \frac{d\lambda'}{\beta(\lambda')}d\lambda' \]
\[ = \exp \int_\lambda^{\bar{\lambda}} -(\lambda')^\alpha d\lambda' \]
\[ = \exp \left[ \frac{1}{1+\alpha} (\lambda^{1+\alpha} - \bar{\lambda}^{1+\alpha}) \right]. \] (3.15)
It is seen that
\[ \bar{\lambda} \rightarrow 0 \quad \text{as} \quad t = \exp \left[ \frac{\lambda^{1+a}}{1+\alpha} \right], \quad (3.16) \]
\[ \bar{\lambda} \rightarrow \infty \quad \text{as} \quad t \rightarrow 0. \quad (3.17) \]

At high momentum, the coupling constant goes to zero, while at low momenta it goes to infinity.

The \( \beta \) function have different behaviours depending on the theory. In QED, the \( \beta \) function leads to a coupling constant of \( \simeq 1/137 \) as \( t \rightarrow 0 \), but the coupling increases with \( t \) to \( \simeq 1/127 \) at the scale of the Z boson mass.

### 3.3 Symmetries of the SM

A symmetry operation leaves the system unaltered (or at least you cannot see the difference) and plays an important role in physics because of Noethers theorem. The theorem states that if a system is invariant under a certain symmetry operation, a conserved quantity is related to that symmetry. These presumed conserved quantities can be tested in experiments and can tell whether the system has the corresponding symmetry or not.

Parts of the SM Lagrangian are assumed to have several symmetries. All are supposed to be invariant under the Poincaré group which gives conservation of momentum and energy. Furthermore, some parts are believed to have charge conjugation where particles are changes to anti-particles, parity transformation which reverses the spatial part of the space-time, and time reversal where the time dimension is reversed. However, the most relevant symmetries of the SM for the content of this thesis are the gauge symmetries described below.

### 3.4 Gauge symmetries

The SM is based on the assumption that Nature possesses gauge symmetries. A theory is said to be gauge symmetric (or gauge invariant) under a certain group if the Lagrangian is unaltered by a gauge transformation of the fields:

\[ \phi_r(x) \rightarrow \phi'_r(x) = \exp[iM^a(x)\Lambda^a]\phi_r(x) = S(x)\phi_r(x). \quad (3.18) \]

Here, \( M^a \) are \( n \times n \) matrices representing generators of the group, \( a \) is a summation index running from 1 to the number of generators of the group, and \( \Lambda^a \) specifies how much \( \phi_r \) is rotated by the corresponding generator \( M^a \). Note that \( \Lambda^a \) depends on \( x \) which means that there is a different rotation to each point in spacetime. The amount of rotation needs not be the same for different fields in the theory.

The gauge transformation is a rotation of the field components in an internal space. Therefore, the mass term in the Lagrangian (3.5) is invariant since it is basically the length of the field. However, the derivative \( \partial_\mu \) does not transform covariantly, i.e in the same way as \( \phi \):

\[ (\partial_\mu \phi)' = S(\partial_\mu \phi) + (\partial_\mu S)\phi, \quad (3.19) \]
hence the kinematic term in the Lagrangian will in general not be gauge invariant. Instead, the normal derivative is replaced by a covariant derivative $D_\mu$ given by

$$D_\mu \phi = \partial_\mu \phi - igM^a A^a_\mu \phi,$$  \hspace{1cm} (3.20)

where $g$ is a coupling constant, and a gauge field $A^a_\mu$ has been introduced for each generator of the group. The gauge fields will couple to the field $\phi$.

Because we require $D_\mu \phi$ to transform covariantly, i.e.

$$(D_\mu \phi)' = SD_\mu \phi,$$  \hspace{1cm} (3.21)

the gauge fields must transform as

$$(A^a_\mu)' = SA^a_\mu S^{-1} - \frac{i}{g} (\partial_\mu S) S^{-1}.$$  \hspace{1cm} (3.22)

It is clear that mass terms for the gauge fields are not gauge invariant and are therefore not included in the Lagrangian. However, gauge invariant kinematic terms for the gauge fields are added.

## 3.5 The electroweak theory

It is a typical textbook example to show that Maxwells equations can be derived by making the Lagrangian for a complex scalar field gauge invariant under a $U(1)$ gauge transformation\(^{2}\) and using the Euler-Lagrange equation on the gauge field. The massless gauge field is then associated with the photon and the coupling constant $g$ with the electric charge $e$. Thus, the photon mediates the force between electrically charged particles, but the photon does not couple to itself since it has no electric charge.

Both the electromagnetic and the weak coupling constants are running due to equation 3.13 and at some point they become equal in strength, see figure 3.3. Glashow, Weinberg and Salam discovered that the electromagnetic and the weak force can be described by a unified $SU(2) \times U(1)$ symmetry. This introduces four massless vector gauge fields, one for $U(1)$ and three for $SU(2)$. Introducing a scalar field with a non-zero vacuum expectation value, the gauge symmetry is spontaneously broken. This is called the Higgs-mechanism, and the scalar field is identified with the newly discovered Higgs boson. Two of the $SU(2)$ gauge fields become massive and linear combinations of these are identified with the $W^\pm$ bosons. Likewise, the $Z^0$ boson is identified with a massive linear combination of the third $SU(2)$ gauge field and the $U(1)$ gauge field, whereas the photon is the massless linear combination. The electric charge of electromagnetism is then given by a combination of the weak hypercharge of the original $U(1)$ symmetry and one of the components of the weak isospin of the $SU(2)$ symmetry.

## 3.6 The strong force

Quarks carry electric and weak charge and hence interact via the electroweak force. However, they also carry color charge which gives rise to interactions via the strong

\(^{2}\) $U(1)$ is the group of all complex numbers $z$ with $|z| = 1.$
force. The theory of the strong force is described as a $SU(3)$ gauge theory and is called \textit{quantum chromodynamics} (QCD). There are 8 generators $F_{1,...,8}$ of $SU(3)$ which can be written as $3 \times 3$ matrices called the Gell-Mann matrices. The three color charges \textit{red, green, blue} are written as color spinors

$$r = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad b = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

(3.23)

and are eigenstates of $F_3$ and $F_8$, whereas the other generators change the color state. The gluons are the 8 gauge fields corresponding to the 8 generators. They carry linearly independent combinations of color and anti-color as can be seen from the fact that they change the color state and color is conserved in an interaction. Since the gluons carry color, they are able to couple to each other. The gluons are massless since the gauge symmetry is not broken as was the case in the electroweak theory.

The coupling constant of the strong force $\alpha_s$ is running. The $\beta$ function (3.13) is negative and decreases faster than $\lambda$. From equation (3.16) this implies that as the momentum goes to zero, i.e. at long distances, the coupling constant goes to infinity. This explains why no isolated color object has ever been observed which is known as \textit{color confinement}: as the distance between the quarks and gluons grows, so does the coupling. Eventually, it will be energetically more favourable that a quark anti-quark pair spontaneously appear. This is illustrated in figure 3.4.

Another consequence of the $\beta$ function in QCD is that the coupling constant goes to zero as the momentum goes to infinity. This is known as \textit{asymptotic freedom} and implies that the validity of perturbation theory increases with increasing momentum, and the quarks and gluons can to a first approximation be considered as free. The asymptotic freedom is used in the factorization of hadronic processes where the hadron-
Figure 3.4: Illustration of color confinement. As the quarks are pulled apart, it becomes energetically more favourable to create a quark-antiquark pair [13].

The hadron cross section $\sigma_X$ for the production of a system $X$ is written as [14]

$$\sigma_X(s, Q^2) = \sum_{a,b} \int_{x_{\text{min}}}^{1} dx_1 dx_2 f_{a/h_1}(x_1, Q^2) f_{b/h_2}(x_2, Q^2) \hat{\sigma}_{ab \rightarrow X}(x_1 x_2 s, M_X^2)$$

Here, $\hat{\sigma}$ is the cross section of the fundamental process between the partons $a$ and $b$ which are able to create the final state $X$. This can be calculated from QCD. The probability that parton $a$ from hadron 1 has the momentum fraction $x_1$ of hadron 1 is given by $f_{a/h_1}$ and is called a parton distribution function (PDF). The lower limit of the integral is the minimum fraction of momentum required to produce the final state, and $Q^2$ is the momentum transfer squared of the process.

The $x$ dependence of the PDFs are determined by lepton-hadron scattering, whereas the $Q^2$ dependence is calculated from the running coupling constant and is therefore dependent on the renormalization scale, $\mu$. The PDFs for a proton at two different mass scales are shown in figure 3.5. The valence quarks (up-up-down) of the proton radiates gluons inside the proton and these gluons can then radiate either gluons or quark-antiquark pairs. As the mass scale increases, the colliding protons are more likely to make a collision between the gluons, quarks or antiquark due to the smaller length scale of the protons and thereby smaller probability to hit the valence quarks. Also, the masses of the quarks become less important. Notice that no $b, \bar{b}$ quarks are visible at $Q^2 = 10 \text{ GeV}^2$ as the masses are too high. The gluons dominate over quarks by roughly a factor $1/\alpha_s$. The gluons are radiated from the valence quarks whereas the quarks are pair produced from the gluon and thereby an extra vertex with a factor $\alpha_s$ is needed. For a fixed mass scale, the $x_1$ and $x_2$ will decrease as the CM energy increases, thus the LHC mostly collide gluons.

As described in the following chapter, diffractive processes are $t$ channel processes with small momentum transfer. Therefore, the process is not expected to factorize like equation (3.24). The colorless object transferred in a diffractive process is called the pomeron and is thought to be a glueball which is a bound state of 2 or more gluons. In this way, the pomeron belongs to the $SU(3)$ gauge group of the strong force even though it is colorless. One can think of other possible descriptions of the pomeron, but as the SM is built upon gauge symmetries, one has to ask to which gauge group the pomeron belongs. The pomeron is expected to be able to interact with another pomeron since they both have an internal color structure. This can lead to specific patterns of the jets produced in diffractive processes and provide information about the non-perturbative region of QCD.
Figure 3.5: The MSTW 2008 NNLO PDFs at $Q^2 = 10 \text{ GeV}^2$ and $Q^2 = 10^4 \text{ GeV}^2$ [15].
4 | Diffraction theory

This chapter gives an introduction to the theory about diffraction which is the processes studied in this thesis. Two definitions of diffraction are discussed and a kinematic analysis shows that they are equivalent. The S-matrix theory is briefly introduced, since this is the foundation on which Regge theory relies. Regge theory is explained in some details. It has proved successful in describing elastic scattering, and the postulated pomeron introduced to describe diffraction is used in the PYTHIA8 event generator [5]. The information in this chapter is based on ref. [16] when nothing else is cited.

4.1 Definition

There exists no unique definition of hadronic diffractive processes. One definition is

1) A reaction in which no quantum numbers are exchanged between the colliding particles is, at high energies, a diffractive reaction.

This definition covers all of the four cases:

- **elastic scattering** where the final state particles are the same as the initial state particles, figure 4.1a.

- **single diffraction** (SD) when one of the incoming particles also leaves the collision while the other decays into a bunch of particles which of course must have the same quantum numbers as the mother particle, figure 4.1b.

- **double diffraction** (DD) when both particles decay into bunches, each of them carrying the quantum numbers of their respective mother particle, figure 4.1c.

- **central diffraction** (CD) when both particles leave the collision and an additional bunch of particles is produced with the quantum numbers of the vacuum, figure 4.1d.

The specification that it should be at high energies is necessary since otherwise it would not be possible to distinguish processes like

\[ p + n \rightarrow p + n \]  

(4.1)

with the exchange of a \( \pi^0 \) in figure 4.2a from that with the exchange of a \( \pi^+ \) in figure 4.2b. The former is a diffractive process while the latter is not since the \( \pi^+ \) carries electric charge. At high energies, the outgoing hadron will be in the forward direction of the incoming mother hadron because the momentum transfer is approximately
constant with CM energy for a given impact parameter. Therefore, the two processes can be distinguished, and the condition of no exchange of quantum numbers will be sufficient to characterize diffractive processes.

Another definition is

2) *A diffractive reaction is characterized by a large, non exponentially suppressed, rapidity gap in the final state.*

This definition is more useful in an experiment since typically some final state particles escape detection due to detector inefficiencies and limited angular coverage, thus it is difficult to know if quantum numbers have been exchanged. Non-diffractive processes may display rapidity gaps, but they are expected to be exponentially suppressed, i.e.

\[
\frac{dN}{d\Delta \eta} \sim e^{-\Delta \eta},
\]

which follows from the assumption of uniformly distributed particles in rapidity and Poisson statistics. This is not the case in diffractive event which becomes clear when the kinematics of such a process is studied.

### 4.2 Kinematics

This section described some of the kinematics in two-body reactions

\[
1 + 2 \rightarrow 3 + 4,
\]

(4.3)
as well as single-inclusive processes

\[
1 + 2 \rightarrow 3 + X,
\]

(4.4)
where $X$ is a system of particles. Single diffraction belongs to the latter. It will be shown that the two definitions of a diffractive process stated in the previous section are in fact equivalent.

### 4.2.1 Two-body processes

The particles can be described by their 4-momentum, $p$. However, not all 16 parameters are independent. The conservation of 4-momentum ($p_1 + p_2 = p_3 + p_4$) gives 4 constraints, the mass shell condition ($p_i^2 = m_i^2$) gives 4 constraints, and fixing a reference frame gives 6 constraints since the 3-momenta of the initial state particles are now determined. The two remaining variables are often chosen to be two of the three Lorentz invariant Mandelstam variables

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2,$$
$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2,$$
$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2.$$ (4.5)

These variables can be interpreted as the CM energies in the $s, t,$ and $u$ channels, respectively,

$$1 + 2 \rightarrow 3 + 4 \quad (s - \text{channel}),$$ (4.6)
$$1 + \bar{3} \rightarrow \bar{2} + 4 \quad (t - \text{channel}),$$ (4.7)
$$1 + \bar{4} \rightarrow \bar{2} + 3 \quad (u - \text{channel}),$$ (4.8)

where a bar indicates an antiparticle with the opposite momentum. They can also be visualized as the diagrams shown in figure 4.3.

The Mandelstam variables fulfil

$$s + t + u = \sum_i m_i^2,$$ (4.9)

which, for equal mass scattering, gives the physical region of the variables in the $s$-channel:

$$s > 4m^2, \quad t < 0, \quad u < 0.$$ (4.10)

In the center-of-mass frame we can write the 4-momenta of the particles as

$$p_1 = (E_1, \mathbf{p}) = (E_1, 0, 0, p_z),$$
$$p_2 = (E_2, -\mathbf{p}) = (E_2, 0, 0, -p_z),$$
$$p_3 = (E_3, \mathbf{p}') = (E_3, \mathbf{p}_T, p_3'),$$
$$p_4 = (E_4, -\mathbf{p}') = (E_4, -\mathbf{p}_T, -p_4').$$ (4.11)

![Figure 4.3: The three Mandelstam variables interpreted as the CM energies in their respective channels. (a) s-channel, (b) t-channel, (c) u-channel.](image-url)
where the coordinate system has been chosen such that the momenta of particle 1 and 2 only have a $p_z$ component. The energy of particle 1 can be written in terms of the masses and Mandelstam $s$:

\[
\begin{align*}
    s &= (p_1 + p_2)^2 \\
    &= (E_1 + E_2)^2 - (p_1 + p_2)^2 \\
    &= E_1^2 + E_2^2 + 2E_1E_2 \\
    &= E_1^2 + |p_2|^2 + m_2^2 + 2E_1\sqrt{m_1^2 + |p_2|^2} \\
    &= E_1^2 + |p_1|^2 + m_1^2 + 2E_1\sqrt{m_2^2 + |p_1|^2} \\
    &= 2E_1^2 + m_1^2 + m_2^2 + 2E_1\sqrt{m_1^2 + E_1^2} - m_1^2, \\
\end{align*}
\]

(4.12)

where it has been used that particle 1 and 2 are on their mass-shell. Same calculation can be done for the other particles which gives

\[
E_{1,2,3,4} = \frac{1}{2\sqrt{s}}(s + m_{1,2,3,4}^2 - m_{1,2,3,4}^2).
\]

(4.13)

From the mass-shell condition $E_i^2 = m_i^2 + p_i^2$ one finds the expressions for the momenta:

\[
\begin{align*}
    p &= E_1^2 - m_1^2 = \frac{1}{4s} \left[ s - (m_1 + m_2)^2 \right] \left[ s - (m_1 - m_2)^2 \right], \\
    p' &= E_3^2 - m_3^2 = \frac{1}{4s} \left[ s - (m_3 + m_4)^2 \right] \left[ s - (m_3 - m_4)^2 \right]. \\
\end{align*}
\]

(4.14)

### 4.2.2 Single-inclusive processes

For a single-inclusive process, the $X$ system in equation (4.4) is not a particle on the mass shell but an unmeasured system of particles, thus there is one less constraint on the kinematic variables. Therefore, the invariant mass of the $X$ system $M^2$ is used. It is given by

\[
M^2 = (\sum_i E_i)^2 - (\sum_i p_i)^2 = (E_1 + E_2 - E_3)^2 - (p_3 - p_1 - p_2)^2 = (p_1 + p_2 - p_3)^2, 
\]

(4.15)

where the sum is over all particles in the $X$ system and energy-momentum conservation has been used in the second step. Using again the CM frame, we get in the limit where $s, M^2 \gg m_1^2, m_2^2, m_3^2$ that

\[
\begin{align*}
    E_1, E_2 &\simeq \frac{\sqrt{s}}{2}, \quad |p| = p_z \simeq \frac{\sqrt{s}}{2}, \\
    E_3 &\simeq \frac{s - M^2}{2\sqrt{s}}, \quad |p'| \simeq \frac{s - M^2}{2\sqrt{s}}. \\
\end{align*}
\]

(4.16)

The relative energy loss $\xi$ is in this limit

\[
\xi \equiv 1 - \frac{E_3}{E_1} \simeq 1 - \frac{|p'|}{p_z} \simeq \frac{M^2}{s}.
\]

(4.17)
Let us look the limit where \( s \to \infty \) and where all particles in the reaction including those of the composite system \( X \) have mass \( m \). In this limit we have \( p_z \to \infty \) and the rapidity reduces to
\[
y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} = \frac{1}{2} \ln \frac{(E + p_z)^2}{E^2 - p_z^2} = \ln \frac{E + p_z}{m_T} \approx \ln \frac{2p_z}{m_T},
\]
where the tranverse mass \( m_T \) is defined as
\[
m_T \equiv \sqrt{m^2 + p_T^2}.
\]
The momentum transfer in hadronic scattering is typically small, hence \( p'_z \approx \lvert p' \rvert \).
Therefore, using equation (4.16) for fixed \( M \), i.e. not taking into account any physics processes, we get
\[
y_3 \approx \ln \frac{2p_z}{m_T} \approx \ln \frac{s - M^2}{m_T} \approx \ln \frac{\sqrt{s}}{m_T}.
\]
The maximum value of \( y_3 \) is reached when \( m_T \) is at minimum, that is when \( p_T = 0 \):
\[
(y_3)_{\text{max}} = \ln \frac{\sqrt{s}}{m}.
\]
The system \( X \) is distributed in a range of rapidity due to the fact that it is a many particle system. The highest rapidity of the \( X \) system in absolute value is obtained when a single particle carries most of the momentum \( p_z \sim \sqrt{s}/2 \) and \( p_T = 0 \Rightarrow m_T = m \), thus
\[
\lvert y_X \rvert_{\text{max}} \approx \ln \frac{\sqrt{s}}{m}.
\]
The lowest rapidity of \( X \) is for a particle with the maximum transverse mass \( \sim M \) and the minimum momentum \( \sim (m/M)\sqrt{s}/2 \) (where it is assumed that all the particles in \( X \) share the \( p_z \) momentum equally), hence
\[
\lvert y_X \rvert_{\text{min}} \approx \ln \frac{m\sqrt{s}}{M^2}.
\]
Since the masses of the particles are small compared to the momentum, the rapidity can be approximated by the pseudorapidity, and we find the relation between the relative energy loss and the rapidity gap between particle 3 and the system \( X \):
\[
\Delta \eta \simeq \Delta y \simeq (y_3)_{\text{max}} + \lvert y_X \rvert_{\text{min}} \simeq \ln \frac{s}{M^2} \Rightarrow \xi \simeq e^{-\Delta \eta},
\]
where \( \xi \) is defined in equation (4.17).

### 4.2.3 Diffractive processes

The longitudinal momentum transfer for a diffractive process characterized by definition 1 is typically [17]
\[
|\Delta p_z| \lesssim \frac{1}{R},
\]
where \( R \) is the size of the target. For at proton with \( R \sim 1 \) fm this corresponds to \( |\Delta p_z| \lesssim 1 \) GeV. Therefore, from eq. (4.16) we have
\[
|\Delta p_z| = |p_z - p'_z| \approx \frac{M^2}{2\sqrt{s}} \lesssim 1 \text{ GeV} \Rightarrow \frac{M^2}{s} \ll 1 \Rightarrow \xi \simeq 0,
\]
and the rapidity gaps will therefore be large in diffractive events where no quantum numbers are exchanged. We have hereby shown that definition 1 implies definition 2.
4.3 \textit{S}-matrix theory

The $S$-matrix theory was thought as an alternative to QFT in the description of the strong interactions and some of these ideas are still useful.

The $S$-matrix transforms the initial state $|i\rangle$ to the final state $|f\rangle$:

\[
S|i\rangle_{t=-\infty} = |f\rangle_{t=\infty},
\]

where $t$ is the time. The probability for the transition is defined as

\[
P_{i\rightarrow f} \equiv |\langle f|S|i\rangle|^2.
\]

Typically one writes

\[
S = 1 + iT \Rightarrow \quad S_{if} \equiv \langle f|S|i\rangle = \delta_{if} + iT_{if} = \delta_{if} + i(2\pi)^4\delta^4(p_f - p_i)A(i \rightarrow f),
\]

where in the last step the four-momentum conservation has been extracted from $T$ to give the relativistic scattering amplitude $A(i \rightarrow f)$.

The $S$-matrix must be relativistically invariant, therefore we want to express it in terms of Mandelstam variables. Furthermore, we assume unitarity, analyticity and crossing symmetry.

\textbf{Unitarity}

Unitarity follows from conservation of probability:

\[
\sum_k p_{i\rightarrow k} = \sum_k |\langle k|S|i\rangle|^2 = \sum_k \langle i|S^\dagger|k\rangle\langle k|S|i\rangle = \langle i|S^\dagger S|i\rangle = 1.
\]

Written in terms of $T$ this gives

\[
(1 - iT^\dagger)(1 + iT) = 1 \Rightarrow \\
i(T^\dagger - T) = T^\dagger T \Rightarrow \\
i\langle f|T^\dagger - T|i\rangle = \sum_n \langle f|T^\dagger|n\rangle\langle n|T|i\rangle \Rightarrow \\
2\text{ Im } T_{if} = \sum_n T_{fn}^*T_{in}.
\]

If $|i\rangle = |f\rangle$, i.e. forward elastic scattering ($t = 0$), we get the optical theorem:

\[
\sigma_{\text{tot}} \simeq \frac{1}{s} \text{ Im } A_{\text{el}}(s, t = 0),
\]

where $A_{\text{el}}$ is the amplitude for elastic scattering.

\textbf{Analyticity}

Analyticity means that $A(s, t)$ is an analytic function of $s$ and $t$ when these are continued to complex values, and the physical $A$ in the $s$-channel process (4.6) is obtained when $s, t \rightarrow \text{real}$, and $s > 4m^2, t < 0$. 

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CHAPTER 4. DIFFRACTION THEORY

Crossing symmetry

Crossing symmetry is postulated and states that the same function $A(s, t, u)$ describes the $s, t$ and $u$ channel reactions in the different physical domains. Knowing $A$ in one channel, it should be possible to analytically continue it to the other channels. In other words: If $A$ is known in the reaction

$$1 + 2 \rightarrow 3 + 4 \quad \text{where} \quad s \geq 4m^2, t < 0, u < 0 \ , \quad (4.33)$$

it should be possible to use this $A$ to find a function describing the reaction

$$1 + \bar{3} \rightarrow \bar{2} + 4 \quad \text{where} \quad t \geq 4m^2, s < 0, u < 0 \ . \quad (4.34)$$

Other consequences

Another important consequence derived from $S$-matrix theory is the Froissart-Gribov bound:

$$\sigma_{\text{tot}}(s) \leq C \cdot \ln^2 s \ , \quad \text{where} \quad C \geq \frac{\pi}{m^2 \pi} , \quad (4.35)$$

which puts an upper limit on the asymptotic behaviour of the total cross section. This bound will be discussed in section 4.4.2.

4.4 Regge theory

Regge theory follows from the assumption of analyticity and crossing symmetry of the $S$-Matrix and therefore of the scattering amplitude $A$. The theory will be shown for elastic two-body reactions and extended to diffractive processes.

The Legendre polynomials $P_l$ form an orthonormal basis for functions in the interval $[-1, 1]$ [18]. Therefore, the $\theta$ dependence, i.e. the scattering angle, in the scattering amplitude $A(s, \cos \theta)$ in the $s$-channel can be written as

$$A(s, z) = \sum_{l=0}^{\infty} (2l + 1) A_l(s) P_l(z), \quad z \equiv \cos \theta = 1 + \frac{2t}{s - 4m^2} , \quad (4.36)$$

where $l = 0, 1, 2, \ldots$ is the angular momentum of the scattered particle. This representation of $A(s, z)$ is well defined when

$$s \geq 4m^2 \quad \text{and} \quad -1 \leq z \leq 1 , \quad (4.37)$$

i.e. in the physical region of the $s$-channel. However, since in $S$-matrix theory one assumes crossing symmetry, we want a representation of $A(s, z)$ valid in the $t$-channel (4.7) where $t \geq 4m^2$ since now $t$ is the CM energy. Here, the representation (4.36) becomes meaningless since $|z| > 1$ which is outside the valid interval for the orthonormal basis of the Legendre polynomials.

The problem is solved by letting the angular momentum take complex values such that the partial wave amplitudes $A_l(s)$ are now $A(s, l)$ where

$$A(l, s) \rightarrow A_l(s) \quad \text{for} \quad l = 0, 1, 2, \ldots \ . \quad (4.38)$$
and let $A(l, s)$ meet some extra requirements. Using the theory about functions of a complex variable\footnote{\cite{ref:19}} and assuming that the singularities of $A(l, s)$ are \textit{poles} we get

$$A(s, t) \sim_{t \to \infty} -\beta(s) \frac{t^{\alpha(s)}}{\sin \pi \alpha(s)} \quad \text{(s-channel)},$$

(4.39)

where $\alpha(s)$ is the location of the pole of $A(l, s)$ in the complex $l$-plane with the largest real component, and $\beta(s)$ is the residue function. The poles are called \textit{Regge poles} and $\alpha(s)$ is called a \textit{Regge Trajectory}.

Starting instead in the $t$-channel and continue to the $s$-channel one gets

$$A(s, t) \sim_{s \to \infty} -\gamma(t) \frac{s^{\alpha(t)}}{\sin \pi \alpha(t)} \quad \text{(t-channel)}.$$

(4.40)

The residue functions $\beta$ and $\gamma$ are completely unknown, however from the crossing symmetry assumption we know that they must have the same analytic form such that the analytic form of $A(s, t)$ in the $t$-channel can be found by interchanging $s$ and $t$ in $A(s, t)$ in the $s$-channel.

The expression (4.40) shows that the singularity in the complex angular momentum plane of the partial wave amplitude with the largest real value in the $t$-channel determines the asymptotic behaviour in the $s$-channel.

The procedure can be extended to include non-zero spin particles, branch cuts in the complex $l$-plane, etc., but that will not be dealt with here.

### 4.4.1 Resonances

Near a Regge pole, the partial wave amplitude is

$$A(l, t) \sim_{t \to \alpha(t)} -\frac{\beta(t)}{l - \alpha(t)}.$$

(4.41)

If we assume that for a real $t_0$ we can write

$$\alpha(t_0) = l + i\epsilon,$$

(4.42)

where $\epsilon$ is real and smaller than 1, we can do a Taylor expansion

$$\alpha(t) = l + i\epsilon + \alpha'(t_0)(t - t_0) + \ldots$$

(4.43)

and we find

$$A(l, t) \sim_{t \to \alpha(t)} \frac{\beta(t)}{\alpha'(t_0)(t - t_0) + i\epsilon/\alpha'(t_0)}.$$

(4.44)

This is the structure of a Breit-Wigner, hence the Regge poles are resonances when $t$ is real and positive and $\alpha(t)$ interpolates these resonances. Therefore, the interpretation of equation 4.40 is that the large $s$-limit of the scattering amplitude is due to the exchange of resonance(s) with the mass $\sqrt{t_0}$. These resonances are found experimentally in elastic scattering experiments and proves the usefulness of Regge theory for those processes.
CHAPTER 4. DIFFRACTION THEORY

Some of the mesonic resonances are shown in figure 4.4. The resonances have quantum numbers and are exchanged in different hadronic processes. For small \( t \) one can write \( \alpha(t) \) as

\[
\alpha(t) = \alpha(0) + \alpha' t, \tag{4.45}
\]

but as seen on the figure, this is valid for a range of \( t \) up to at least 6 GeV\(^2\) with \( \alpha(0) \simeq 0.5 \) and \( \alpha' \sim 1 \text{ GeV}^{-2} \).

Figure 4.4: The mesonic trajectories with the largest value of \( \alpha(0) \) [20].

In Regge theory, the elastic cross section is given by

\[
d\sigma_{\text{el}} \over dt = F(t) s^{2\alpha(t)-2} = F(t) s^{2\alpha(0)-2} e^{-2\alpha'|t| \ln s}, \tag{4.46}
\]

from where it can be seen that as \( s \) grows, the width in \( t \) shrinks, i.e. the scattering angle decreases. This is indeed observed in data and lends another support to the validity of Regge theory.

4.4.2 The pomeron trajectory

At large \( s \) only one pole dominates the scattering amplitude, equation (4.40). Via the optical theorem one can write the total cross section in the presence of only one pole as

\[
\sigma_{\text{tot}} \overset{s \to \infty}{\sim} \frac{1}{s} \text{Im} A(s, t = 0) \overset{s \to \infty}{\sim} s^{\alpha(0)-1}. \tag{4.47}
\]

From figure 4.4 we have \( \alpha(0) \simeq 0.5 \) and this leads to a decrease in the cross section when the energy is increased. In the 1960’s it was expected that the cross section was constant in the asymptotic limit and a Regge trajectory with \( \alpha(0) = 1 \) called the pomeron \( P \) was introduced to explain this. Therefore, in the high energy limit the pomeron will dominate elastic and diffractive processes, where no quantum numbers are exchanged in the \( t \)-channel, thus the pomeron must have quantum numbers of the vacuum. The SD process can then be described by the Feynman diagram shown in figure 4.5.
Fits to the total cross sections for different types of elastic scattering shows that

\[ \alpha_P(0) = 1.0808 \quad \text{and} \quad \alpha_p' \approx 0.25 \text{ GeV}^{-2}, \quad (4.48) \]

i.e. a total cross section increasing faster than \( s \). With this value of \( \alpha_P \), the total cross section will violate the Froissart-Gribov bound (4.35) at large enough \( s \), i.e. unitarity will be violated, and the theory must be corrected for at some point. Also, the elastic data can be fitted equally well with a logarithmic growth, thus the existence of the pomeron is not necessarily the correct solution.

No resonances are observed on the pomeron trajectory, hence the pomeron does not have a mass. The pomeron is expected to be a glueball, i.e. exchange of at least two gluons since it must have quantum numbers of the vacuum, see section 3.6.

### 4.4.3 SD cross sections

When \( s \gg M^2 \gg |t| \), Regge theory predicts that the SD differential cross section is given by

\[ \left. \frac{d\sigma_{SD}}{dM^2dt} \right|_{t=0} \sim \frac{1}{(M^2)^{\alpha_P(0)}}. \quad (4.49) \]

What will be measured in chapter 10 is the cross section as a function of \( \xi \). Assuming \( \xi = M^2/s \) from equation (4.17), this gives

\[ \left. \frac{d\sigma_{SD}}{d\xi dt} \right|_{t=0} \sim \frac{s}{(s\xi)^{\alpha_P(0)}}, \quad (4.50) \]

which, with the value of \( \alpha_P(0) \) in equation (4.48), gives a decreasing cross section as a function of \( \xi \). However, usually a value of \( \xi \) of 0.05 or 0.1 is thought to be the upper limit for which Regge theory still applies.

The \( t \) dependence on the cross section is assumed to be

\[ \frac{d\sigma_{SD}}{dt} \sim e^{(B_0 + 2\alpha_p' \ln s)t}, \quad (4.51) \]

which favours small values of \( t \) (remember that \( t \) is negative in the physical \( s \)-channel).

Regge theory also predicts that

\[ \frac{\sigma_{SD}}{\sigma_{tot}} \sim s^{\alpha_P(0)-1}, \quad (4.52) \]

i.e. a rise with energy since \( \alpha_P(0) > 1 \). However, data shows that the ratio is decreasing, and a renormalization of the SD cross section has been suggested to account for this decrease.
4.5 Diffraction at the LHC

Clearly, the Regge theory of diffraction processes is not complete. Predictions about the elastic cross sections are observed in data while the diffractive results are more ambiguous. More data will hopefully help to refine (or reject) the theory. At the LHC, the total cross section are studied at higher $s$ which will provide data closer to the asymptotic behaviour. Also, the large $s$ leads to higher $M$ of the dissociated system and $d\sigma^{SD}/dM^2$ can be tested in a new regime. The possibility to track the proton in the ALFA detector and thereby reconstruct the energy loss gives an indirect measure of $M$ without the need for corrections of the direct measure of $M$ in ATLAS.
5 The Large Hadron Collider

This chapter gives an overview of the Large Hadron Collider (LHC) at CERN. The LHC provides the colliding protons to the experiments, but normally, the specific conditions of the proton beams have no interest except for the center-of-mass energy and the luminosity. For diffractive physics with the ALFA detector, however, the beam conditions are crucial in the understanding of the surviving proton.

5.1 The CERN accelerator complex

The information in this section is based on ref. [21].

At the LHC, protons are colliding with a centre of mass energy up to 14 TeV. In order to reach this high energy, the protons are passed through a series of accelerators. The CERN accelerator complex is shown in figure 5.1.

![CERN's accelerator complex](image-url)

**Figure 5.1:** The CERN accelerator complex [22].
Hydrogen molecules are passed through a strong electric field in order to strip off the electrons. The remaining protons are now injected to the Linear accelerator where radiofrequency (RF) cavities accelerate the protons to energies of 50 MeV. The protons then enters several synchrotron rings (the Proton Synchrotron Booster, the Proton Synchrotron and the Super Proton Synchrotron) before injection in the LHC at an energy of 450 GeV.

The LHC is a synchrotron ring of 26659 m in circumference. It is buried 50-175 m underground and crosses the borderline between France and Switzerland. The main aim of the LHC is to accelerate protons\(^1\) and collide them with a centre of mass energy of up to 14 TeV.

The LHC consists of two beam pipes each with a diameter of about 3 cm. Beam 1 has protons moving clockwise as seen from above and beam 2 has particles moving counter-clockwise. The beam pipes have four crossing points where the beams are colliding and the ATLAS, ALICE, CMS and LHCb experiments are located at these crossing points. The beam pipes have an ultrahigh vacuum of \(\sim 10^{-7} \text{ Pa}\) in order to reduce the collision probability of beam particles with air molecules in the beam pipe. Such a collision brings the colliding protons out of their orbit and a beam halo is created.

Each beam pipe has 8 radiofrequency (RF) cavities located between ALICE and CMS. The RF cavities are used to accelerate the protons. They operate at a frequency of 400 MHz and the RF voltage per beam is increased from 8 MV at injection to 16 MV at collision with \(\sqrt{s} = 14 \text{ TeV}\).

The LHC has a total of 9593 magnets to guide the protons in the beam pipe. 1232 of these are superconducting dipole magnets cooled to a temperature of 1.9 K in order to obtain a magnet strength of up to 8-8.5 T. These dipoles are used to bend the protons in the circular orbit while the rest of the magnets in the LHC are dipoles, quadrupoles, and higher order magnets used for corrections. Just as for the RF cavities, the magnets are ramped as the proton energy is increased.

5.2 Accelerator theory

This section gives a brief theoretical description of the purpose of the main elements in the LHC. The magnetic lattice plays an important role in the kinematic reconstruction of protons described in chapter 9. More details on accelerator optics can be found in ref. [23] and [24].

5.2.1 RF cavities

At injection in the LHC, the protons have an energy of 450 GeV and therefore must be accelerated further before collision at up to \(E_{\text{proton}} = 7 \text{ TeV}\). Since the protons are charged particles, electromagnetic fields will act on the protons according to the Lorentz force

\[
F = q(E + v \times B),
\]

where \(q\) is the charge of the proton and \(v\) is the velocity, \(E\) is the electric field and \(B\) is the magnetic field.

\(^1\)Also heavy ions are accelerated in special runs, but this will not be considered here.
The RF cavities use a longitudinal electric field to accelerate the protons. A proton following the design orbit, i.e. the ideal path, will always hit the electric field in the same phase and thus get the same energy transfer.

At low energies, protons with more energy than the design proton reach the RF cavity earlier due to their higher speed and thus get a smaller energy transfer, while slower moving protons get a larger energy transfer, see figure 5.2. In this way, the RF cavities reduce the energy divergences of the beam protons during the acceleration. Furthermore, if a proton is too fast (slow) it will get a kick larger (smaller) than the design proton and may instead start to oscillate around another design proton further ahead (behind) in the beam pipe. Thus, the protons will be grouped into bunches where the protons oscillate around the design proton.

At injection in the LHC, the energies of the protons are already so high that all protons roughly have the speed of light. This gives a situation opposite to the one sketched in figure 5.2 since more energetic protons have a larger orbit due to their higher momenta, and therefore they arrive later than the less energetic protons. The RF cavities are then synchronized such that the design proton hits the cavity on the right hand side of the maximum instead of the left hand side shown in figure 5.2. Hereby, a more energetic proton arrives later and thus gets a smaller boost than the design proton. The process of bunch grouping still applies.

5.2.2 Dipole magnets

Superconducting dipole magnets are used to bend the protons to obtain the circular orbit. Already at injection in the LHC, the protons have \( v \approx c \), hence from (5.1) it is seen that a transverse magnetic field will be much more efficient in bending the protons than an electric field. A magnetic field in the \( y \) direction perpendicular to the \((x, z)\)-plane of the accelerator orbit acts on the protons with a force \( F_x = -qvB_y \) in the horizontal plane and thereby bends the protons.

Dipole magnets can also be used to focus protons deviating from the design orbit. We can define a co-moving system shown in figure 5.3 where \( \rho \) is the radius of the LHC, \( x \) is the radially outward displacement from the design orbit\(^2\), \( y \) is the upward

\(^2\)Actually, the LHC coordinate system is defined with \( x \) pointing towards the center of the ring, but this detail does not change the results discussed here.
displacement, $\theta$ is the angle measured from the center of the LHC, and we define $R = \rho + x$.

Since the energy variation of the protons in the dipole magnet is negligible, we have $d\gamma/dt \approx 0$. Considering a monochromatic beam of proton, i.e. all protons have the same momentum, the horizontal equation of motion is

$$\frac{dp}{dt} = \frac{d}{dt}(\gamma m \ddot{x} - \frac{v^2}{R}) = -qvB_y(R),$$  \hspace{1cm} (5.2)

where the term $\gamma mv^2/R$ is the acceleration due to the circular motion of the design particle. The right hand side of the equation is the resulting force given by the Lorentz force (5.1) where we let the magnetic field be a function of $R$.

Since $dv/dt \approx 0$ and $\theta = v \cdot t/R$, we get

$$\gamma m \left( \frac{d^2\theta}{dt^2} \frac{d^2x}{d\theta^2} - \frac{v^2}{R} \right) = \gamma m \left( \frac{v^2}{R} \frac{d^2x}{dt^2} - \frac{v^2}{R} \right) = -qvB_y(R) \Leftrightarrow$$  \hspace{1cm} (5.3)

$$\frac{d^2x}{d\theta^2} + \left( \frac{qB_y(R)}{\gamma mv} R - 1 \right) R = 0.$$  \hspace{1cm} (5.4)

The variation of $x$ is zero for a particle following the design orbit, hence from eq. (5.2) we get

$$\gamma m \frac{v^2}{\rho} = qvB_y(\rho) \Leftrightarrow B_y(\rho) \equiv B_0 = \frac{\gamma mv}{q\rho}. \hspace{1cm} (5.5)$$

A Taylor expansion of eq. (5.4) in $x \ll \rho$ gives

$$\frac{d^2x}{d\theta^2} + \left( \frac{qB_y(R)}{\gamma mv} R - 1 \right) R = \frac{d^2x}{d\theta^2} + \left( \frac{qB_y(R)}{\gamma mv} \rho \left( 1 + \frac{x}{\rho} \right) - 1 \right) \rho \left( 1 + \frac{x}{\rho} \right) = 0 \Rightarrow$$

$$\frac{d^2x}{d\theta^2} + \left[ \frac{1}{B_0} \left( B_y(\rho) + \frac{\partial B_y}{\partial x} x \right) \left( 1 + \frac{x}{\rho} \right) - 1 \right] \rho \left( 1 + \frac{x}{\rho} \right) \approx 0 \Rightarrow$$

$$\frac{d^2x}{d\theta^2} + \left[ \left( 1 + \frac{1}{B_0} \frac{\partial B_y}{\partial x} x \right) \left( 1 + \frac{x}{\rho} \right) - 1 \right] \rho \left( 1 + \frac{x}{\rho} \right) \approx 0,$$  \hspace{1cm} (5.6)

and since $\frac{1}{B_0} \frac{\partial B_y}{\partial x} \ll 1$ (i.e. the magnetic field varies only slowly with $x$) and $\frac{x}{\rho} \ll 1$, only terms linear in these are kept and we get

$$\frac{d^2x}{d\theta^2} + \left( \frac{x}{\rho} + \frac{1}{B_0} \frac{\partial B_y}{\partial x} \right) \approx 0.$$  \hspace{1cm} (5.7)
The same derivation applies for the vertical direction except that there is no centripetal force. Defining now the field index
\[ n = -\frac{\rho}{B_0} \left( \frac{\partial B_y}{\partial x} \right)_{x=0}, \tag{5.8} \]
we can write the equations for the transverse motion as
\[ \frac{d^2 x}{d\theta^2} + (1 - n)x = 0 \quad \frac{d^2 y}{d\theta^2} + ny = 0. \tag{5.9} \]
For \( 0 < n < 1 \) we have focusing in both horizontal and vertical direction and the protons will oscillate around the design orbit in the plane transverse to the beam direction. This is called betatron oscillations.

Equation (5.9) can be extended to protons with different momenta and the solution can be written in the matrix form
\[ \begin{pmatrix} x \\ x' \\ \delta p \\ p \end{pmatrix} = M(\theta) \begin{pmatrix} x_0 \\ x'_0 \\ \delta p_0 \\ p_0 \end{pmatrix}, \tag{5.10} \]
where the transfer matrix, \( M \), is given by
\[ M(\theta) = \begin{pmatrix} \cos (\sqrt{1-n} \theta) & -\frac{\rho}{\sqrt{1-n}} \sin (\sqrt{1-n} \theta) & \frac{\rho}{\sqrt{1-n}} [1 - \cos (\sqrt{1-n} \theta)] & -\frac{\sqrt{1-n}}{\rho} \sin (\sqrt{1-n} \theta) \\ -\frac{\rho}{\sqrt{1-n}} \sin (\sqrt{1-n} \theta) & \cos (\sqrt{1-n} \theta) & 0 & -\frac{\sqrt{1-n}}{\rho} \sin (\sqrt{1-n} \theta) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{5.11} \]

5.2.3 Quadrupole magnets

It is seen from equation (5.11) that the betatron amplitude scales with \( \rho \), i.e. with the radius of the synchrotron ring. Therefore, instead of having a field index \( n \) in the dipole field, quadrupoles are used to focus the beam of protons in the plane transverse to the beam direction. If only bending dipoles were used, it would be necessary to have a very large beam pipe and hence also large magnets, which are very expensive.

The field of a quadrupole can be described as the gradient of a potential:
\[ B = -\nabla V, \quad V(x,y) = gxy, \tag{5.12} \]
hence from the Lorentz force we have
\[ F_x = q (v \times B(x,y))_x = -qvB_y(x,y) = qvgx, \tag{5.13} \]
\[ F_y = q (v \times B(x,y))_y = qvB_x(x,y) = -qvgy. \tag{5.14} \]
The force in the horizontal direction only depends on \( x \) and vice versa for \( y \). Therefore, the motion in horizontal and vertical plane of a section containing only dipole and quadrupole magnets are decoupled. This will be important for the parametrization of the proton impact in ALFA at \( z = 240 \) m in chapter 9.

It is seen from equation (5.13) that a quadrupole with \( g < 0 \) will take a particle with negative \( x \) and move it outwards and a particle with positive \( x \) will be moved inwards. This is in contrast to the dipole magnet, where they are just bend less and
more than the design particle, respectively. This makes the focusing more efficient in the horizontal plane, but it will have a defocusing effect in the vertical plane, equation (5.14).

A transfer matrix can be written just as in the case for dipole magnets. In the limit where the focal length of the quadrupoles $f$ satisfies

$$f = \frac{1}{kl} \gg l,$$

(5.15)

where $k$ is the normalized focusing strength of the quadrupole and $l$ is the length, the transfer matrix simplifies to

$$M_x(y) = \begin{pmatrix} 1 & 0 & 0 \\ \pm \frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

(5.16)

where $+1/f$ is for a defocusing quadrupole and $-1/f$ is for a focusing. The net effect on a proton passing through first a focusing quadropole then a drift space and then a defocusing quadrupole with $f_{\text{focus}} = -f_{\text{defocus}} = f$ is

$$M_{\text{Doublet},x} = M_{\text{defocus},x} M_{\text{drift}} M_{\text{focus},x}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ -\frac{1}{f} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 + \frac{1}{f} & l & 0 \\ -\frac{1}{f^2} & 1 - \frac{1}{f} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

(5.17)

i.e. there is a net focusing effect in both $x$ and $y$. Typically, synchrotron rings as the LHC have FODO cells consisting of a Focusing-Drift-Defocusing-Drift magnet system.

### 5.2.4 Higher order magnets

Higher order magnets are used to correct for the fact that the focal length of the quadrupoles depends on the momentum of the protons. Without the higher order magnets, the spread in momentum of the beam protons would cause fluctuations from the design orbit. Also unavoidable errors in the dipoles and quadrupole, e.g. misalignment or gradient errors, are corrected for. These higher order magnets will not be described further as none of them are located between the IP and ALFA and hence have no influence on the diffractively scattered protons measured in this thesis.

### 5.2.5 Twiss parameters

Considering only particles with momentum $p$, i.e. no momentum dispersion, the motion in the transverse plane can be written as

$$y'' + K(z) = 0, \quad K(z) = K(z + L),$$

where $y$ stands for both $x$ and $y$, and $L$ is the circumference of the LHC. The function $K(z)$ is calculated from the magnet system, and the periodicity follows from the fact that the orbit of the design particle is closed. Equation (5.18) is called Hill’s equation. The transfer matrix reduces to a $2 \times 2$ matrix, because $\delta p = 0$, and can be written in the Twiss form

$$M = \cos \mu + J \sin \mu,$$

where $J = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}$ and $\cos \mu \neq \pm 1$.  

(5.18)
Stable solutions occur when \( \det(M) = 1 \), since the elements of \( M^n \) need to be finite. Otherwise, the protons would be lost after a number of revolutions. In the case of stable solutions, \( \alpha \) and \( \gamma \) can be expressed through the \( \beta \)-function which is periodic in \( L \). Using the Hill’s equation and the periodicity of \( K(z) \) one finds that the solution is given by \( \beta(z) \) alone:

\[
\begin{align*}
y(z) &= \sqrt{\epsilon \beta(z)} \cos[\Phi(z) - \delta], \\
y'(z) &= -\sqrt{\frac{\epsilon}{\beta(z)}} (\sin[\Phi(z) - \delta] + \sqrt{\epsilon} \cos[\Phi(z) - \delta]) ,
\end{align*}
\]

where \( \Phi(z) = \int_0^z \frac{ds}{\beta(s)} \), \( \delta \) is an arbitrary phase, and \( \epsilon \) is the transverse emittance described below. The number of betatron oscillations per revolution, \( Q = \Phi(L)/2\pi \), must not be an integer, otherwise a defect in a magnet will act on the proton in the same phase for every revolution. This will cause the betatron amplitude to grow and in the end a loss of the proton. If \( Q \) is irrational, these contributions will average out.

The proton trajectory maps out an ellipse in the phase space of \( y \) at a given point in the ring because \( \Phi \) changes per revolution. This ellipse is shown in figure 5.4. The \( \beta \)-function, and hence the shape of the ellipse, changes with \( z \), but the area of the ellipse is constant.

The transverse emittance, \( \epsilon \), is defined as the area divided by \( \pi \) of the ellipse containing a certain fraction of the beam protons. When the protons are not accelerated in the longitudinal direction, the emittance is an invariant determined by the initial conditions at injection. The beam pipe must have a strong vacuum in order to reduce the probability of a collision between a beam proton and atmospheric molecules in the beam pipe, whereby the proton is scattered and lost. Therefore, the emittance should be as low as possible to have a small beam pipe.

When the particles are accelerated in the longitudinal direction in the RF cavities, the beam divergence, and hence the emittance, shrinks because the transverse momentum components are invariant. Therefore, one defines the normalized emittance by

\[
\epsilon_N = \left( \frac{p_0}{m_0 c} \right) \epsilon ,
\]

which is constant during acceleration of a proton beam.\(^3\) The shrinkage of the emit-

---

\(^3\)Electrons in a storage ring will loose energy due to synchrotron radiation and the normalized
tance is the reason why the proton beam enters several accelerators before injection in the LHC. If the protons should be accelerated from rest to 7 TeV in the LHC, the beam pipe should have been much larger.

5.2.6 Optics

The value of the $\beta$-function at the interaction point is called $\beta^*$. The LHC magnet system can be tuned to give different $\beta^*$ and hence different conditions for the colliding beams.

In normal LHC runs, an optics is chosen with a low $\beta^*$ in order to get as small beams and hence as large luminosity as possible. Therefore, $y'$ will be large at the interaction point, and strong quadrupoles must be located close to the IP to focus the beam afterwards. Furthermore, a crossing angle between the beams are implied to avoid that one bunch of beam 1 collides with several bunches of beam 2 before the beams are again split to distinct beam pipes.

$\sqrt{s} = 8$ TeV, $\beta^* = 90$ m optics

The optics used for the data collection analysed in this thesis has $\beta^* = 90$ m. This gives large beams but small angles of the beam protons. The small angles are essential for the measurement of a small scattering angle since the incident angles of the colliding protons are negligible. As will be described in chapter 7, the ALFA detector is build to measure elastic scattering at small angles, hence the high $\beta^*$ is unavoidable.

The large beams give low luminosity which reduces the amount of pile-up (multiple proton-collisions per bunch crossing) necessary for the track reconstruction in ALFA, see section 7.6. The average number of collisions per bunch crossing, $\mu$, can be calculated from the beam dimensions, the number of protons in a bunch and the total cross section. Assuming that the number of collisions are Poisson distributed, the amount of pile-up can be calculated from $\mu$. For run 206881, which is analysed in chapter 10, the $\mu$-parameter had the value $\mu \simeq 0.07$ [25].

Only few bunches are used which decreases the luminosity further. The crossing angle between the beams are not necessary due to the limited number of bunches. Furthermore, the optics features a parallel-to-point focusing in the $y$ direction for protons with the beam energy. This means that the $y$ coordinates of the track pattern in ALFA is independent of the $y$ coordinate of the IP, hence the $y$ component of the $t$-spectrum of elastic scattering can be measured directly from the proton impact in ALFA.

5.3 Beam transport simulation

The knowledge about the magnetic lattice of the LHC can be used to simulate the trajectories of particles in the beam pipe. The standard software package is called MadX [26]. It allows for the possibility of a thick lens approximation of the LHC magnets and not the thin lens approximation described in section 5.2.3. FPTracker is another software package developed for the transportation and uses the thin lens approximation, thus it is faster than MadX. It represents each element of the LHC emittance will not be constant.
lattice by a matrix, and the matrices are then multiplied together in order to obtain the complete transfer matrix. MadX is well-tested and a comparison between FPTracker and the thin lens version of MadX validates FPTracker [27].

FPTracker is used by the package ALFA_BeamTransport which combines the event generation with beam transport. ALFA_BeamTransport will be used in chapter 9 to transport the protons in order to obtain a parametrization of the proton trajectory from the interaction point in the ATLAS detector to the tracking in the ALFA detector at $z = 240$ m.
6 | The ATLAS detector

A Toroidal LHC ApparatuS (ATLAS) is a general purpose detector installed at IP1 of the LHC. It is designed to study most of the hopefully new phenomena at the TeV scale. In diffractive studies, ATLAS is used to measure the properties of the dissociated system. The ALFA detector is a forward detector to ATLAS and measures the diffractively scattered protons. A description of ALFA is given in chapter 7.

As shown in figure 6.1, ATLAS is build of several sub-detectors and this chapter gives an overview of the different elements and the physics behind. The main emphasis is on the inner detector (ID) and the calorimeters. The ID may be able to reconstruct a vertex which will be considered as the interaction point. This vertex plays an important role for the resolution of the reconstructed kinematics for the proton in ALFA discussed in chapter 9. The calorimeters are crucial in the measurement of the invariant mass of the dissociated system and of the rapidity gaps characteristic for diffractive processes, chapter 4. Because of the expected rapidity gaps, the $\eta$ coverage of the individual sub-detectors is important.

A full description of the detector elements can be found in ref. [28] and the physical explanation is explained in great details in ref. [29].

Figure 6.1: A sketch of the ATLAS detector [30].
6.1 Inner Detector

The Inner Detector (ID) measures the positions of charged particles at different radii, whereby the particle trajectories can be fitted. The primary vertex of a collision can be found by associating tracks of particles to a particular vertex candidate. Also, the transverse momenta of the charged particles produced in the collision can be determined. A central solenoid magnet surrounding the ID provides an axial magnetic field of 2 T such that the charged particles are bent in the plane transverse to the beam direction according to the Lorentz force, eq. (5.1). Measuring how much the momentum vector has been rotated therefore gives the transverse momentum component, and the direction of the rotation gives the sign of the charge of the particle. A high magnetic field will bend the particles more and thereby increase the momentum resolution, but also a high resolution of the track coordinates is required in order to obtain a high momentum resolution. Furthermore, a good spatial resolution minimizes the chance of having a pile-up event in the same detector element as the interesting object.

The track efficiency depends on the transverse momentum of the particle and is shown in figure 6.2. The plot is made only for particles with $p_T > 0.5$ MeV, but particles with $p_T$ as low as 100 MeV can leave a track but with a lower efficiency.

![Figure 6.2: The track efficiency of the ID as a function of transverse momentum of the particle [31].](image1)

![Figure 6.3: The resolution of the $x$ coordinate of the reconstructed primary vertex in the ID as a function of the number of associated tracks [32].](image2)

The spatial resolution of the reconstructed vertex depends on the number of tracks associated to the vertex. The resolution for the $x$ coordinate of the vertex is seen on figure 6.3, and the $y$ resolution is identical since the ID is symmetric in azimuthal angle. The resolution in $z$ is about 2-3 times worse.

The ID consists of three individual sub-detectors complementing each other: the pixel detector, the Semiconductor Tracker (SCT), and the Transition Radiation Tracker (TRT), each of them having a barrel part and two identical end-caps. The $\eta$ coverage is seen on figure 6.4.
6.1.1 The pixel detector

The pixel detector is closest to the beam and uses semi-conductor pixels. The silicon is doped with a material with 5 valence electrons on one side and another material with 3 valence electrons on the other side. Thereby, a depletion zone free of charge carriers is obtained, and an applied external voltage of $\sim 150$ V increases the depletion zone further. The leakage current from + to − is now very small. A charged particle traversing the depleted zone loses energy due to ionization and electron-hole pairs are created. These will then drift towards the electrodes and give a hit.

Each pixel is 50 $\mu$m in the $R - \phi$ direction and 400 $\mu$m in $z$ direction, and the total number of pixels is approximately 80.4 millions. The B and C mesons and the $\tau$ lepton have a decay length of $\gtrsim 100$ $\mu$m [1], hence the pixel size is sufficient to find secondary vertices from these decays. The pixels are arranged in three cylinders of radius 50.5 mm, 88.5 mm, and 122.5 mm in the barrel and three disks with a radial coverage of $88.8 \text{ mm} < R < 149.6 \text{ mm}$ in the end-cap. The pixel detector covers $|\eta| < 2.5$, and typically a particle crosses three pixel layers, hence the pixel detector provides three points to the track reconstruction.

6.1.2 The Semiconductor Tracker

The SCT uses silicon strip detectors which are 80 $\mu$m in $R - \phi$ direction and 6.4 cm in $z$ direction. The physics principle behind the straws is the same as for the silicon pixels. The barrel consists of 4 layers at radii 299 mm, 371 mm, 443 mm, and 514 mm.
Each layer is double-sided with an angle of 40 µrad between the silicon strips in the two sides. The end-cap consists of 9 wheels where the strips run radially. The SCT covers |\eta| < 2.5, and typically the SCT provides four points to the track reconstruction with good resolution in the transverse plane.

### 6.1.3 The Transition Radiation Tracker

The TRT uses straws with a diameter of 4 mm and length of 144 cm in the barrel and 37 cm in the end-cap. Each straw is filled with a gas mixture of 70 % Xe, 27 % CO\textsubscript{2} and 3 % O\textsubscript{2}, and in the middle there is a 30 µm Tungsten wire plated with gold. The wires are kept at ground potential whereas the straws are typically at a voltage of \(~\sim\)−1500 V. Charged particles traversing a straw ionizes the gas, and the free electrons drift to the wire. This leads to further ionization and induces a detectable electric pulse in the wire. The drift time of the electrons are used to obtain a precision of 130 µm per straw-hit.

In the barrel part, the straws are parallel to the beam, whereas the straw hits are arranged radially in wheels in the end-caps. The TRT covers |\eta| < 2, and typically 36 tracks are provided to the track reconstruction.

The material between the straws make ultra relativistic particles emit transition radiation in form of X-rays which is then detected in the straws. Transition radiation is emitted when a particle crosses the interface of two media with different dielectric constant. This provides an electron/pion separation, since the electron is much lighter and therefore more relativistic.

### 6.2 Calorimeters

The calorimeters measure the energy of the particles by absorbing them. ATLAS has both an electromagnetic calorimeter and a hadronic calorimeter because the energy deposit of a particle traversing a material depends on the type of particle. Electrons and photons interact via the electromagnetic force with the atoms in the detector material and are measured by the electromagnetic calorimeter, whereas hadrons (which are mainly pions) only interact via the strong force with the atomic nucleus of the material. The ATLAS calorimeters are shown in figure 6.5.

#### 6.2.1 The electromagnetic calorimeter

The electromagnetic calorimeter (ECal) is located just outside the central solenoid surrounding the ID. It is a sampling calorimeter where passive lead absorber layers and active liquid argon layers are sandwiched. A relativistic electron traversing one radiation length of lead emits about half of its energy as Bremsstrahlung when it is accelerated by scattering with the atomic nuclei. The emitted photons then convert to electron-positron pairs which can also emit Bremsstrahlung. This procedure repeats such that a shower develops until the energy of the electrons and positrons reach the critical energy where the energy loss due to Bremsstrahlung equals the loss due to ionization. The critical energy depends on the atomic number of the absorber. The same occurs for an incoming photon, except that it starts with the pair production. Lead is used as the shower material since Bremsstrahlung increases with the atomic number. At the critical energy, the number of electrons and positrons are at maximum,
and they roughly have the same energy. The energy of the incoming electron or photon is then given by

\[ E_{\text{incoming}} = E_{\text{critical}} \cdot N_p, \]

(6.1)

where \( N_p \) are the number of particles. These particles are sampled in the liquid argon layers where they ionize the argon atoms, and the ions are then converted to an electric signal. The noise level in the ECal ranges from 10 to 50 MeV [33].

The accordion geometry shown in figure 6.6 is used for the layers and provides full \( \phi \) covering since the space needed to read out the signal does not cause any area where energy can escape undetected.

The barrel covers \( 0.1 < |\eta| < 1.475 \) and the end-caps covers \( 1.375 < |\eta| < 3.2 \).

Figure 6.5: The ATLAS calorimeter system [28].

Figure 6.6: The accordion geometry used in the ATLAS electromagnetic calorimeter [34].
The barrel is split in two components at $z = 0$ with a separation of 4 mm which is the reason why there is a gap in the pseudorapidity coverage at $|\eta| < 0.1$. The barrel has a thickness of at least 22 radiation length which increases with $\eta$ as the particles do not traverse perpendicular to the beam axis. Both the barrel and the end-caps are segmented in $\phi$, $\eta$ and three layers in the longitudinal direction in order to determine the direction of the incoming particle. This makes it possible to connect the signal in the calorimeter to a track in the ID. The $\Delta \eta \times \Delta \phi$ granularity is about $0.025 \times 0.025$, but it varies between barrel and end-caps and in the longitudinal segmentation. The longitudinal segmentation also serves as a measure of how much energy was not absorbed in the calorimeter. If most of the energy was deposited in the outermost layer, it can be expected that the incoming particle escaped the calorimeter. A presampler of liquid argon is placed in front of the ECal to measure the amount of energy lost before the ECal.

### 6.2.2 The hadronic calorimeter

The hadronic calorimeter (HCal) is located outside the ECal. The hadronic barrel calorimeter is a sampling calorimeter with absorber layers made of steel and scintillating tiles used as the active material (which is why it is also called the tile calorimeter). The hadrons interact with the nuclei in the steel absorber via the strong force whereby mostly pions are produced. The neutral pions decay into two photons which are detected as showers just as in the ECal. A charged pion can produce another pion in the reaction $\pi + p \rightarrow \pi + \pi + p$ but the threshold energy is much larger than in an electromagnetic shower. Therefore, the number of particles at shower maximum in the HCal is much lower than in the ECal, and the relative uncertainty on the energy measurement is larger. The tile calorimeter (central and extended barrel) covers $|\eta| < 1.7$ and has a radial depth of about 7.4 interaction lengths.

The Hadronic End-cap Calorimeter (HEC) uses copper as the absorbing material and liquid argon as the active material because the radiation level is too high to use scintillating tiles. Each HEC consists of two independent wheels segmented in two in the radial direction and in 32 in the azimuthal direction. The HEC covers $1.5 < |\eta| < 3.2$.

Both the HCal and the HEC are segmented in $\phi$, $\eta$ and three and four layers in the longitudinal direction, respectively. The segmentation is about $\Delta \eta \times \Delta \phi = 0.1 \times 0.1$, but it varies with the layers and $\eta$. The larger segmentation in the HCal and the HEC results in a noise level about 10 times higher than in the ECal [33].

### 6.2.3 The Forward Calorimeter

The Forward Calorimeter (FCal) has three longitudinal compartments with a total of approximately 10 interaction lengths. The first uses copper optimized for electromagnetic shower, and the other two use tungsten for the hadronic shower. Liquid argon is used as the active material. The FCal covers $3.1 < |\eta| < 4.9$.

### 6.3 The Muon system

Muons above about 5 GeV escape through the calorimeters due to their higher mass compared to the electron and therefore less Bremsstrahlung. In the muon system, the
muons are bent by large toroid magnets, and the tracks are measured by Monitored Drift Tubes. For $2 < |\eta| < 2.7$, additional Cathode Strip Chambers are used to measure the tracks.

6.4 The Minimum Bias Trigger Scintillator

ATLAS has a trigger system to select the interesting events since the amount of data is too large to be stored on tape, and most of the events are not the new physics ATLAS has been build to discover. ATLAS has three trigger levels, each one using only a limited amount of the collected data per bunch crossing to make a decision on whether or not to keep the event. These triggers will unavoidable make a bias in data, therefore also a Minimum Bias Trigger Scintillator (MBTS) is installed as part of ATLAS.

The MBTS covers $2.09 < |\eta| < 3.82$ and triggers on almost all inelastic events. It consists of scintillator tiles with 16 on each side of the IP at $|z| = 3560$ mm, i.e. between the ID and the ECal end-cap. The counters on each side are divided in 2 segments in $\eta$ and 8 segments in $\phi$.

6.5 Signature of the SM particles in ATLAS

Figure 6.7 shows how the measurements in the ATLAS sub-detectors are combined to identify the different kinds of particles:

- The photon and the electron (and positron) are both absorbed in the ECal, but they can be distinguished using the ID since the electron has a track due to its electric charge.
- The charged pions and protons/anti-protons have tracks in the ID and are absorbed in the HCal, and they are therefore indistinguishable. The neutron, on the other hand, does not have a track in the ID.
- The neutral pion decays rapidly into two photons, which are measured in the ECal, and the invariant mass of the photons should then correspond to the pion mass.
- The muon has a track in the ID, almost no energy deposit in the calorimeters, and a track in the muon system.
- Neutrinos are not measured directly, but because of energy conservation in a collision, the neutrinos are reconstructed from missing transverse energy (the longitudinal energy component is poorly measured due to particle escape along the beam pipe).
- $b$ and $c$ quarks and the $\tau$ lepton are identified using a secondary vertex in the ID.
- The top quark, $Z$, $W$ and Higgs bosons must be identified through their decay products.
Gluons and quarks hadronize into jets before the detector elements are reached. A jet from a gluon/quark is seen as several tracks in the ID as well as energy deposit in both ECal and HCal. Jet algorithm are developed to find these jets.

Figure 6.7: A computer generated picture showing how the different particles are measured in ATLAS [35].
The ALFA detector

This chapter gives a description of the ALFA detector. The ALFA detector measures protons in the extreme forward direction which gives the possibility to tag diffractively scattered protons. Special emphasis has been put on the spatial resolution of the ALFA detector and the alignment to the beam since this is crucial for the kinematic reconstruction of the protons described in chapter 9. The information in this chapter is based on ref. [36] and [37] unless otherwise stated.

The ALFA (Absolute Luminosity For ATLAS) detector was build to measure elastically scattered protons with small transferred four momentum $t$, i.e. small scattering angle, to determine the elastic cross section in the forward direction. This cross section is related to the total cross section through the optical theorem:

$$\sigma_{\text{tot}} = \frac{4\pi}{k} \text{Im} [A_{\text{el}}(t = 0)] ,$$

(7.1)

where $\sigma_{\text{tot}}$ is the total cross section, $k$ is the wave number of the incident proton, and $A_{\text{el}}$ is the elastic scattering amplitude. The optical theorem was mentioned in section 4.3 as a consequence of unitarity, and a proof can be found in ref. [16]. The absolute luminosity can then be inferred from the measurements of elastic and total cross sections. It turns out that the luminosity can be well determined by other methods without ALFA, therefore the main aim for ALFA is now to determine the total cross section.

ALFA is a tracking detector positioned about 240 m from the IP as shown in figure 7.1. It is able to measure protons only few millimetres from the beam in order to reach the small $t$ regime. The detectors measure the proton impact in the plane transverse to the beam. On each side of the IP there are two stations, each consisting of an upper detector approaching the beam from above and a lower detector approaching the beam from below. Detector 1+3 are called armlet 1, 2+4 armlet 2, 5+7 armlet 3 and 6+8 armlet 4. An elastic event would then have tracks in armlet 1+4 or 2+3. The two stations on each armlet make it possible to determine not only the position of the proton but also the local angle in the armlet. The ALFA main detector has a tracking resolution of 30 $\mu$m necessary for the measurements at the small $t$ regime and to eliminate background events.

7.1 Main detector

The main detector (MD) used for tracking the protons is built by layers of 64 scintillating fibers. A traversing proton excites some of the electrons in the material of the fiber, and a light pulse is emitted when the electrons return to the ground state. The
size of a fiber is $0.5 \times 0.5 \text{ mm}^2$, and they are coated with aluminum to reduced the amount of light propagating from one fiber to a neighbouring fiber.

The MD has 10 planes, and each plane is formed by two perpendicular layers of fibers glued to a titanium substrate. The titanium substrate is shaped such that it does not overlap with the tracking area where instead, the fibers are glued together with a material with a significantly smaller density. Thereby the interaction of the proton with passive detector material is minimized. The layers are rotated 45 degrees wrt. the $(x,y)$-plane of the LHC. In order to obtain a larger tracking area close to the beam, 40 out of the 64 fibers in each layer are cut at 45 degree while the remaining fibers are cut at 90 degrees. This gives the horizontal detector edge seen in figure 7.2 with a width of 28 mm.

A single fiber has a spatial resolution of

$$\text{RMS} = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\int_{-d/2}^{d/2} (x - x_0)^2 p(x) dx} = \sqrt{\int_{-d/2}^{d/2} \frac{x^2}{d} dx} = \frac{d}{\sqrt{12}}, \quad (7.2)$$

where $d$ is the width of the fiber. A flat probability distribution has been assumed such that $p(x) = 1/d$ and $x_0 = 0$. The $(x,y)$ coordinate system is rotated 45 degrees
wrt. the fibers such that \( x, y = \sqrt{a^2 + b^2} \), hence
\[
\delta x = \sqrt{\left( \frac{dx}{da} \delta a \right)^2 + \left( \frac{dx}{db} \delta b \right)^2} = \sqrt{\frac{a^2}{a^2 + b^2} \delta a^2 + \frac{b^2}{a^2 + b^2} \delta b^2} = \delta a ,
\]
where in the last step it has been used that \( \delta a = \delta b \). Thus, the resolution of \( x \) and \( y \) is also given by \( d/\sqrt{12} \). The fiber width is 500 microns, hence from (7.2) the resolution of one fiber is
\[
\text{RMS} = \frac{500 \, \mu\text{m}}{\sqrt{12}} = 144 \, \mu\text{m} .
\]
By staggering the planes with each plane shifted 1/10 of the width of a fiber, the resolution should in principle be 14.4 \( \mu\text{m} \). In reality, the resolution is 30 \( \mu\text{m} \), since the layer efficiency is only about 90%, and the positions of the fibers are not perfect.

### 7.2 Overlap detector

The Overlap detectors (OD) are placed on each side of the main detector as shown in figure 7.2 and are used to align the upper and lower detector in each station. There are three planes with 30 scintillating fibers glued to the titanium substrate on both the left and the right side of the MD. The fibers are placed horizontally, since the OD is only used to align in vertical direction. The fibers need to be bend and therefore the 30 fibers of each plane are mounted with 15 fibers on the front and 15 fibers on the back side of the titanium substrate in order to maximize the bending radius. The planes are staggered to improve resolution just as the MD, except here each plane is shifted with 1/3 of the width of a fiber.

The relative position between OD and MD is found with a high resolution telescope shown in figure 7.2 in a test beam and does not exceed 8 \( \mu\text{m} \).

The distance between the ODs is found by subtracting the \( y \) coordinates for a particle with hits in both the upper and lower overlap detector. The resolution of one event is
\[
\text{RMS} = \frac{d}{\sqrt{12}} = \frac{500 \, \mu\text{m}/3}{\sqrt{12}} = 48 \, \mu\text{m} ,
\]
but the resolution can be significantly improved by using many hits with the method sketched in figure 7.3. The grid \( p \) in the ODs is 166 \( \mu\text{m} \), \( n \) is the number of particles going through one cell, and \( d \) is the distance between cells in the two overlap detectors. Using the fraction of particles going through each cell in the lower overlap detector for a given cell in the upper overlap detector, a resolution better than 8 \( \mu\text{m} \) can be expected when the number of hits is larger than \( \sim6000 \) [38]. Due to staggering imperfections, the resolution of 8 \( \mu\text{m} \) is not obtainable in reality. A combination of the OD-to-OD and OD-to-MD resolution gives a MD-to-MD resolution of 20 \( \mu\text{m} \) using some thousand events.

### 7.3 MultiAnode PhotoMultiplier Types

The fibers from the MD and OD are connected to MultiAnode PhotoMultiplier Types (MAPMTs) outside the Roman pot, in which the detector is located, in order to convert the light to an electrical signal. The MAPMTs have 64 channels corresponding to the
number of fibers in each layer of the MD. The fibers are connected to the MAPMTs such that no neighbouring fibers are neighbours in the MAPMT. In this way, the cross talk between the fibers are reduced. About 4 photo-electrons is produced by a fiber hit giving a fiber efficiency of about 98 %. The MAPMTs have a rise time of 1 ns, hence this will not cause any problem for the separation of signals to different bunch crossings.

### 7.4 Trigger detector

ALFA uses 3 mm thick plastic scintillator tiles and not the MD itself to trigger useful events in order to obtain $\sim 100\%$ trigger efficiency. The trigger efficiency is studied in ref. [39]. The scintillator tiles cover the entire area of the MD, and each MD has two separate scintillator tiles in order to make coincidence. The scintillator tiles are painted with special white reflecting paint to obtain the highest light gain. The ODs have scintillator tiles as triggers too. Only one scintillator tile for each OD is needed, since the coincidence is obtained from signal in the upper and the lower overlap trigger. Clear fibers glued to the scintillator tiles guides the light to photomultiplier tubes outside the Roman pot.

### 7.5 Roman pots

The ALFA detectors operate inside the LHC beam pipe. Therefore, the ALFA detectors are placed inside Roman pots in order to separate the detectors from the ultra high vacuum in the beam pipe. Otherwise, the detectors would destroy the vacuum. A picture of an ALFA Roman pot is shown in figure 7.4.

Inside the Roman pot there is a secondary vacuum. The wall, through which the protons pass before hitting the detector, must be as thin as possible to reduce the probability of interaction with the protons, and a thickness of 500 $\mu$m is chosen. The window has a thickness of 200 $\mu$m. It should also be as thin as possible in order to have the ALFA detector close to the beam, however, it must be strong enough not to break if either the vacuum of the beam pipe or of the Roman pot is lost. The secondary vacuum in the Roman pot reduces the bending of the window and allows
the Roman pot to get closer to the beam. The distance between the window and the detector should be as small as possible.

The Roman pots are connected to a mechanical system which is able to move them up and down in steps of 5 µm. The Roman pots must be moveable since the beam has different sizes and locations in the beam pipe in different runs. Also, the detector should be moved out of the beam at high luminosity runs in order to reduce the radiation damage. The machinery is made such that in case of a power cut, the Roman pots will be moved out of the beam pipe.

### 7.6 Track reconstruction

The track of a proton in ALFA is reconstructed using the assumption that the proton hits ALFA perpendicular to the 20 layers of scintillating fibres in the MD. The track is reconstructed in the fibre coordinate system (CS) and afterwards rotated 45 degrees to the (x, y) CS.

The principle of the track reconstruction is shown in figure 7.5. One takes advantage of the fact that the 10 parallel fibre layers overlap, and that several layers have one or more hits. The fibre width is set to 480 µm, and a one dimensional histogram with a bin width of 1 µm is filled for each fiber hit. This means that 480 bins are filled for each fiber hit. The bin contents will then be up to 10 which is the number of layers in each of the perpendicular directions. The maximum of the histogram is a plateau, and the position of the reconstructed track is given as the middle of this...
plateau. A requirement of at least three layers with maximum three fibre hits is used since otherwise the reconstruction will not be clean enough.

If two protons hits the fibre layers, there will be two maxima in the histogram. Two maxima for each of the directions in the fibre CS gives four possible reconstructed tracks. All four candidates are saved in the data. An upper limit of 9 reconstructed tracks is used. The low average number of collisions per bunch crossing for the special ALFA runs described in section 5.2.6 is therefore crucial to reduce the number of events with multiple tracks in the detector.

7.7 Alignment

The position of the ALFA Roman pots to the beam needs to be known very precisely in order to make use of the tracking precision of 30 $\mu$m in the MD. A beam based alignment is done by moving the Roman pots one at the time into the beam in steps of 10 $\mu$m until there is a signal above threshold in the Beam Loss Monitors. The Beam Loss Monitors are installed in the LHC to measure the beam properties and dump the beam if something is wrong such that the beam will not quench the LHC magnets. This procedure gives a vertical alignment of 200 $\mu$m.

After data taking, the alignment of the detectors are found using the tracks from elastic-like events, i.e. events with hits in armlet 1+4 or 2+3. The offset in $x$ is found as the mean of a Gaussian distribution fitted to the $x$ coordinates of the elastic events since the elastic hit pattern is symmetric with respect to the beam in the horizontal direction (there is no preferred azimuthal scattering direction). In the installation of the detectors it cannot be avoided that the ($x$, $y$) detector CS is rotated slightly wrt. the LHC ($x$, $y$) CS. The amount of rotation is found by fitting a straight line to the ($x$, $y$) track pattern in the detector. A perfect alignment would result in a line parallel to the $y$ axis and the actual angle between the ($x$, $y$) track pattern and the $y$ axis determines the rotation. After the correction of the rotation, a Gaussian fit is again used on the $x$ coordinates to correct for a remaining offset.

The vertical alignment cannot be done in the same way as in the horizontal direction due to the gap between the detectors. Instead, the $y$ coordinates are aligned wrt. one of the stations using that the scattering angle in A-side is equal to minus the scattering angle in C-side when beam divergences are neglected.
Systematic uncertainties on the horizontal alignment arise from the selection of the elastic event. In the run 191373 with a $\sqrt{s} = 7$ TeV, $\beta^* = 90$ m optics, the systematic uncertainties are estimated to be 2-8 $\mu$m by varying the selection cuts and fit ranges. The statistical uncertainties are below 1 $\mu$m. Uncertainties on the vertical alignment is dominated by the distance measurement between the MDs in an ALFA station. They do not exceed 6 $\mu$m. Work is still in progress for this alignment, and the alignment in run 206881, which is analysed in chapter 10, the alignment is still preliminary.

Notice that the alignment is done using data and symmetry assumptions of the elastic scattering wrt. the beam. However, the beam center at the ALFA stations are not necessarily at (0, 0, 240 m) in the LHC CS. Therefore, the final coordinates are actually given in the beam CS. This needs to be taken into account when ALFA and ATLAS signals are combined in the event reconstruction, see section 9.1.3.
8  |  ALFA acceptance in $\beta^* = 90$ m optics at $\sqrt{s} = 8$ TeV

As described in chapter 7, the ALFA detector does not cover all azimuthal angles.$^1$ Therefore, some diffractively scattered protons will not be measured. Acceptance plots obtained through MC simulations show how large a fraction of events one can expect to measure. These plots give insight in the physics one can analyse with ALFA data, and furthermore they can give a hint on which cuts one can apply to the proton kinematics in a full MC production.

The package ALFA_BeamTransport described in section 5.3 is used to make the acceptance plots. However, ALFA_BeamTransport does not take the LHC collimators into account, hence all results presented here are preliminary.

The proton kinematics at the IP can be described with three variables: The relative energy loss, $\xi$, the transverse momentum, $p_T$, and the azimuthal angle, $\phi$. Figure 8.1 shows the acceptance plots of the diffractively scattered protons at $z = 237$ m as a function of two of the three variables, where a flat distribution has been used for the last parameter. A flat distribution of $\phi$ is assumed to reflect data, since the diffraction process is of course independent of the chosen coordinate system. Flat distributions of $\xi$ and $p_T$, however, are not expected in data, hence the acceptance plots 8.1b, 8.1c, 8.1e and 8.1f are highly depending on physics assumptions. The kinematic regions with zero acceptance are zero for any chosen distributions of $\xi$ and $p_T$ since the entire range has been used in these plots. However, the areas with non-zero acceptance in these four figures can only be used to compare the LHC acceptance with the ALFA acceptance.

The protons are generated at IP= (0, 0, 0) and transported through the magnetic lattice of LHC to $z = 237$ m by the package ALFA_BeamTransport described in section 5.3. The LHC acceptance shows how large a fraction will actually reach $z = 237$ m. For some configurations of the proton kinematics, the magnets will bend the protons out of the beam pipe and the protons will be lost. In ref. [40] there are plots showing the proton trajectory from IP to ALFA for different combinations of the proton kinematics. There, it can be seen where the protons are lost.

The ALFA acceptance takes into account the acceptance of LHC and the area of the beam pipe at $z = 237$ m covered by the ALFA main detector and will therefore reduce the acceptance further. A distance from the detector edge to the beam center of 8 mm has been used as well as the assumption that ALFA is located symmetrically.

$^1$The ATLAS detector also has a limited acceptance region described in chapter 6, but this has no influence on the acceptance for the diffractively scattered protons. The influence of the ATLAS acceptance on the selection cuts of diffractive events is described in chapter 10.
around $x = 0$. The 8 mm is a preliminary estimate of the distance found in ref. [41].

The hitmap at $z = 237$ m shown in figure 8.2 helps the understanding of the acceptance plots. As the energy loss grows, the protons hits ALFA at higher $x$ values, which is why the acceptance plots are not symmetric in the horizontal direction. Also, the spread in $y$ is much larger than in $x$, which is why the acceptance is lowest for $\phi = \pm \frac{\pi}{2}$ is the LHC acceptance plots.

Figure 8.1a shows that the LHC acceptance is up to 100% for $\xi < 0.17$ and $p_T < 0.3$ GeV and figure 8.1c shows that the acceptance is uniform in $\phi$ for this region of $p_T$. This is the region where most of the diffractive protons will be in, see the discussion in section 4.4.3, but figure 8.1d shows that the acceptance is significantly reduced by the ALFA acceptance. No protons with $p_T < 0.1$ GeV hit ALFA since the protons will simply be too close to the beam. ALFA also reduces the acceptance for $\phi \approx 0$ and $\phi \approx \pm \pi$ significantly, since ALFA approaches the beam in the vertical direction and has only overlap detectors on the horizontal sides of the beam. These cannot track protons in the $(x, y)$-plane and are not included in the acceptance plots.

The acceptance plots will be further discussed in chapter 9 where a deeper understanding of the proton trajectories from IP to ALFA is given.

A PYTHIA8 simulation generating the kinematics of SD events followed by transport to ALFA shows that the total ALFA acceptance for SD events is $(39.4 \pm 0.1)\%$, where the uncertainty reflects how the acceptance changes depending on the chosen IP distribution. As will be shown in chapter 9, an offset in IP from $(0, 0, 0)$ changes...
the \((x, y)\) position of the protons in ALFA and hence also the acceptance. However, the beam centre will also move due to this offset and since the ALFA detectors are aligned wrt. the beam, the IP offset does not have an important influence on the acceptance. The dominant contribution to the uncertainty on the acceptance arises from the uncertainty on the distance from detector edge to the beam center. Setting the distance to 6 mm instead of 8 mm gives an acceptance of 48.5%. The change in distance does not alter the LHC acceptance, and the ALFA acceptance plots are only slightly modified. The smaller distance increases the acceptance region for low \(p_T\).

For a MC production, the only meaningful cut to make is that \(p_T > 0.1\) GeV since the acceptance below this cut is zero. However, an important thing to note is that the acceptance plots are made with IP = (0, 0, 0) which is not the case in data. Here, the beam sizes makes it possible for collisions to happen away from (0,0,0) and hence, some protons with \(p_T < 0.1\) GeV may hit ALFA.

Although maybe physically improbable, figure 8.1d shows that ALFA can tag protons in single diffractive events where the dissociated system has an invariant mass of

\[
M_{SD} \leq 8000 \text{ GeV} - (1.0 - 0.3) \cdot 4000 \text{ GeV} = 5200 \text{ GeV} ,
\]

while for central diffractive events the dissociated system, there is an upper limit of

\[
M_{CD} \leq 8000 \text{ GeV} - 2(1 - 0.3) \cdot 4000 \text{ GeV} = 2400 \text{ GeV} .
\]

Thus, there is enough energy available to allow all SM particles to be produced.
Kinematic reconstruction

The ALFA detector measures the position of protons in the plane transverse to the beam direction at $z = 237$ m and $z = 241$ m. When looking at diffractive events, one can use the ALFA detector as a trigger since having a signal in both ALFA and in ATLAS indicates a diffractive event. However, if it is possible to use the coordinates measured by ALFA to reconstruct the kinematics of the proton at the interaction point (IP), ALFA can provide further information to the understanding of diffraction than just as triggering for ATLAS data.

This chapter describes a method developed to perform the kinematic reconstruction. The procedure is done in two steps:

1. The ATHENA package ALFA_BeamTransport$^1$ described in section 5.3 uses information about the setup of the LHC magnet system to transport protons from the IP in ATLAS to the ALFA stations at $z = 237$ m and $z = 241$ m. The magnets between ATLAS and ALFA are shown in figure 9.1, and as described in section 5.2, the proton impact in ALFA depends on the proton kinematics and the position of the IP, and this relationship is parametrized.

2. The measured IP in ATLAS and the proton impact in the two ALFA stations at $z = 237$ m and $z = 241$ m are given as input to the fitting program Minuit, which numerically inverts the parametrization and finds the kinematics of the proton.

The following procedure is developed for a $\beta^* = 90$ m optics at $\sqrt{s} = 8$ TeV since the data sample obtained with this optics is the one where the largest number of diffractive events has been triggered. Physics performance of the reconstruction code and systematic errors are studied, and the code is validated by a comparison with a similar code made in Cracow.

Figure 9.1: Sketch of the magnets between ATLAS and ALFA. Quadropoles are labelled with a $Q$ and dipoles with a $D$ [28].

$^1$After this section has been made, a new package called ForwardTransportFast [42] has been developed, which makes ALFA_BeamTransport obsolete. However, the results presented in this section are independent of the package used.
9.1 Parametrization

9.1.1 Parametrizing the proton kinematics

The kinematics of the scattered proton can be described with several different variables. Initially, this section uses the variables energy, $E$, the momentum transverse to the beam, $p_T$, and the azimuthal angle, $\phi$, describing the direction of $p_T$. The angle takes the values $[-\pi, \pi]$ with $\phi = -\pi$ being $p_T$ along negative $x$ direction, i.e. pointing away from the center.

To find the dependence on the azimuthal angle $\phi$, protons with fixed $E$ and $p_T$ but different values of $\phi$ are transported. The positions for these protons in ALFA at $z = 237$ m are shown in figure 9.2 for 12 different values of $\phi$ uniformly distributed in $[-\pi; \pi]$.

The geometry of an ellipse shown in figure 9.3 is used to parametrize $\phi$ since the proton coordinates in figure 9.2 for fixed $E$ and $p_T$ all fulfill the ellipse equation

$$\left(\frac{x}{R}\right)^2 + \left(\frac{y}{r}\right)^2 = 1,$$

where $R$ and $r$ are the semi-axes of the ellipse in $x$ and $y$ direction, respectively. The $x$ coordinate in ALFA can be written as the center of the ellipse, which is independent of $\phi$, plus the displacement from the center and vice versa for $y$. Therefore, the $\phi$ dependence is given by

$$x(E, p_T, \phi) = C_x(E, p_T) + \cos(\phi) \cdot R(E, p_T),$$

$$y(E, p_T, \phi) = C_y(E, p_T) + \sin(\phi) \cdot r(E, p_T),$$

where $C_x$ and $C_y$ are constants.
Figure 9.4: The coordinates in the transverse plane to the beam from simulation at 
\( z = 237 \, \text{m} \) as a function of \( p_T \) for two different energies overlayed with linear fits. (a) The 
\( x \) coordinate and (b) the \( y \) coordinate.

where \((x, y)\) is the proton impact in the ALFA stations and \((C_x, C_y)\) is the center of 
the ellipse. This parametrization applies for both ALFA stations at \( z = 237 \, \text{m} \) and 
\( z = 241 \, \text{m} \) but the center, radii and proton impact are different.

Figure 9.4 shows the \( p_T \) dependence of \( x \) and \( y \) at two different energies and a 
fixed value of \( \phi = \pi/21 \). The azimuthal angle has been chosen such that the same 
\( p_T \) interval can be used for both coordinates but is otherwise arbitrary. The linear 
fit in \( p_T \) describes the data well, hence a change of variables to \( p_x = \cos(\phi) \cdot p_T \) and 
\( p_y = \sin(\phi) \cdot p_T \) reduces the parametrization to

\[
\begin{align*}
x(E, p_x) &= C_x(E) + p_x \cdot a(E), \\
y(E, p_y) &= C_y(E) + p_y \cdot b(E).
\end{align*}
\]

The energy is now the only variable of which both \( x \) and \( y \) depend. The separation 
of the motion in \( x \) and \( y \) is expected from the discussion of magnets in section 5.2, 
since the magnet system of LHC between ATLAS and ALFA only includes dipoles and 
quadrupoles, see figure 9.1.

The equations (9.3) are of the form \( y = ax + b \), therefore only two protons with 
different values of horizontal and vertical momentum are needed to be transported to 
determine the values of the centre and radii of the ellipse for a given energy:

\[
\begin{align*}
x(E, p_{x,1}) &= C_x(E) + p_{x,1} \cdot a(E) \\
x(E, p_{x,2}) &= C_x(E) + p_{x,2} \cdot a(E) \} \Rightarrow \\
a(E) &= \frac{x(E, p_{x,1}) - x(E, p_{x,2})}{p_{x,1} - p_{x,2}} \quad \text{and} \quad C_x(E) = x(E, p_{x,1}) - p_{x,1} \cdot a(E)
\end{align*}
\]

and likewise for the parameters in the \( y \) direction.

From figure 8.1a and 8.1b it is seen that in order to get the largest possible energy 
range for the parametrization, values of \( p_T \) around 1 GeV and \( \phi \sim -\pi \) (or \( \phi \sim \pi \)

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which is of course equivalent) should be used. With the values

\[
\begin{align*}
  p_{x,1} &= -1.3 \text{ GeV}, \quad p_{y,1} = 0.3 \text{ GeV}, \\
  p_{x,2} &= -1.1 \text{ GeV}, \quad p_{y,2} = 0.1 \text{ GeV},
\end{align*}
\]

(9.5)

energies as low as 2610 GeV can be obtained. These values might be completely different in other optics, thus for each new optics studied, corresponding values must be found.

One might argue from figure 8.1d that protons with an energy below \( \sim 2800 \text{ GeV} \) will never hit ALFA anyway. However, since the ALFA detector has a limited precision of 30 \( \mu \text{m} \), the kinematic reconstruction of the protons gives Gaussian distributed energies around the true value. Hence, the parametrization should allow for protons with energies less than the limit indicated in the acceptance plots as well as unphysical energies above the beam energy. The upper limit is chosen to be 4300 GeV which is more than 7\( \sigma \) above the beam energy, see figure 9.16. Thus, it should cover all proton configurations.

The parameters of the ellipse are calculated in the entire energy range in steps of 5 GeV and are plotted in figure 9.5 for protons hitting ALFA at positive \( z \). They are also calculated for negative \( z \) but not shown since the difference is small. The parametrization includes the term for \( C_y \) even though it is very small and could safely be set to zero, but this might not be the case in other optics. The deviation from a smooth curve is likely due to limited numerical accuracy in ALFA_BeamTransport. The vertical radius \( b \) is an order of magnitude larger than the horizontal radius \( a \) which is in agreement with the elongated ellipse of figure 9.2. This is also consistent with figure 8.1f where the acceptance region has a much larger span in \( p_x \) than in \( p_y \). The horizontal center coordinates become large for low energies which means that diffractively scattered protons with a large energy loss will hit ALFA at larger \( x \) values.

The differences between the first and second ALFA station are completely given by the proton momentum at the first station, since there are no magnets between the stations. Hence, another and completely equivalent way of parametrization for the second station is therefore to use the proton impact and angle in the first station. The energy dependence of the differences between \( a, b \) and \( C_x \) at \( z = 237 \text{ m} \) and \( z = 241 \text{ m} \) turns out to have an important effect described in section 9.3.

Interpolators with the \textit{akima} method [44] are used for the parametrization of the energy since the analytic function is unknown. For a given interval between two neighbouring data points, an interpolator uses neighbouring data points to fit a polynomial to describe the interval. This is done for all data points in such a way that an interpolator is a piecewise function composed of polynomials going through all the data points. The akima method uses polynomials with degree of maximum 3.

These interpolators fits the data points more precisely than a 4. degree polynomial fitted over the whole range, hence the interpolators give a more precise reconstruction. By eye it seems that the 4. degree polynomials fit the data quit well but when looking closer, the discrepancy between fit and data points is \( \sim 20 \mu \text{m} \) and is comparable with the 30 \( \mu \text{m} \) resolution of ALFA. This discrepancy will cause a systematic error of about 2 GeV in the energy reconstruction which should be eliminated if possible.

Of course higher degree polynomials could be used, but it turns out that polynomials in \( 1/|\mathbf{p}| \), where \( \mathbf{p} \) is the momentum vector of the proton, fits the data points much better than polynomials in energy. This is expected from the discussion in section 5.2 since the bending of the proton by the magnets depends on \( 1/p \). A faster but
Figure 9.5: Simulation of the parameters of the ellipse describing a proton impact on the ALFA stations for positive \( z \) as a function of the proton energy and 4. degree polynomial fits to data points for \( z = 237 \) m. (a) is the horizontal radius, (b) is the vertical radius, (c) is the horizontal centre coordinate and (d) is the vertical centre coordinate.

A less precise alternative to the interpolators is therefore to use 4 degree polynomials in \( 1/p \). This gives discrepancies between fit and data points of \( \sim 1 \mu \text{m} \), which will of course have a smaller systematic error than 2 GeV. Both methods are implemented in the code and the user should specify which one to use. Interpolators are the default method since this is still the most precise, and all results presented in this chapter are obtained using the interpolators. The polynomials are faster to use than the interpolators by about 2 % but gives also a numerical accuracy in the reconstruction which is about a factor 20 worse than for the interpolators. The numerical accuracy when using interpolators is found in section 9.2.4.


9.1.2 Parametrizing the IP

So far it has been assumed that the IP is at exactly $(0,0,0)$. This is an idealized situation since the beams have a certain size, hence the IPs are distributed around the beam spot. In a given interaction, ATLAS can provide a vertex if there are $\geq 2$ reconstructed tracks in the ID. This vertex will be considered as the IP, thus including the IP in the parametrization, may give a more precise reconstruction of the kinematics.

The parametrization (9.3) is modified in order to include the IP dependence:

$$
\begin{align*}
  x(E,p_x,v_x,v_y) &= C_x(E) + \Delta C_x(E) \cdot v_x + p_x \cdot a(E, v_x, v_y), \\
  y(E,p_y,v_x,v_y) &= C_y(E) + \Delta C_y(E) \cdot v_y + p_y \cdot b(E, v_x, v_y).
\end{align*}
$$

(9.6)

By transporting protons with different value of $v_x$ and the other parameters held fixed, it is found that $x$ does not depend on $v_y$ and vice versa for $y$. Thus, again there is no mixing between $x$ and $y$.

Figure 9.6 shows the dependence of $v_x$ on $x$ and $v_y$ on $y$. The constant value for the difference between coordinates with different values of $p_x$ and $p_y$ (green lines) show that $p_x$ and $p_y$ does not depend on $v_x$ and $v_y$, respectively. The linear fits in $v_x$ and $v_y$ describe their respective data well, hence the parametrization is reduced to

$$
\begin{align*}
  x(E,p_x,v_x,v_y) &= C_x(E) + \Delta C_x(E) \cdot v_x + p_x \cdot a(E), \\
  y(E,p_y,v_x,v_y) &= C_y(E) + \Delta C_y(E) \cdot v_y + p_y \cdot b(E).
\end{align*}
$$

(9.7)

where $C_x$ and $C_y$ are the same as in equation (9.3). The $\Delta C_x$ and $\Delta C_y$ terms describe the correction to the center of the ellipse due to deviations of the IP from $(0,0,0)$ and are shown in figure 9.7. Notice that the correction in the vertical direction crosses zero for protons at around the beam energy because of the special parallel-to-point optics useful for the elastic scattering analysis, see section 5.2.6.

The $z$ position of the IP is not included in the parametrization. An offset of 100 mm from $v_z = 0$ for a proton with $E = 4000$ GeV and $p_x = 1$ GeV can not be

---

**Figure 9.6:** The coordinates in the transverse plane to the beam at $z = 237$ m as a function of the IP coordinates for two different values of momentum overlayed with linear fits. (a) The $x$ coordinate as a function of $v_x$ and (b) the $y$ coordinate as a function of $v_y$. 
Figure 9.7: Simulation of the correction of the center coordinates as a function of proton energy. (a) is the correction in horizontal direction and (b) is the correction in the vertical direction. The $\Delta C_x$ and $\Delta C_y$ have no unit since they are multiplied by the IP deviation from (0,0).

distinguished from an offset in $v_x$ of

$$v_x = \frac{1 \text{ GeV}}{4000 \text{ GeV}} \cdot 100 \text{ mm} = 0.025 \text{ mm}.$$  \hspace{1cm} (9.8)

This effect will not be significant when the resolution of $v_x$ from ATLAS is taken into account and from figure 9.7a it is seen that it only corresponds to a deviation of $\sim 3 \mu\text{m}$ in ALFA, which is an order of magnitude less than the resolution in ALFA. Furthermore, the effect of adding another variable to Minuit will at least cost some extra computing time and may also cause other problems due to the strong correlations between the IP coordinates. The effect of a displacement in $v_z$ will never correspond to an offset in $v_y$ or $v_x$ of more than 0.1 mm even for optics with a crossing angle between the beams of 142.5 $\mu$rad which is the case in normal runs.

### 9.1.3 Coordinate system transformation

The parametrisation of the IP and the proton impact in the ALFA detector is done with ALFA BeamTransport which uses the LHC coordinate system (CS). Therefore, the coordinates must be transformed to the LHC CS before use in the kinematic reconstruction.

The ATLAS ID measurement of the IP should be transformed from ATLAS CS to LHC CS. Unfortunately, the information about ATLAS in the LHC CS is rather poor. One needs to measure the distances between ATLAS and the LHC aperture, and it is stated that the two coordinate systems are within few hundreds of microns[45] but no more information has been found. Therefore, the coordinate transformation is not possible and one simply has to use the ATLAS ID measurement as if it was given in the LHC CS. This will result in a systematic error on the reconstructed kinematics but may possibly be eliminated using elastic events, see section 9.7.2.
The ALFA coordinates are given in the beam CS using elastic events as described in section 7.7. Therefore, we need to know the beam position at the location of ALFA in order to find the transformation from beam CS to LHC CS. This can be determined from the parametrization (9.7) for a particle with the beam energy and no transverse momentum, where the beam spot (BS) measured in the ATLAS (≈ LHC) CS is used as the IP. The transformation needed is therefore

\[
x_{\text{LHC}}^{\text{ALFA}} = x_{\text{beam}}^{\text{ALFA}} + C_x^{\text{beam}}(\text{beam energy}) \cdot x_{\text{BS}}^{\text{LHC}},
\]

\[
y_{\text{LHC}}^{\text{ALFA}} = y_{\text{beam}}^{\text{ALFA}} + C_y^{\text{beam}}(\text{beam energy}) \cdot y_{\text{BS}}^{\text{LHC}},
\]

where the superscript indicates the CS used for the measurement and the subscript indicates the z position. As the beam CS is determined using the elastic data over an entire run, the weighted average of the BS for elastic events over the entire run must be used.

## 9.2 Reconstruction

The proton impact in the two ALFA stations at \( z = 237 \) m and \( z = 241 \) m is parametrized as a function of the energy, horizontal momentum, vertical momentum and \((x,y)\) of the IP. What data gives is the ALFA coordinates and a vertex in the ATLAS ID, all given an uncertainty. These coordinates and their corresponding uncertainties are used to construct the following \( \chi^2 \) function:

\[
\chi^2 = \frac{(x_{237}^m - x_{237}^r(p))^2}{\sigma_{x_{237}}^2} + \frac{(y_{237}^m - y_{237}^r(p))^2}{\sigma_{y_{237}}^2} + \frac{(x_{241}^m - x_{241}^r(p))^2}{\sigma_{x_{241}}^2} + \frac{(y_{241}^m - y_{241}^r(p))^2}{\sigma_{y_{241}}^2} + \frac{(v_{x}^m - v_{x}^r)^2}{\sigma_{v_{x}}^2} + \frac{(v_{y}^m - v_{y}^r)^2}{\sigma_{v_{y}}^2}.
\]

(9.10)

Here, \( m \) are the measured coordinates and \( r \) are the reconstructed coordinates which depend on the vector of the five parameters \( p = (E, p_x, p_y, v_x, v_y) \) through equation (9.7). The minimum of this \( \chi^2 \) function yields the five parameters which gives the best agreement with data.

The reconstructed IP is in itself of limited interest. One could also choose to use the IP as input to the parametrization of the ALFA coordinates but without including the IP terms in the \( \chi^2 \) function, i.e. assuming that the errors on the IP coordinate measurements are zero. This possibility will be further discussed in section 9.10.

### 9.2.1 Minuit

Minuit[46] is used to minimize the \( \chi^2 \) function (9.10) in order to find the five parameters \( E, p_x, p_y, v_x, v_y \). An iterative technique to find the minimum has to be used, since the equations (9.7) are non-linear in \( E \). Minuit requires an initial guess of the five parameters, \( p_0 \), and then calculates the gradients of the \( \chi^2 \) wrt. each of the parameters as

\[
g_r(p_0) \equiv \left. \frac{\partial \chi^2}{\partial p_r} \right|_{p=p_0} = \sum_{i=1}^{6} -\frac{2}{\sigma_i^2} [y_i - f(x_i; p_0)] \frac{\partial f_i(p_0)}{\partial p_r},
\]

(9.11)

where \( p \) is the vector of the five fitting parameters, \( r \) is the parameter index running from 1 to 5, \( y_i \) are the measurements and \( \sigma_i \) are the errors, and \( f_i \) are the reconstructed
coordinates using equation (9.7). At the minimum, all components of the gradient are zero, hence Minuit tries to find an increment \( \delta p \) such that

\[
g_r(p_0 + \delta p) = \left. \frac{\partial \chi^2}{\partial p_r} \right|_{p=p_0+\delta p} = 0 \quad \text{for all } r. \tag{9.12}
\]

Minuit uses a Taylor expansion around \( p_0 \) keeping only the zeroth and first order terms

\[
g_r(p_0 + \delta p) \approx g_r(p_0) + \sum_{s=1}^{5} \frac{\partial g_r}{\partial p_s} \delta p_s = g_r(p_0) + \sum_{s=1}^{5} \frac{\partial^2 \chi^2}{\partial p_r \partial p_s} \delta p_s \tag{9.13}
\]

to iteratively find the minimum. When the gradients are below a certain value given by the user, Minuit returns \( p \) as the solution.

Though the minimization procedure looks simple, many problems may arise where decisions have to be made, e.g. what should be done if the \( \chi^2 \) gets larger during the iteration. Different algorithms are available, but the default method MIGRAD is chosen since this gives better performance than the other possibilities.

### 9.2.2 MC sample for test of numerical performance

The ideal case with no detector smearing is set up to test the numerical performance of the reconstruction. For this purpose, a Monte Carlo (MC) sample with the maximal range of kinematics is generated.

The energy range is chosen to be \( E \in [2.61; 4.3] \text{ TeV} \) as this is the interval covered by the parametrization. Outside this interval, the parametrization would rely on extrapolation with no guarantee on the validity. Protons with energies larger than the beam energy are of course unphysical, but since detector smearing may make a proton with \( E < 4.0 \text{ TeV} \) look like one with \( E > 4.0 \text{ TeV} \), this energy range needs to be tested as well. The same argument applies for the lower limit which is below the acceptance region of ALFA, 8.

A large range in the IP of \( v_x \in [-3.0; 3.0] \text{ mm} \) and \( v_y \in [-3.0; 3.0] \text{ mm} \) is chosen. The large range in the IP is not only used to include the spread of IPs due to the beam sizes, but also to cover situations where the beam spot is not at \((v_x, v_y) = (0, 0)\) in the LHC CS.

The momentum distributions are only limited by the LHC acceptance and not the parametrization. With the given distribution of IP, the LHC acceptance gives the limits \( p_x \in [-2.3; 2.3] \text{ GeV} \), \( p_y \in [-1.7; 1.7] \text{ GeV} \). The \( p_x \) and \( p_y \) intervals are larger than was shown in figure 8.1b and 8.1c since the IP and momentum is correlated in the parametrization (9.7), and an IP deviating from zero therefore enhances the maximal range of momentum.

### 9.2.3 Minimization procedure

Minuit is able to handle free parameters allowed to take on any value as well as parameters with limits. However, the internal Minuit routine can only handle free parameters, therefore Minuit transforms external parameters \( P_{\text{ext}} \) with limits to internal parameters \( P_{\text{int}} \) in the Minuit routine given by

\[
P_{\text{int}} = \arcsin\left(2\frac{P_{\text{ext}} - a}{b - a} - 1\right), \quad P_{\text{ext}} = a + \frac{b - a}{2} (\sin P_{\text{int}} + 1), \tag{9.14}
\]
where \( a \) is the lower limit and \( b \) is the upper limit of \( P_{\text{ext}} \). In this way, the external parameter will always be limited between the lower and upper parameter regardless of the internal parameter. This transformation enhances the numerical inaccuracy especially when the external parameter is close to a limit, therefore parameters with limits should be avoided if possible. The \( p_x, p_y, v_x \) and \( v_y \) parameters are given as free parameters since the parametrization itself does not need any limits. However, the energy needs to be limited in \([2.61; 4.3] \text{TeV}\) since the interpolators only cover this range.

Minuit requires starting values for all the five parameters, and these are chosen independent of the measured coordinates in ALFA. The correlations seen in figure 9.8 between the impact in ALFA and the kinematics of the proton are not useable to determine reasonable start values for the parameters. The strong correlation seen as the dark line between \( p_y \) and \( y_{237} \) is only for \( E > 3400 \text{ GeV} \) where \( b_{237} \) is flat, see figure 9.5b, and hence can only be used when the energy is known. Figure 9.8 can only be used to see if the reconstructed kinematics of the proton is actually physically possible.

The starting values for \( v_x \) and \( v_y \) are just the real MC values since ATLAS will provide a vertex in the real data. For the vertical momentum, \( p_y = 0 \) is chosen since this will be the average value, even though no protons will hit ALFA with this vertical momentum. For every event, the fit is performed five times with different starting values for \( E \) and \( p_x \):

- \( p_x = 0 \text{ GeV} \) and \( E = 0.5 \cdot E_{\text{lower bound}} + 0.5 \cdot E_{\text{upper bound}} \);
- \( p_x = -1.5 \text{ GeV} \) and \( E = 0.15 \cdot E_{\text{lower bound}} + 0.85 \cdot E_{\text{upper bound}} \);
- \( p_x = -1.5 \text{ GeV} \) and \( E = 0.85 \cdot E_{\text{lower bound}} + 0.15 \cdot E_{\text{upper bound}} \);
- \( p_x = 1.5 \text{ GeV} \) and \( E = 0.15 \cdot E_{\text{lower bound}} + 0.85 \cdot E_{\text{upper bound}} \);
- \( p_x = 1.5 \text{ GeV} \) and \( E = 0.85 \cdot E_{\text{lower bound}} + 0.15 \cdot E_{\text{upper bound}} \);

and the fit giving the lowest value of \( \chi^2 \) is chosen. This procedure is necessary since situations with multiple minima occurs and \( E \) and \( p_x \) are highly correlated, see figure
9.9. The correlation between $E$ and $p_x$ is expected from figure 9.5a and 9.5c, because $E$ has a large influence on $C_x$ which can be compensated for by $a$. The correlation between $E$ and $p_y$ is not as strong since $C_y$ on figure 9.5d is essentially zero. If only one starting value is used for $E$ and $p_x$, Minuit will sometimes find a local minimum and not the global one.

All five parameters are fitted simultaneously even though Minuit gives the user the opportunity to fix some parameters while fitting the others. By fixing parameters, a problematic parameter can be fixed at a reasonable starting value while fitting the others. Afterwards, the fixed parameter can be released and a second fit performed but now varying all parameters and using the fit result from the first step as starting values. None of the tried combinations of fixing and releasing parameters gave better results than the simultaneous fit. Minuit is able to find $p_y$ and $v_y$ even with bad fixed values of the other three provided that $E_{\text{true}} > 3400$ GeV because the correlation between $p_y$ and the vertical impact in ALFA is strong here. However, when $E, p_x$ and $v_x$ are released, no improvement is obtained compared to the simultaneous fit.

In addition to the $\chi^2$ in eq. (9.10), also the analytically calculated derivatives of the $\chi^2$ wrt. the five parameters are given as input to Minuit. In this way, the computing time is reduced since otherwise Minuit has to find the derivatives itself by calculating the $\chi^2$ different places around the starting point.

In the first iteration of the minimization, Minuit uses the first and second derivatives of the $\chi^2$ wrt. the parameters together with a step size given by the user to search for a better minimum. These step sizes are chosen to be 1 GeV, 5 MeV and 5 MeV for $E, p_x$ and $p_y$ respectively which is roughly 10 % of the resolution for these variables (figure 9.16, 9.17 and 9.18) and $\sigma_{v_x}/10, \sigma_{v_y}/10$ for the IP, where $\sigma_{v_x}$ and $\sigma_{v_y}$ are the resolutions of the IP coordinates given by ATLAS. If the step sizes are too large, Minuit may jump around in the five dimensional parameter space without converging and if the step sizes are too small, Minuit will simply end up almost where it started. After the first iteration, Minuit uses the obtained information from the first iteration to find step sizes.
9.2.4 Numerical performance

The minimization procedure is used on the maximal kinematic range, and the resolutions of the five parameters are shown in figure 9.10. It is seen that the means are close to zero, hence on average there is no offset. The RMS of the distributions are not a good measure for the resolutions, since few outliers have a significant influence. However, by eye it looks like the uncertainties are all of the order $\sim 10^{-5}$ relative to the ranges of the parameters.

The cut $|y_{237}|, |y_{241}| > 4$ mm has been applied since there will be rotational symmetry in the ellipse when the radii get too small which causes the fit to fail in some events. The cut in 4 mm is less than the actual distance of the ALFA detector to the beam for both the $\sqrt{s} = 7 \text{ TeV}, \beta^* = 90 \text{ m}$ and $\sqrt{s} = 8 \text{ TeV}, \beta^* = 90 \text{ m}$ runs and the value is chosen in order to allow for any effects which may be visible in data. The cut may be chosen to be even lower before the problems occur. With this cut, only a fraction of $(6.0 \pm 2.4) \cdot 10^{-5}$ of the protons will have a reconstructed energy more than 3 GeV from the true value, and all of these protons has a true energy within 40 GeV from the upper limit of the fit range for the energy. Such rare cases are therefore not physical, an they may only be seen in data due to limited detector resolution.

The small relative uncertainties and the negligible number of failed events serves as a confirmation on the validity of the parametrization and the efficiency of the reconstruction. The numerical imprecision will not have any importance compared to the effect arising from the detector resolution which will be discussed in the section 9.6.

9.3 Realistic setup to test physics performance

The numerical precision of the kinematic reconstruction gives relative uncertainties of $\sim 10^{-5}$. In this section, the effects from detector smearing are simulated but still without taking any a priori knowledge of data into account except for the ALFA acceptance. The IP coordinates and ALFA coordinates are smeared with Gaussian numbers with a width of 0.3 mm and 0.03 mm, respectively.

The MC sample described in section 9.2.2 is slightly modified such that the energy interval is now $E \in [2800; 4000]$ GeV, but the ranges of the other parameters are unchanged. True energies below 2800 GeV are not possible due to the ALFA acceptance (figure 8.1d), and true energies above 4000 GeV are unphysical. These regions were tested in section 9.2 in order to check the validity of the parametrization. Now, the reconstructed energies can only take values in this range due to the detector smearing.

When using the minimization procedure described in section 9.2.3 on this MC sample, the five reconstructed parameters always gives a lower $\chi^2$ than the true parameters. This is easily understood since the true parameters will on average give $\chi^2 = 6$ due to the smearing of the coordinates, whereas the minimization procedure can find slightly modified parameters lowering the value of $\chi^2$.

The double minimum structure, which was present in section 9.2 even without detector smearing and was the reason for five fits with different starting points per proton, is still an issue. Unfortunately, the global minimum, which the algorithm is sure to find with the five starting points, are now not always the physically correct minimum. In $(1.67 \pm 0.04)%$ of the events, a local minimum has kinematics closer to the true kinematics than the global minimum.
Figure 9.10: Simulation of the resolution of the five fit parameters due to the accuracy of the parametrization and numerical precision of Minuit.
A careful study has been done in order to minimize the amount of events where the reconstructed parameters are in a physically wrong minimum. It turns out that if out of the five minima found from the five starting points we take the one with the lowest value of energy, then the amount of failed events is reduced to only $(0.43 \pm 0.02)\%$. About $1/3$ of these $0.43\%$ failed events have a local minimum with an even lower energy which is the physically correct minimum. Using other fitting procedures, one can be sure to find the lowest lying minimum in energy, but these fitting procedures are not used since about $2/3$ of the failed events need a minimum with a higher energy.

A minimization procedure giving an even smaller amount of events with a minimum, which is physically wrong, has not been found. In the following, a detailed description is given of the $(1.67 \pm 0.04)\%$ of the events where the global minimum is no longer the physically correct one because of the detector smearing. It will be explained why the minimization procedure choosing the minimum with lowest value of energy works well. The reason for this turns out to be a complicated interplay between the different behaviours of the ellipse parameters in figure 9.5 and 9.7 in the two ALFA station at $z = 237$ m and $z = 241$ m.

### 9.3.1 Origin of the double minimum structure

The smearing of 0.03 mm in $y_{237}$ and $y_{241}$ have the most significant effect on the double minimum structure. Therefore, this smearing will be used to illustrate the effect of detector smearing on the double minimum structure of the $\chi^2$ function.

Figure 9.11 shows two hitmaps in the first ALFA station at $z = 237$ m. Figure 9.11a has no smearing at all, while figure 9.11b shows the effect of a displacement of 0.10 mm in $y_{237}$ which is about three times the resolution of ALFA. This gives three different cases: An event is classified as "no double minimum" if Minuit returns the same kinematics for all five starting points, "double minimum, global is physically right" if Minuit returns different kinematics but the one with lowest $\chi^2$ is the one where the reconstructed kinematics is closest to the true, and finally "double minimum, global is physically wrong" if a local minima and not the global one is closest to the true kinematics. The double minimum structure, which is present even without smearing, mainly occurs for large $x$ which in general are events with low energy, see figure 9.8a. Both blue and green points can turn red, i.e. events initially without as well as events initially with a double minimum structure can develop a physically wrong global minimum. Note that the physically wrong minima only occur for the lower ALFA station when the displacement in $y_{237}$ is positive. Likewise, a negative displacement will give rise to wrong global minima in the upper detector. For larger displacements, the effect will be in both stations, but 0.1 mm is already a $\approx 3\sigma$ displacement, thus these cases are expected to be rare.

Figure 9.12 is a sketch of a typical event in the lower detectors where a positive displacement of $y_{237}$ results in a double minimum structure.

In the favoured solution, the slope in vertical direction between the ALFA stations is almost unchanged, but the vertical radius of the ellipse in the parametrization is smaller and the vertical center coordinates are lowered. The lower centres are obtained by a larger $v_y$ since the $\Delta C_y$'s in figure 9.7b are negative. From the growth of the difference between $b_{237}$ and $b_{241}$ in figure 9.5b it can be concluded that the smaller radius but the same slope can be obtained with an increase in $E$ and a smaller $|p_y|$.

In the unfavoured case, Minuit does not have a unique procedure in the way the
Figure 9.11: The hitmap at $z = 237$ m from simulation showing the positions for the three different analytic forms of the $\chi^2$ function. (a) $y_{237}$ is given to the reconstruction. (b) $y_{237}$ displaced with 0.10 mm is given to the reconstruction, but $y_{237}$ is displayed in the hitmap.

Figure 9.12: A sketch of an event in the lower ALFA detectors with a double minimum structure. The $y$ coordinate at $z = 237$ m has been shifted by 0.1 mm, hence Minuit tries to find kinematics such that the proton trajectory goes through $y_{237,smeared}$ and $y_{241,\text{true}}$. The solution with true kinematics goes through $y_{237,\text{true}}$ and $y_{241,\text{true}}$. The green trajectory has kinematics closest to the truth and hence is the favoured solution.

radii and centres are changed. However, all unfavoured solutions have a reconstructed energy much higher than the true energy of about $400 - 1000$ GeV. The reconstructed solutions all have $E > 3600$ GeV, hence from figure 9.5b the difference between the radii in $y$ at 237 m and 241 m is at maximum, and the corrections of the centres are minimal. The vertical radii and centres of the ellipse are then changed individually from event to event in order to minimize the $\chi^2$, and no overall logic has been found.
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Figure 9.5a and 9.5c shows that the large positive shift in $E$ means that $p_x$ is negatively shifted to correct the position in $x$ (when $E$ grows, $|p_x|$ has to grow, and the majority of events with positive $x$ have $p_x < 0$, figure 9.8b). The shift in $E$ explains why the double minimum structure occurs mainly for hits in ALFA with positive $x$: As seen on figure 9.8a the negative $x$ value is only possible for large energies where the difference between $b_{237}$ and $b_{241}$ is maximal.

Concluding the $y$ smearing scenario, the correlation between the energy and $y$ component of the radii and centres of the ellipse used in the parametrization gives the possibility of a solution with a completely wrong kinematics but a smaller $\chi^2$.

Details on the measurements smearing effects

The advantage of taking the fit with the lowest energy instead of the lowest $\chi^2$ depends on which one of the coordinate measurements is poorly measured. Table 9.1 lists the number of events where the minimum with the lowest energy finds a different kinematics from the minimum with the lowest $\chi^2$. This is given for all the different measurements used by the reconstruction. Also, for each of the reconstruction methods is listed how many times it gives the better result.

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Events with double minima</th>
<th>low $E$ best</th>
<th>low $\chi^2$ best</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_{237}$, up</td>
<td>3282</td>
<td>3281</td>
<td>1</td>
</tr>
<tr>
<td>$y_{237}$, down</td>
<td>3225</td>
<td>3225</td>
<td>0</td>
</tr>
<tr>
<td>$y_{241}$, up</td>
<td>3504</td>
<td>3504</td>
<td>0</td>
</tr>
<tr>
<td>$y_{241}$, down</td>
<td>3579</td>
<td>3579</td>
<td>0</td>
</tr>
<tr>
<td>$x_{237}$, up</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$x_{237}$, down</td>
<td>612</td>
<td>171</td>
<td>441</td>
</tr>
<tr>
<td>$x_{241}$, up</td>
<td>520</td>
<td>130</td>
<td>390</td>
</tr>
<tr>
<td>$x_{241}$, down</td>
<td>3</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>$v_y$, up</td>
<td>137</td>
<td>131</td>
<td>6</td>
</tr>
<tr>
<td>$v_y$, down</td>
<td>148</td>
<td>143</td>
<td>5</td>
</tr>
<tr>
<td>$v_x$, up</td>
<td>31</td>
<td>31</td>
<td>0</td>
</tr>
<tr>
<td>$v_x$, down</td>
<td>1137</td>
<td>592</td>
<td>545</td>
</tr>
</tbody>
</table>

Table 9.1: The table shows the effects of different kinds of detector smearing on a sample with 50000 event. Up (down) for $y$ and $x$ means an added value of 0.1 mm (-0.1 mm) while for $v_y$ and $v_x$ the added value is 1.0 mm (-1.0 mm). These offsets correspond to about three times the resolutions of the detectors.

$y$ smearing: As expected, there is clear symmetry between a shift upwards and downwards in $y$ direction for both the IP and the ALFA coordinates since the optics is symmetric in $p_y$. The downward shift in $y_{241}$ has the same effect as described for the upward shift in $y_{237}$. For $y_{237}$, down and $y_{241}$, up the effect is the same, but now it occurs in the upper ALFA detector.

$v_y$ smearing: The double minimum structure for $v_{y,237}$ and $v_{y,241}$ occurs for small energies where the difference between $\Delta C_{y,237}$ and $\Delta C_{y,241}$ is largest, see figure 9.7b. The wrong minimum has a much larger energy such that the difference between $\Delta C_{y,237}$ and $\Delta C_{y,241}$ is negligible, but where there is a large difference
in the vertical radius for the two stations, figure 9.5b. Instead of large centres and negligible radii, the wrong minimum has negligible centres and large radii.

\textbf{x smearing}: The optics is symmetric in positive and negative \( p_x \) just as for \( p_y \), but the energy dependence of \( C_x \) breaks the symmetry in \( x \) direction. However, the structure of the minimum is symmetric whether \( x_{237} \) is moved up (down) or \( x_{241} \) is moved down (up).

The shift in \( x_{237,\text{down}} \) and \( x_{241,\text{up}} \) are the only ones where the method of lowest \( \chi^2 \) gives the best results. The double minimum structure occurs for events where the smearing makes the difference between \( x_{237} \) and \( x_{241} \) larger, i.e. where \( x_{237,\text{true}} < x_{241,\text{true}} \). Therefore, Minuit can either find a solution, where the larger difference between \( x_{237} \) and \( x_{241} \) is obtained through the centres (the low energy solution), or where the difference is obtained through the radii (the high energy solution). Both cases involves the strong correlations between the energy and the horizontal radii and center coordinates of the ellipse used in the parametrization. On average, the higher energy solution is 120 GeV above and the lower energy solution is 170 GeV below the true energy.

The opposite cases of \( x_{237,\text{up}} \) and \( x_{241,\text{down}} \) give only few events where the two methods disagree. This is because \( C_x,237 < C_x,241 \) for all events and therefore there are much fewer events with \( x_{237,\text{true}} > x_{241,\text{true}} \) where the smearing would make the difference larger.

\textbf{v\textsubscript{x,up} smearing}: The double minimum structure occur when the true energy is below 2850 GeV. Here, \( a_{237} \) and \( a_{241} \) have different signs and the horizontal center coordinate is much larger in \( z = 241 \text{ m} \) than in \( z = 237 \text{ m} \), figure 9.5a and 9.5c. The increase in \( v_{x,\text{up}} \) enhances the difference, therefore a wrong minimum occurs when Minuit finds the solution with larger radii and negligible centres. This feature is absent for \( v_{x,\text{down}} \) since in that case the smearing diminishes the difference between \( C_x,237 \) and \( C_x,241 \).

\textbf{v\textsubscript{x,down} smearing}: The double minimum for \( v_{x,\text{down}} \) occurs for events where the effect of \( v_{x,\text{down}} \) is an almost equal enhancement of the horizontal center coordinates in the two stations. Again, one solution corrects for this through the radii, while the other solution is a change of the centres in the two stations. Here, the different reconstructed energies are not as far away from each other as for \( v_{x,\text{up}} \), and the two methods are equally good.

\textbf{Concluding remarks on the double minimum structure}

In all cases, the issue arises on whether the smearing should be corrected for by the radii or the centres of the ellipse used in the parametrization. Both cases may have a small \( \chi^2 \), but one of them can have a kinematics far from the true. It turns out that the low energy solution is more often the correct one. However, it should be noted that this is a feature of this specific optics, and further investigations are needed to handle a possible double minimum structure in other optics configurations.
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9.4 Final reconstruction algorithm

The final reconstruction algorithm uses the parametrization (9.7) and Minuit to minimize the $\chi^2$ function (9.10). Five fits are performed for each event with different starting values of energy and horizontal momentum. The minimum with the lowest reconstructed energy is chosen as the best estimate of the true parameters.

The algorithm is developed for a $\sqrt{s} = 8$ TeV, $\beta^* = 90$ m optics, but the validity of the procedure has also been tested for a $\sqrt{s} = 7$ TeV, $\beta^* = 90$ m optics. The algorithm is intended to be used by others and a user manual can be found in appendix B.

9.5 Validity of reconstructed errors

In addition to the five parameters which minimizes the $\chi^2$, Minuit also returns the errors on the five fit parameters. The errors describe the uncertainties on the parameters well as seen on figure 9.13 where the pull distribution for the energy is shown and a Gaussian fit in $\pm 2$ is overlayed. The pull distributions for the other four parameters are shown in appendix C.1.

![Pull distribution from simulation for the reconstructed energy, where $\sigma_E$ is the error returned by Minuit. The IP and ALFA coordinates have been smeared with a Gaussian number with a width of 0.3 mm and 30 $\mu$m, respectively, and a mean of zero. The values for $\chi^2$, $\mu$ and $\sigma$ are for a Gaussian fit in the range $[-2; 2]$.](image)

Table 9.2 summarizes the parameters from the Gaussian fits. The relatively small values of $\chi^2/ndf$ show that the core of the pull distributions are well described by Gaussian distributions. The widths are reasonably close to 1 and only for $v_x$ is the mean more than 3$\sigma$ from 0.

9.6 Physics performance

The ALFA detector has a resolution of 30 $\mu$m and the ATLAS ID has a resolution depending on the number of tracks. This section describes the influence on the kinematic...
reconstruction resolution due to the two detector resolutions. The ALFA detector in the second station at $z = 241$ m is given a resolution of $40 \, \mu m$ to account for multiple scattering in the first station [37]. This information about the multiple scattering effect came too late to be used in the development of the minimization procedure described in section 9.2 and 9.3, where a resolution of $30 \, \mu m$ was assigned to both stations. It will most likely change some of the numbers in those sections, but not the procedure. Also, the distance from ALFA to the beam centre is about 8 mm in the 8 TeV run, thus the cut in $y$ has been modified to $|y_{237}|, |y_{241}| > 8$ mm. This does not have a significant effect on the resolution but only in the ALFA acceptance as described in chapter 8.

### 9.6.1 MC sample for test of physics performance

The kinematic reconstruction algorithm has been developed without any a priori knowledge of data except for the acceptance region of the LHC. However, the best estimate of the performance, when it is used on data, is obtained using a MC sample resembling data as much as possible. Therefore, a priori knowledge about the kinematics in SD events as well as the measured IP distribution in the data are taken into account.

The particle generator Pythia8 is used to generate kinematic distributions of SD events. Of course the true kinematic distributions in data are unknown, but a particle generator as Pythia8 is as close as we can get. The distribution of $E$ has a sharp peak at $E = 4000$ GeV and falls off rapidly. The $p_x$ and $p_y$ distributions are of course identical, since the physical process is independent of the chosen coordinate system. The two distributions are independent of each other and are both well described by a Gaussian distribution with mean of zero and width of 0.27 GeV.

The IP distribution of the MC used to test the physics performance has been chosen based on the data shown in figure 9.14 since run 206881 is the one with the highest number of events with diffractive triggers. In data, the beam spot (BS) for a given luminosity block is given as the average of all reconstructed vertices in the luminosity block. The width of the BS, however, takes into account that ATLAS gives different uncertainties on the reconstructed vertices. The width of the beam spot is given as the convolution of the resolution of the vertices and the width of the vertex distribution [48]. Therefore, the IP distribution depends on the event-by-event resolution of vertices and can not be given a constant width.

The distribution of resolution for the $x$-coordinate of the reconstructed vertices in the ATLAS ID is shown in figure 9.15. The shape is the same for all luminosity blocks, but the upper limit is given by the BS $\sigma_x$ in figure 9.14. In the MC sample,
the distribution of vertex resolutions is approximated to be flat between 0.03 mm and \( \sigma_x \), and the width of the IP distribution in the MC sample is calculated as

\[
\text{width}_{\text{IP}} = \sqrt{\text{width}_{\text{beam spot}}^2 - \text{resolution}_{\text{vertex}}^2},
\]

i.e. not a constant. This approach is used for both \( v_x \) and \( v_y \). The width of the BS is chosen to be the average value over the run, i.e. 0.16 mm for \( v_x \) and 0.135 mm for \( v_y \). Also, there is a linear correlation between the resolution of \( v_x \) and \( v_y \) which has been taken into account. The BS is chosen at \((-0.27, 0.715)\) which is a rough estimate of the average BS in data.

The \( v_z \) distribution is simply chosen as a Gaussian distribution with a mean of -7.0 mm and a width of 58 mm since this is what figure 9.14 shows. The error on \( v_z \) (and \( v_z \) itself) is not given to the reconstruction code, hence there is no need to use the same approach as for \( v_x \) and \( v_y \). The \( v_z \) coordinate does not play an important role in the resolution (see section 9.1.2) but is included for completeness.

### 9.6.2 Definition of resolution

The distributions of the true minus the reconstructed kinematics are not Gaussian, hence the sigma of a Gaussian fit does not resemble the true resolution. Instead, the resolutions of the reconstructed parameters are given as the RMS of the distributions of the reconstructed minus the true parameter because the RMS can be used for all kinds of distributions.

The \( \xi \) resolution (or equivalently the \( E \) distribution) can be well described by a Gaussian fit, but this is not the case for \( p_x \) and \( p_y \). The shape of the \( p_{x, \text{reco}} - p_{x, \text{true}} \) distribution is like the distributions shown in figure 9.10. The \( p_{x, \text{reco}} - p_{x, \text{true}} \)
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Figure 9.15: Resolutions of the reconstructed vertices in the ATLAS ID for luminosity blocks 490-510 in run 206881.

distribution has a Gaussian distribution with mean zero, but for $p_x \lesssim -1.5$ GeV an additional Gaussian distribution with mean $\sim 1$ GeV appears, i.e. the reconstructed $p_x$ is about $-2.5$ GeV. This is due to the double minimum structure described in section 9.3. As described in section 9.6.1, the $p_x$ distribution in data is expected to have a Gaussian distribution with a width of 0.27 GeV, hence events with $p_x \leq -1.5$ GeV only constitutes $\sim 1 : 10^7$. Resolutions of such low $p_x$ values are merely given for completeness.

Other choices than the RMS could have been used such as an iterative Gaussian fit in $\pm 1.5\sigma$, which is the ATLAS standard, whereby the long tails of $p_y$ and the extra Gaussian in $p_x$ are ignored. This procedure is not used as it makes an event selection. Also, no cut in the $\chi^2$ of the reconstruction fit has been made as it is up to the user to decide whether events with a high $\chi^2$ should be used in the analysis. Anyway, the effect of a cut in $\chi^2$ will not change the resolution significantly.

9.6.3 Resolution

$\xi$ resolution

The resolution of $\xi$ is plotted in figure 9.16 as a function of $\xi$ for the different types of detector smearing. The resolution of the relative energy loss $\xi$ has been plotted instead of the energy, since $\xi$ is the parameter used in the literature as it is independent of the beam energy. The MC for the $\xi$ resolution plot has $p_x$ and $p_y$ distributed according to PYTHIA8 at the IP, but the transport to ALFA makes an event selection due to the acceptance of ALFA. There are almost no events for $|p_y| < 0.1$ GeV, and for high $\xi$ the $p_x$ distribution is shifted to negative values. However, this is also expected to be the case in data as the real events are of course influenced by the same acceptance.

The resolution of $\xi$ is quoted both for the case where the IP has been smeared according to the vertex resolution and the case where the IP has been smeared with the width of the BS. This corresponds to the case where we have a reconstructed vertex in the ATLAS ID and the case where no reconstructed vertex is available and the BS
has to be used as input to the reconstruction. In both cases, the true IP is given by equation (9.15). Clearly, a reconstructed vertex gives a better resolution than using the BS. For the case with a reconstructed vertex, the dominant effect is from the ALFA detector resolution. This difference is diminished when using the BS.

The resolution gets better for all types of smearing with increasing $\xi$. The reason is that the difference between $C_{x,237}$ and $C_{x,241}$ and between $\Delta C_{x,237}$ and $\Delta C_{x,241}$ grow with $\xi$ (figure 9.5c and 9.7a), thus the smearing of 0.03 mm in the $x$ measurements in ALFA has a smaller effect, but the sensitivity to $\xi$ grows.

**Transverse momentum resolution**

The transverse momentum $p_T$ and the azimuthal angle $\phi$ is used in the literature instead of $p_x$ and $p_y$ because the physical processes are independent of the chosen coordinate system. However, the resolution plots for the momentum in the transverse plane are chosen to be given in terms of $p_x$ and $p_y$ since the optics and acceptance is not symmetric in $\phi$.

The $p_x$ and $p_y$ resolution plots are made for protons with fixed energy of 4000 GeV and 3500 GeV. Using the PYTHIA8 simulated energy distribution would give almost the same as for $E = 4000$ GeV because of the strong fall off in the cross section with increasing energy loss, equation (4.50). With two plots at different energies it is possible to see how the resolution changes with energy.

The resolution for $p_x$ is shown in figure 9.17. The effect of the double minimum structure at low $p_x$ is clearly visible. It is most dominant at high energy as expected from the discussion in section 9.3. Whether the IP smearing or the ALFA smearing is the dominating effect depends on both the energy and $p_x$. The ranges of $p_x$ at the two energies are different since the ALFA acceptance changes.

The resolution for $p_y$ is shown in figure 9.18. The $p_y$ resolution is about a factor 100 better than the $p_x$ resolution due to the larger vertical radii in the two ALFA stations, see figure 9.5a and 9.5b. Also, the optics has a stronger correlation of the energy with the horizontal direction than with the vertical direction, hence the $p_y$ reconstruction
Figure 9.17: The resolution of $p_x$ as a function of $p_x$ from simulation at (a) $E_{\text{proton}} = 3500$ GeV and (b) $E_{\text{proton}} = 4000$ GeV. The resolution is given as the RMS and each bin contains 5000 events.

The resolution due to the ALFA smearing decreases with increasing energy since the radius grows (figure 9.5b). Also, the resolution from IP smearing decreases with energy due to the parallel-to-point optics for $E = 4000$ GeV, hence only the smearing of the $x$ coordinate of the IP influences the resolution as this will slightly change the energy. The gap around $p_y = 0$ is due to the ALFA acceptance. At $E \simeq 4000$ GeV it is symmetric around $p_y = 0$ since the optics is symmetric in $y$ and $\Delta C_y(4000 \text{ GeV}) \simeq 0$. At $E = 3500$ GeV this is no longer the case since $\Delta C_y(3500 \text{ GeV}) \neq 0$.

9.7 Systematic uncertainties

The resolutions of the reconstructed kinematics on event-by-event basis has been estimated in the previous section. However, as the reconstruction uses coordinates measured by ALFA and ATLAS, an offset in these detectors are expected to give a systematic effect on the reconstruction.

9.7.1 ALFA detector offset

The positions of the ALFA detectors in the beam CS is known with $\sim 5 \mu m$, section 7.7. An offset in the ALFA detectors causes systematic uncertainties on the reconstructed kinematics. The most significant effect of the offsets on the reconstructed $\xi$ is obtained if the inner ALFA station has an offset in the horizontal direction opposite to the horizontal offset in the outer station. This is understandable from the principle discussed in figure 9.12 and the strong correlation between $\xi$ and the $x$ coordinates. The effect is shown in figure 9.19 where no other smearing has been applied. The other possible combinations of offsets show similar behaviour and but the effects are smaller.
Figure 9.18: The resolution of $p_y$ as a function of $p_y$ from simulation at (a) $E^{proton} = 3500$ GeV and (b) $E^{proton} = 4000$ GeV. The resolution is given as the RMS and each bin contains 10000 events.

The horizontal ALFA offset causes an offset and a smearing on the reconstructed $\xi$ decreasing with $\xi$. The offset in reconstructed kinematics scales with the offset of the detector. The effect is smaller by a factor of $\sim 10$ or more compared to the resolution due to detector resolution shown in figure 9.16. The smaller effect is expected as the detector position offset is much smaller than the detector resolution.

The effects of detector position offset on the horizontal and vertical momentum reconstruction is shown in figure C.2 and C.3 in appendix C.2. Again, the systematic uncertainty is smaller by a factor $\sim 10$ compared to the resolution due to detector resolution.

9.7.2 ATLAS detector offset

The coordinate system transformation described in section 9.1.3 also gives a systematic uncertainty on the reconstruction. The uncertainty on the overall beam spot position of the run measured in the ATLAS CS is sufficiently small to safely neglect this contribution. However, the effect of a discrepancy between the ATLAS and the LHC CS of $\sim 200 \mu m$ [45] is not negligible as it will have direct impact on the IP input to the reconstruction.

Figure 9.20 shows the effect on the $\xi$ reconstruction of an ATLAS CS offset of 200 $\mu m$ in both horizontal and vertical direction wrt. the LHC CS. No other smearing has been applied. The offset in $\xi$ decreases with $\xi$. It is comparable to the resolution from detector resolution in figure 9.16, and the offset divided by the resolution seems to be more or less constant. From elastic data it should be possible to give an estimate of the ATLAS offset by measuring the offset in $\xi$ (which should be zero) and compare it to the beam energy. This will mainly work for the offset in $x$ as this has the largest correlation with the energy.

The effects on the horizontal and vertical momentum reconstruction is shown in figure C.4 and C.5 in appendix C.2 and again the effect is not negligible. The $p_x$ offset
may be eliminated by the ATLAS offset estimate from the elastic energies, just as for the offset in $\xi$. The $p_y$ offset, on the other hand, is so small that the elastic data will be of no use.

### 9.8 Reconstruction of central diffractive protons

The ALFA detector makes it possible to tag both protons in a central diffractive (CD) event, therefore the reconstruction algorithm is developed to be used also on such events. In CD events, the two protons are reconstructed simultaneously in order to find the same IP for both of them. One could also choose to reconstruct them separately using the one-proton fit, but then the reconstructed IPs for the two protons would not be the same, see figure 9.21. The slightly stronger correlation in $v_y$ is expected due to a general better reconstruction in the $y$ direction. Different IPs for the two protons are of course not physically possible.

The procedure for two protons are very similar to the case with one proton. There are now 8 parameters to be reconstructed, $\{E_A^+, p_A^+, E_C^+, p_C^+, v_x^+, v_y^+\}$, and there are 10 measurements, $\{x_{A,m}^{237}, y_{A,m}^{237}, x_{A,m}^{241}, y_{A,m}^{241}, x_{C,m}^{237}, y_{C,m}^{237}, x_{C,m}^{241}, y_{C,m}^{241}, v_x^m, v_y^m\}$, where superscripts $A$ and $C$ indicates A-side and C-side of ATLAS. The function to minimize
Figure 9.21: The correlation between the reconstructed IPs from simulation for two protons coming from the same collision when the protons are fitted individually. (a) The horizontal IP coordinate and (b) the vertical IP coordinate.

The fit is performed in three iterations for each event because the double minimum structure is also present for the two-proton-fit:

**First iteration:** The five starting points given in section 9.2 are used for both protons, where the fit with lowest $\chi^2$ is saved.

**Second iteration:** The kinematics of the first proton is fixed at the values returned by the first iteration while the five starting points are used to find the solution with lowest energy of the second proton.

**Third iteration:** The kinematics of the second proton from the second iteration is fixed and the five starting points are used to find the solution with lowest energy of the first proton.

The difference between reconstructed and true energy for the two protons for 2000 events after the first iteration is shown in figure 9.22a, where the presence of the double minimum structure is seen by the long tails at higher energies. For most of the cases, only one of the protons has a reconstructed energy far from the true energy. This is expected since the kinematics of the protons are only correlated through the
IP, hence the probability of both protons having a global minimum, which is physically wrong, is small - about $1.7\% \times 1.7\%$, see section 9.3. The second and third iterations are used to find the physically correct minimum for the two protons individually. The minimization procedure gives wrong kinematics in a large amount of events if just one iteration with the five starting points is used to find the lowest energy solution for both protons simultaneously. Figure 9.22b shows the result after the three iterations.

![Figure 9.22: The difference between reconstructed and true energy from simulation for a two-proton-fit for 2000 simulated events. (a) is after the first iteration and (b) is after the third iteration. The three iterations are described in the text.](a) (b)

A fourth iteration, where the starting points for the 8 parameters are chosen to be the reconstructed values from the third iteration, may give even better results if the step sizes are sufficiently small such that Minuit does not revert back to the global minimum of iteration 1. This iteration has not been included as the additional computation time will be too large compared with the possible gain and danger of this iteration.

Table 9.3 shows the resolution of the IP both for the method where each proton is fitted individually and the method where the protons are fitted simultaneously. There is a significant decrease in the $v_x$ width, but the offset in the mean slightly increases. The $v_y$ performance does not seem to be affected, but the two protons now have the same IP which is physically meaningful.

Also for the kinematics of the protons, the differences between the one- and two-proton fits are negligible. Again, this is expected due to the small correlation between

<table>
<thead>
<tr>
<th></th>
<th>A-side proton</th>
<th>C-side proton</th>
<th>both protons</th>
</tr>
</thead>
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<td>$v_x$ mean</td>
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<td>-9±2µm</td>
<td>-22±2µm</td>
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<td>260±2µm</td>
<td>237±2µm</td>
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<td>$v_y$ mean</td>
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<tr>
<td>$v_y$ width</td>
<td>298±4µm</td>
<td>296±4µm</td>
<td>295±4µm</td>
</tr>
</tbody>
</table>

Table 9.3: Parameters for the distributions of the reconstructed minus the true IP coordinates. A-side proton and C-side proton are fitted individually.
the protons. Thus, the resolution plots in section 9.6 also applies for the two-proton fit.

9.8.1 Reconstruction of elastic events

An extension to the two-proton fit suitable for elastic analysis has not been made. It could be done simply by fixing the energies of the two protons to the beam energy and requiring that the sums of horizontal and vertical momenta of the protons add up to zero. This will simplify the parametrization (9.7) to equations linear in only four parameters: One horizontal momentum component, one vertical and two IP coordinates. The full machinery of the kinematic reconstruction would therefore be redundant.

9.9 Effective $\sqrt{s} = 7$ TeV, $\beta^* = 90$ m optics

The design optics is based on knowledge about the positions and field strengths of the LHC magnets. However, the magnet positions have not been measured at run conditions and may be displaced, and the LHC operator sets the current of the magnets and not the field strength, hence the calibration between current and field strength needs to be known. This affects the optics on which both the elastic analysis and the kinematic reconstruction of the protons rely.

The elastic data in run 191373 taken with a $\sqrt{s} = 7$ TeV, $\beta^* = 90$ m optics has been used to find a correction to the design optics, the effective optics. Assuming approximately zero transverse momentum of the beam protons in the IP and in addition that the elastic scattering angle in uniformly distributed in azimuth gives constraints on the transport matrix describing the proton trajectories from ATLAS to ALFA. These constraints are used to fit the longitudinal positions and field strengths of the quadrupoles which yields the effective optics. More details on the effective optics can be found in ref. [37].

This section investigates how the effective optics affects the kinematic reconstruction. No effective optics has yet been produced for the $\sqrt{s} = 8$ TeV, $\beta^* = 90$ m optics, thus the results only apply for 7 TeV. However, as the design optics for 7 TeV and 8 TeV are very similar, an effective optics in 8 TeV is expected to have a comparable effect. Therefore, the difference between the design and effective optics serves as an estimate of the systematic uncertainties on the kinematic reconstruction due to the optics.

9.9.1 MC comparison

Figure 9.23 shows the energy resolution as a function of energy due to the optics. A MC sample with kinematics as described in section 9.6 is generated and transported to ALFA with the design optics. However, the simulation is done without applying any detector smearing in IP or ALFA coordinates. Figure 9.23a shows the resolution when the parametrization, and hence also the reconstruction, is done with design optics, whereas figure 9.23b shows the resolution when the parametrization and reconstruction are done with effective optics, i.e. when transport and parametrization are done with different optics.

In figure 9.23a, the average value of each bin is approximately zero and the error bars depicting the RMS correspond to $\sim 5$ MeV. Since no detector smearing has been
applied, this plot is essentially just the energy resolution in figure 9.10 as a function of energy (but now at 7 TeV). Thus, the RMS reflects the precision of the parametrization (9.7) and the numerical accuracy of Minuit and nothing else.

In figure 9.23b there is an offset at low energies with small RMS whereas for high energies the offset is negligible but the RMS is larger. However, both the offset at low energies and the RMS at higher energies are about 0.001 which is only ~ 10% of the combined resolution from detector effects (see figure 9.16 which is for the 8 TeV optics, but is comparable with 7 TeV optics).

The plots for the optics effect on $p_x$ and $p_y$ are shown in figure 9.24 and 9.25, respectively. The plots are made for same values of $\xi$ as the resolution plots in section 9.6.3 in order to make a comparison even though the beam energy is different. At low $p_x$ the offset due to the optics is comparable with the resolution due to detector smearing while at higher $p_x$ the offset is about ~ 40% of the resolution. The double minimum structure comes into play at $p_x < -1$ GeV in figure 9.24b resulting in offsets around $-0.15$ GeV but with error bars around 0.2 GeV. The offset in $p_y$ at $\xi = 0.15$ is comparable with the resolution due to detector smearing while at $\xi = 0$ the offset is about a factor 3 higher than the resolution.

In conclusion one can say that MC simulations show that the optics effect on the energy reconstruction is small whereas it is a significant effect in the $p_x$ reconstruction and the dominating effect in the $p_y$ reconstruction. An effective optics for $\sqrt{s} = 8$ TeV, $\beta^* = 90$ m has not yet been found, but one might expect an optics effect not far from the one discussed here.
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Figure 9.24: The $p_x$ resolution as a function of $p_x$ from simulation when design optics in used for transport and effective optics is used for reconstruction. (a) is the $p_x$ effect for $E_{\text{proton}} = 3062.5$ GeV and (b) is the effect for $E_{\text{proton}} = 3500$ GeV. Error bars indicate the RMS of the distributions and the errors on the two leftmost bins in (b) are $\approx 0.2$ GeV. Each bin contains 5000 events, and no detector smearing has been applied.

Figure 9.25: The $p_y$ resolution from simulation as a function of $p_y$ when design optics in used for transport and effective optics is used for reconstruction. (a) is the $p_y$ effect for $E_{\text{proton}} = 3062.5$ GeV and (b) is the effect for $E_{\text{proton}} = 3500$ GeV. Error bars indicate the RMS of the distributions and each bin contains 10000 events. No detector smearing has been applied.
9.9.2 Optics dependence in the $\sqrt{s} = 7$ TeV data

Since the effective optics has been obtained by analysing the elastic events in $\sqrt{s} = 7$ TeV, $\beta^* = 90$ m data, it will be interesting to see the differences (if any) in the reconstructed kinematics. The elastic selection cuts\cite{37} are applied on the data. These selection cuts are:

- bunch crossing ID,
- luminosity blocks,
- reconstructed tracks in ALFA detector 1,3,6,8 or 2,4,5,7,
- tracks in A- and C-side are back-to-back,
- tracks must be more than 60 microns from the edge of the detector,
- beam screen cut on the upper limit of the $|y|$ coordinates of about 20 millimetres,
- $x$ versus $\theta_x$ must be within a given ellipse, where $\theta_x$ is the $x$ angle of the proton between the ALFA stations and not the angle at the IP.

The proton reconstruction is done with the two-proton fit described in section 9.8 with the beam spot and beam spot error used as IP input. A coordinate transformation as described in section 9.1.3 has been performed on the ALFA data.

The kinematic reconstruction returns a $\chi^2$ which gives a measure of the goodness of the fit. A probability for the fit can be calculated from the $\chi^2$ and the number of degrees of freedom which is 2 for the two-proton fit. The probability distributions of the reconstruction using the two different optics are shown in figure 9.26a where the goodness of the fit decreases with increasing values on the $x$ axis. The probability distributions are not able to tell whether the effective optics is better than the design optics, since no significant differences between the distributions are present. This is true also on an event by event basis, since the correlation factor between the probability for design and effective optics is close to one as seen on figure 9.26b, i.e. an event with a bad fit using one optics will also have a bad fit using the other optics and vice-versa for a good fit.

The badly reconstructed events have a smaller correlation between $y$ and $\theta_y$ compared to the well reconstructed events. Based on this observation it could be argued that the elastic analysis should impose an extra selection cut in $y$ versus $\theta_y$ just as for $x$ versus $\theta_x$. This selection cut has been studied at length by the elastic analysis group but it turned out that the cut does not have much impact for the analysis.

The reconstructed energy distributions for the elastic protons on A-side are shown in figure 9.27a for different regions of probability of the fit. Both optics have a tail of energies below 3400 GeV. The tails are outside a 95 % confidence interval in the goodness of the fit, and the differences in the tails between the two optics are within the statistical errors. A zoom of the differences between the two distributions are shown in figure 9.27b where it is clear that the differences in the tails at the higher energies can not be attributed to statistics nor bad reconstructed fits. The black curve is clearly not consistent with zero for $E > 3560$ GeV, and the red curve constitute a large fraction. Thus the effective optics gives a smaller tail of events with too high energy. From figure 9.27b it is also clear that these events end up in the peak at 3500 GeV. The effective optics therefore gives an overall more narrow energy distribution.
Figure 9.26: (a) The probabilities reflecting the goodness of the fits for elastic protons in run 191373 when the proton reconstruction is done with design and effective optics and (b) the correlation between the probabilities for the two optics.

Figure 9.27: Energies for elastic protons in run 191373 reconstructed with design and effective optics. (a) shows the distributions and (b) shows the differences normalized to the total number of events.

A Gaussian fit to the red curve for effective optics gives a mean of \( \mu = 3500.09 \pm 0.04 \) GeV and a width of \( \sigma = 32.03 \pm 0.03 \) GeV. The mean can not be used as a measure of the beam energy since it is highly dependent on the horizontal alignment of the ALFA detectors. The alignment is done with elastic event which all have the beam energy, hence the reconstructed elastic events should all have energy of the nominal beam energy. The mean of exactly the postulated beam energy only serves as a confirmation that the reconstruction works at least for elastic protons. It also gives a hint of the correctness of the coordinate transformation described in section 9.1.3,
since the mean is $\mu = 3504.07 \pm 0.05$ GeV when the coordinate transformation of the ALFA data is not used. There is no sign of an offset due to an offset of the ATLAS detector. The width is in agreement with the resolution of the reconstruction and is not due to smearing of the beam energies which has a relative RMS of about $10^{-4}$ [21].

Similar plots are done for $p_x, p_y, v_x$ and $v_y$ and can be seen in appendix C.3. The conclusion is that the effective optics gives more narrow distributions for $E, p_x$ and $v_x$ whereas the differences in $p_y$ and $v_y$ are less pronounced. The scattering angle in $x$ and $y$ direction, which is what the elastic analysis measures, can be found from the reconstruction by dividing $p_x$ and $p_y$ with $E$. The distributions are very much alike the $p_x$ and $p_y$ distributions since $E$ is almost constant. Hence, the reconstruction of elastically scattered protons show that the effective optics gives smaller scattering angle in $x$ and unchanged scattering angle in $y$ which is what was expected and the reason why the effective optics were made.

### 9.10 Comparison with Cracow reconstruction code

Another tool for the kinematic reconstruction of protons in ALFA has been developed by a group in Cracow [49]. This section shows a comparison between the performance of the code described above (Copenhagen code) and the Cracow code.

The Cracow code uses the transport program MadX mentioned in section 5.3, and not FPTracker implemented in ALFA BeamTransport used for the Copenhagen code, to transport the protons from IP to ALFA. The parametrization is done with polynomials where the user can choose the degrees. The unfolding is similarly done by minimizing the $\chi^2$ function:

$$
\chi^2(p, \mathbf{vx}) = \frac{(x_{237}^m - x_{237}^r(p, \mathbf{vx}))^2}{\sigma_{237}^2} + \frac{(y_{237}^m - y_{237}^r(p, \mathbf{vx}))^2}{\sigma_{237}^2} + \frac{(x_{241}^m - x_{241}^r(p, \mathbf{vx}))^2}{\sigma_{241}^2} + \frac{(y_{241}^m - y_{241}^r(p, \mathbf{vx}))^2}{\sigma_{241}^2},
$$

(9.17)

where $p$ is the momentum of the proton at the IP and $\mathbf{vx} = (v_x, v_y, v_z)$ is the IP. The Cracow code uses the IP as input to calculate the ALFA coordinates but the IP is not included in the $\chi^2$ function and hence not reconstructed. The Cracow code reconstructs only one proton at a time, therefore only single proton reconstruction is compared.

The comparison is done in the 8 TeV, $\beta^* = 90$ m optics. The MC sample used for the comparison has been produced with the following distributions:

- $v_x$: Gaussian distribution with $\mu = 0.0$ mm and $\sigma = 0.2$ mm.
- $v_y$: Gaussian distribution with $\mu = 0.0$ mm and $\sigma = 0.2$ mm.
- $v_z$: Gaussian distribution with $\mu = 0.0$ mm and $\sigma = 54$ mm.
- $p_x$: Gaussian distribution with $\mu = 0.0$ mm and $\sigma = 0.5$ GeV.
- $p_y$: Gaussian distribution with $\mu = 0.0$ mm and $\sigma = 0.5$ GeV.
- $p_z$: Uniform distribution between 2800 GeV and 4000 GeV.
These distributions do not reflect a MC for SD events, but since this is simply a test of the overall performance of the codes, it does not matter. The specific values for the distributions are chosen from an example file in the Cracow code package except for the $p_z$ distribution where the lower limit has been changed from 3000 GeV to 2800 GeV. The particles are transported to ALFA using FPTracker, since this is the option in the example file in the Cracow code package. Some particles will be lost during the transportation from IP to ALFA, hence the sample, on which the codes are compared, does not have exactly the distributions listed above. A cut in $|y_{237,241}| > 4$ mm has been applied.

Different combinations of detector smearing are tested to give a more complete picture of the differences between the codes. In every plot, the default degrees of the polynomials in the Cracow code are used. Furthermore, the Copenhagen code has been modified such that the last two terms of the $\chi^2$ function (9.10) are removed, i.e. the IP is taken as input but not fitted just as for the Cracow code. In this way, the two codes will be compared on an equal footing. The effect of fitting the IP in the Copenhagen code is discussed in section 9.10.5.

### 9.10.1 Check of the parametrization

In order to compare the parametrization and reconstruction alone without any detector smearing, the true ALFA and IP coordinates are given as input. Figure 9.28 shows the $\xi$ differences as a function of the true $\xi$.

![Figure 9.28](image)

**Figure 9.28:** Simulation of the $\xi$ differences as a function of true $\xi$ with no detector smearing applied. Right plot is a zoom of the left.

The left plot shows that the Cracow code fails for some of the events at very high $\xi$. Also, the difference $\xi_{\text{cra}} - \xi_{\text{true}}$ exceeds 0.001 for $\xi_{\text{true}} > 0.2$. The reason is likely that this is outside the tested intervals for the polynomials in the Cracow code parametrization, hence the polynomials are just extrapolated to this region. These failed events will probably only constitute a small fraction in data since the SD cross section is expected to fall rapidly with $\xi$, see equation (4.49). However, the large $\xi$ events are the most interesting in a physical perspective since the postulated pomeron is more energetic.
The zoom in the right plot shows that the precision of the Copenhagen code is below $10^{-5}$ in agreement with figure 9.10. The limited precision of the Cracow code is clearly visible, and the shape suggests that it is due to the use of polynomials in the parametrization and not the difference between MadX and FPTracker. The use of interpolators in the Copenhagen code gives an advantage in the precision, however when detector smearing is included, this will only be important at high $\xi$.

The failed events in the Cracow code at very high $\xi$ are also visible in the $p_x$ and $p_y$ reconstruction where the differences for these events are large. Also, the events at $\xi_{\text{true}} > 0.2$ have differences up to 0.1 GeV in $p_x$ and $p_y$.

### 9.10.2 IP smearing

The effect of IP smearing in the $\xi$ reconstruction is shown in figure 9.29 where (0, 0, 0) is given as input for the IP instead of the true IP. The failed events at $\xi \approx 0.3$ visible in figure 9.28 are not shown here. Clearly, the IP smearing affects the reconstruction, but notice that the Copenhagen and Cracow codes are in agreement for almost every event with $\xi < 0.2$. With $\xi > 0.2$ the Copenhagen code works best. Of course the IP smearing also affects the reconstruction of $p_x$ and $p_y$, but again the two codes agree for the majority of events.

![Figure 9.29](image)

**Figure 9.29:** Simulation of the $\xi$ differences as a function of true $\xi$ with IP smeared with 0.2 mm and true ALFA coordinates given as input to the reconstruction algorithms.

### 9.10.3 ALFA smearing

The result of smearing the ALFA coordinates with 30 $\mu$m while giving the true IP as input is shown in figure 9.30 for the reconstruction of $\xi$ and $p_x$. There is a clear discrepancy between the codes for some of the events with $\xi < 0.15$ and $-1$ GeV < $p_x < 0$ GeV where the Cracow code finds solutions with too high energy and too low $p_x$. This is the double minimum structure described in section 9.3 which is not dealt with in the Cracow code. For the kinematic distributions chosen for this comparison, the fraction of events where the Cracow code finds wrong kinematics is $\sim 1\%$ but is likely to be different in real data.
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Figure 9.30: Simulation of the differences as a function of the true parameter with true IP and ALFA coordinates smeared with 30 µm given as input to the reconstruction algorithms. (a) shows the $\xi$ differences and (b) shows the $p_x$ differences for $\xi < 0.2$.

9.10.4 All detector smearing effects

The combined effect of using $(0, 0, 0)$ as IP input instead of the true IP and a smearing of the ALFA coordinates are not much different from the effect of ALFA smearing alone. The differences get larger but the shape is much like fig. 9.30.

9.10.5 IP fitting

The Copenhagen code and the Cracow code are in very good agreement for the vast majority of events as can be seen by the blue points in figure 9.28-9.30. If the resolutions of the reconstructed to the true kinematics are quoted as the width of a Gaussian distribution fitted iteratively to $\pm 1.5\sigma$, the differences are within the error bars. When using the default fitting procedure in the Copenhagen code, a difference in resolution arising from IP smearing could have been expected since the Copenhagen code treats the IP information as fitting parameters whereas the Cracow code uses the IP information only as input to the parametrization. This turns out not to be the case. Making the resolution plots in section 9.6.3, but now with a fixed vertex, gives almost identical plots. The smearing of the IP is simply not large enough.

On an event-by-event basis, it is possible to see an improvement by fitting the vertex instead of fixing it. Table 9.4 shows the mean and RMS of the distributions of true minus reconstructed kinematics for the case where the IP is fixed and the case where the IP is fitted. Instead of giving $(0, 0, 0)$ as input to the reconstruction, an IP of $(0.6, 0.6, 0)$ mm has been given as input, i.e. a 3 sigma displacement in $x$ and $y$ from the average IP. It is clear that the method where the IP is fitted both gives a smaller average offset from the mean and also a smaller RMS. The difference in mean of the $\Delta \xi$ distributions is small compared to the resolution of $\xi$ in figure 9.16 whereas the differences in mean of $p_x$ and $p_y$ are rather large effects.

The difference between the two methods of handling the IP is not visible when
looking on an entire MC sample with vertex distributions and corresponding uncertainties resembling data. However, on an event-by-event study, the IP fitting gives better results when the offset of the IP is large.

<table>
<thead>
<tr>
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<th>fixed IP</th>
<th>fitted IP</th>
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<tbody>
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<td>∆ξ RMS</td>
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<td>∆px mean</td>
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<td>$-16.84 \pm 0.09$ MeV</td>
</tr>
<tr>
<td>∆py RMS</td>
<td>$31.47 \pm 0.07$ MeV</td>
<td>$29.38 \pm 0.07$ MeV</td>
</tr>
</tbody>
</table>

Table 9.4: The values of the distributions of true minus reconstructed kinematics for the two different ways of handling the IP in the Copenhagen code.

9.10.6 Conclusion on the comparison

The Copenhagen code and the Cracow code are in excellent agreement for the vast majority of event. The interpolator versus polynomial approach in the parametrization is not visible when detector smearing is taken into account, except for events with $\xi > 0.2$ where the Copenhagen code gives the best reconstruction. The different approaches concerning how to handle the IP information do not cause differences in the resolution plots but only in event-by-event studies. There is a significant difference between the codes for some of the events with $\xi \gtrsim 0.3$ and for some events with $\xi < 0.15$ and $p_x < 0$ GeV, since the Cracow code fails for some of these events.
10  |  Data analysis

This chapter is devoted to an analysis of the data collected by the ALFA and ATLAS detectors. The presented results are very preliminary since

1) the final alignment of the ALFA detector has not yet been found,

2) the work needed to find modifications to the design optics in order to obtain an effective optics has not begun,

3) the work on a full Monte Carlo simulation is still in progress.

Nevertheless, the kinematic distributions of the protons in single diffractive (SD) events are found with the kinematic reconstruction algorithm developed in chapter 9 and compared to a toy model MC simulation. Momentum correlations between the proton in ALFA and the tracks in the ATLAS ID are also investigated to see if the ID measurement can be used to guide the kinematic reconstruction of the proton.

The reconstruction code is able to handle the central diffractive events as well and the same investigations can be made as for SD events. However, this will not be done due to time limitations.

10.1 Data

The data is from run 206881 taken 11 July 2012 at the LHC in Geneva, Switzerland. Mainly elastic ALFA triggers was used in lumiblock 1-337, while for the rest of the run mainly diffractive triggers were used [25]. The two most important diffractive triggers are trigger signals in both ALFA stations in an armlet, and trigger signals in both stations in an armlet combined with hits in the MBTS in the opposite side to the armlet.

The integrated luminosity for the entire run was 24.11 nb$^{-1}$ [25]. The CM energy of the run was $\sqrt{s} = 8$ TeV and the used optics had a beta function value at the IP of $\beta^* = 90$ m. The physical consequences of this special optics was described in section 5.2.6. Run 206884 and 206885 also had this optics and diffractive ALFA triggers, and these runs can be included in the analysis of diffraction at a later stage to gain more statistics.

10.2 Elastic events

The elastic events are used to align the ALFA detectors (section 7.7) and to find the coordinate transformation from beam CS to LHC CS (section 9.1.3). Therefore, elastic events are crucial also for the diffractive analysis. Elastic events are selected with the
CHAPTER 10. DATA ANALYSIS

<table>
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<th>width [GeV]</th>
<th>$\chi^2$/ndf</th>
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<td>Armlet 4</td>
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<td>33.00 ± 0.02</td>
<td>15889 / 424</td>
</tr>
</tbody>
</table>

Table 10.1: The values for a Gaussian fit to the energy distributions in elastic data.

same cuts as was used in section 9.9.2 for the $\sqrt{s} = 7$ TeV run. Specific cuts for $\sqrt{s} = 8$ TeV deviating from the $\sqrt{s} = 7$ TeV cuts will probably be found when people start to work on the elastic analysis, but for now the $\sqrt{s} = 7$ TeV cuts are the best available.

A weighted mean of the beam spots for all the elastic events gives

$$(x_{\text{ATLAS BS}}, y_{\text{ATLAS BS}}) = (-0.275, 0.712) \approx (x_{\text{LHC BS}}, y_{\text{LHC BS}}),$$

and this coordinate set is used to do the coordinate transformation of the ALFA measurements.

The energies of the elastic events are reconstructed, and the results of a Gaussian fit are listed in table 10.1 for the events fulfilling $1 - \text{Prob}(\chi^2, \text{ndf}) < 68.27\%$ for the goodness of the reconstruction fit. All four armlets have a width which is in agreement with the resolution (figure 9.16), but the means are too low. The beam energy has been measured in ref. [54] to be $3988 \pm 5\text{(stat)} \pm 26\text{(syst)}$ GeV, but as was discussed in section 9.9.2, the reconstructed energy of elastic events should still be at the nominal beam energy of 4 TeV due to the alignment of ALFA. The large $\chi^2$/ndf for the armlets show that the distributions are not perfectly Gaussian. However, this does not change the fact that the offset in the mean energies are too large to be ignored. This may be due to the ALFA detector alignment, the elastic selection cuts, an offset of the ATLAS detector wrt. the LHC CS, or contamination by CD events. An offset of the ATLAS detector seems unlikely since this offset should then have had an influence on the $\sqrt{s} = 7$ TeV elastics, which was not observed in section 9.9.2. Further investigation are needed to determine the cause. For this analysis, the offset will simply be ignored.

10.3 Single Diffraction selection cuts

An SD event is characterized by a proton scattered at a small angle and a bunch of particles produced by the excited proton. A series of cuts in data have to be made in order to find these events. This section describes the cuts used to obtain a clean sample of SD events. An investigation of the consequences of the cuts on the number of event in data has not been performed since is has no interest for this analysis.

10.3.1 Luminosity blocks

The data run is divided into luminosity blocks. A luminosity block (LB) is a time interval where the conditions of the beams, detectors and triggers are assumed to be unchanged. In this way, data in specific LBs, where there has been failures, can be excluded from the data analysis. The duration of a good LB is just above a minute whereas a bad one may be shorter. The requirements for a good LB is that ALFA
must be in the final position in the beam pipe, the dead-time of the ALFA and ATLAS
detectors must be less than 5 %, the duration of the LB is larger than 60 seconds, and
ATLAS ready for physics. The bad LBs are found in ref. [50] and are excluded from
the analysis.

10.3.2 Bunch crossing ID

Some bunches are unpaired, i.e. a bunch in one of the beams did not have a collision
with a bunch in the other beam. The unpaired bunches may nevertheless give a signal
in the ALFA detector due to beam halo or other background sources. The bunch
crossing ID can be found in ref. [25], and signals from the unpaired bunches are
removed.

10.3.3 Hits in ALFA

The weakest requirement on the signal in ALFA is to demand that both triggers in
one armlet has fired. The stronger requirement of a reconstructed track in both main
detectors of the armlet is implied in order to perform the kinematic reconstruction of
the proton.

A veto on the other 3 armlets is used. This removes most of the elastic events,
and it enhances the probability that the signal in ATLAS and in the ALFA armlet
comes from the same collision. Otherwise, one have to determine which one of the
fired armlets (if any) actually contains the information about the SD event.

The multiplicity of tracks in ALFA must be equal to 1. A higher multiplicity is
possible if two scattered protons hit the same armlet, or if a beam halo particle coincide
with the signal. This gives rise to combinatoric issue on how to combine the tracks
in the two sub-detectors of the armlet in the right way, and these issues are avoided
in this analysis. At a later stage, this cut may be dropped in order to increase the
number of events.

Data from unpaired bunches show a characteristic ellipse in a $x_{237}$ vs. $\theta_x$ plot, where
$\theta_x$ is the $x$ angle of the proton between the ALFA stations. Events in this region, which
does not overlap with the pattern of SD events found from MC, are excluded. One
could choose to exclude the entire ellipse, but this will remove a significant amount
of the SD events, hence it is chosen to keep these background events. This cut is
investigated in ref. [51].

10.3.4 Signal in ATLAS

Diffractive events produce a bunch of particles in the forward direction of the exited
proton as described in section 4.2. Therefore, the requirement of a signal in ATLAS
reduces the number of events due to the limited acceptance region of ATLAS but it is
the most powerful way to cut away signal in ALFA arising from beam halo.

Signal in the MBTS

The MBTS is not used in the event selection. A signal in the MBTS above a certain
threshold in the opposite side to the signal in ALFA is sufficient to say that there was
a collision and thereby reduce the probability that the signal in ALFA is beam halo.
The beam spot can be used as IP input to the kinematic reconstruction algorithm.
However, the reconstruction resolution will be worse than if a reconstructed vertex is given as input (see figure 9.16) and in this analysis we are trying to find as clean SD sample and as precise a reconstruction as possible.

Signal in the ATLAS inner detector

We therefore demand a reconstructed vertex in the ATLAS inner detector whereby a more precise reconstruction can be obtained. Furthermore, the fragments of the exited proton can be studied. Only events with exactly one reconstructed vertex is chosen since there is no way to determine which one of them belongs to the proton in ALFA.

The number of reconstructed vertices in ATLAS for events satisfying the cuts in the ALFA signal is shown in figure 10.1. A large fraction of events does not have a reconstructed vertex and may only be analysed using the MBTS requirement.

A significant amount of events have 2 or more reconstructed vertices. One could hope that the true vertex for the diffractive process could be found by reconstructing the proton using the different vertices and then take the vertex with the best fit. Unfortunately, the reconstruction is not able to do this. Figure 10.2 shows the difference between the probabilities of the reconstruction fits for primary and secondary vertex in events with 2 reconstructed vertices. Separation indicates the distance between the vertices.

\[
\text{Separation} = \sqrt{\left( \frac{v_{x,\text{prim}} - v_{x,\text{sec}}}{\sigma^2 v_{x,\text{prim}} + \sigma^2 v_{x,\text{sec}}} \right)^2 + \left( \frac{v_{y,\text{prim}} - v_{y,\text{sec}}}{\sigma^2 v_{y,\text{prim}} + \sigma^2 v_{y,\text{sec}}} \right)^2}, \tag{10.2}
\]

where the primary vertex is chosen as the one with the highest energy sum of the tracks. For the majority of events, the difference in probability is too small to safely select one vertex over the other. The ability to distinguish the vertices does not increase significantly with the separation of the vertices. Notice, that the distributions...
are symmetric around zero meaning that on average does the primary vertex not give better fit results than the secondary. The differences in reconstructed kinematics using the two different vertices can not be used as they are too small. Also, such a choice would rely on a priori knowledge about what we are trying to measure.

The vertex associated with the diffractive process may be chosen as the one with the largest rapidity gap to the proton. However, this will make a strong bias in the event sample and this possibility is not used.

The events with two or more reconstructed vertices may be used to analyse the kinematic distributions of the protons in the same way as for the MBTS requirement. Here, the beam spot can be used as vertex input to the reconstruction code.

**Signal in the calorimeters**

The ATLAS calorimeters have a pseudorapidity coverage of $|\eta| < 4.9$, which is much larger than the ID coverage of $|\eta| < 2.5$. Hence, demanding only a signal in the calorimeters will enhance the number of observed diffractive processes. The calorimeters do not provide a reconstructed vertex, but the beam spot may be used as input to the kinematic reconstruction of the proton just as in the case with the MBTS.

Unfortunately, the calorimeter data shows strange behaviour. Especially around $|\eta| = 2$, the noise level is too high \cite{52}. Whether the problem can be solved at a later stage is unknown, but the calorimeters are useless for this analysis.

### 10.3.5 Background sources

Even with the cuts mentioned above, the SD sample will not be free of background. Possible background sources are:

- An elastic event where one of the protons does not have a reconstructed track in ALFA while a non-diffractive event gives signal in ATLAS.
- A central diffractive (CD) event, where one of the protons does not have a reconstructed track in ALFA.
- A beam halo particle reconstructed in ALFA while a non-diffractive (ND) event gives signal in ATLAS.
- An SD event measured in ALFA but without signal in ATLAS, and an ND event giving signal in ATLAS.
- A beam halo particle reconstructed in ALFA and an SD or CD event giving signal in ATLAS but not in ALFA.
- An ND event with a charged particle produced with kinematics allowing it to hit ALFA.

The cases where two collisions happen in the same bunch crossing will be suppressed compared to one collision by a factor

$$
\frac{P(\text{one collision})}{P(\text{two collisions})} = \frac{e^{-\mu_1^1/1!}}{e^{-\mu_2^2/2!}} \approx \frac{e^{-0.07\cdot0.07^1/1!}}{e^{-0.07\cdot0.07^2/2!}} \approx 30, \quad (10.3)
$$
where $\mu$ is the average number of collisions per bunch crossing described in section 5.2.6. However, a true estimate of the amount of background requires knowledge about the different cross sections, their event shapes and the acceptances and efficiencies of the ALFA and ATLAS detectors for the different events, and an estimate of the beam halo. This will not be done here, but it will simply be assumed that the SD sample is pure enough for the current analysis.

### 10.4 Monte Carlo

A full simulated Monte Carlo sample including acceptances and efficiencies of the different sub-detectors is not yet available. Work is in progress, but due to problems with the transport of protons from IP to ALFA, it has not been finished.

Instead, a simple toy model MC will be used in this analysis to compare the reconstructed kinematic distributions of the single diffractive protons in ALFA with expectations. PYTHIA8 is used on generator level and ALFA_BeamTransport is used to transport the protons to ALFA from an IP distribution as obtained in data, see section 9.6. The ALFA coordinates are smeared with 30 $\mu$m and 40 $\mu$m in the inner and outer detector, respectively, and the proton kinematics is reconstructed. The MC Pythia8, SD does not have any requirement on a signal in ATLAS, whereas the MC Pythia8, SD with vertex has a requirement of $\geq 2$ charged particles with $p_T \geq 100$ MeV in the pseudorapidity range $|\eta| < 2.5$. This is used to simulate a reconstructed vertex in the ATLAS ID. No beam smearing effects or tracking efficiencies are taken into account, hence the results presented for the MC are only indications and may be altered significantly in a full simulation.

Inspired by ref. [53], a non-diffractive (ND) sample has been generated as an estimate of background. Mainly protons, neutrons, pions and kaons are generated with kinematics making it possible to hit ALFA. However, only the protons are transported to ALFA. Neutrons do not give a signal in ALFA since they have no electric charge, and the amount of pions and kaons are small enough to be neglected in this simple toy model MC. The vertex requirement is also used for the ND sample. The ND sample has been scaled wrt. the MC Pythia8, SD with vertex sample, where the cross sections for ND and SD has been taken into account.

Central diffractive (CD) events with only one of the protons tagged by ALFA is also a background source. PYTHIA8 estimates the ratio between CD and SD to be about 10 % at $\sqrt{s} = 8$ TeV, but unfortunately a CD sample has not been simulated due to time limitations. However, a quick look at a small PYTHIA8 generated CD sample shows similar kinematic distributions for CD as for SD events. Thus, the CD sample may be irrelevant in a toy model MC as it may simply scale the SD generated sample when no vertex requirement is used. The track distribution in the CD sample has not been investigated, hence the effect of a vertex requirement is unknown.

### 10.5 SD kinematic distributions

The $\xi, p_x$ and $p_y$ distributions of the events satisfying the ALFA selection cuts and the requirement of a reconstructed vertex in the ID are studied. These distributions provide information on the effect of the selection cuts and possible background when compared to MC.
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Reconstructed relative energy loss

The $\xi$ distribution for SD events in data is shown in figure 10.3 for different ranges of the goodness of the reconstruction fit. It is seen that a cut in the goodness of the reconstruction fit does not have a significant effect on the shape of the distribution. Regge theory predicts that the cross section should fall rapidly with $\xi$ (equation (4.49)), thus many events with $\xi \approx 0$ are expected. These are seen as the peak at $\xi \approx 0$.

![Figure 10.3: Distribution of the reconstructed $\xi$ of SD protons in data for different ranges of the goodness of the reconstruction fit.](image1)

![Figure 10.4: Distribution of the reconstructed $\xi$ of SD protons in data and MC. The MC distributions are normalised to data. Only protons where $1 - \text{Prob}(\chi^2, \text{ndf}) < 68.27\%$ are plotted.](image2)

The events with reconstructed $\xi$ above 0.3 are regarded as background. The ALFA acceptance gives an upper limit on the true $\xi$ of 0.3 (figure 8.1d), but detector resolution has a smearing effect on the reconstruction (figure 9.16) and reconstructed SD events with $\xi > 0.3$ can therefore not immediately be considered as background. Beam halo or protons colliding with molecules in the beam pipe on the way from IP to ALFA are not restricted by this cut in $\xi$ since they do not originate from the IP. However, as figure 10.3 shows, there is an empty region around $\xi = 0.3$ which lowers the probability that events with $\xi > 0.3$ are SD events. Effects from detector resolution cannot explain why the SD distribution would have such a gap. Furthermore, almost all events with a reconstructed $\xi$ above 0.3 have $1 - \text{Prob}(\chi^2, \text{ndf}) > 95.45\%$, i.e. a poor reconstruction.

Figure 10.4 shows a comparison between data and MC. The ND MC distribution have a sharp cut off at $\xi = 0.3$ just like the SD events in figure 10.3. This enlarges the probability that the events in data with reconstructed $\xi > 0.3$ are beam halo.

The MC sample of SD protons with no requirement of a reconstructed vertex in the ID clearly has a peak which is too high at $\xi \approx 0$, i.e. protons with the beam energy. The requirement of a vertex in ID lowers the peak and gives more protons with higher $\xi$. It is a clear indication that requiring a vertex favours SD protons with a larger energy loss. This is expected from the discussion of single-inclusive processes in section 4.2.2 where it was shown that a larger energy loss gives a smaller rapidity gap, equation (4.24). Thus the fraction of particles in the acceptance region of the ID $\eta$ of $|\eta| < 2.5$ increases with $\xi$. The peak is still too large compared to data, but
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<table>
<thead>
<tr>
<th></th>
<th>mean $\times 10^3$</th>
<th>width $\times 10^3$</th>
<th>$\chi^2$/ndf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data, SD</td>
<td>$3.509 \pm 0.019$</td>
<td>$8.473 \pm 0.022$</td>
<td>200 / 77</td>
</tr>
<tr>
<td>MC PYTHIA8, SD</td>
<td>$1.053 \pm 0.007$</td>
<td>$7.243 \pm 0.010$</td>
<td>221 / 78</td>
</tr>
<tr>
<td>MC PYTHIA8, SD with vertex</td>
<td>$2.673 \pm 0.011$</td>
<td>$7.640 \pm 0.013$</td>
<td>262 / 78</td>
</tr>
</tbody>
</table>

Table 10.2: The values for a Gaussian fit in the range $[-0.01; 0.01]$ of the $\xi$ distributions in data and MC simulations.

when efficiencies are taken into account in the full simulation, the peak is expected to decrease further.

Table 10.2 shows the results of a Gaussian fit in $[-0.01; 0.01]$ where 0.01 is about one sigma in resolution at $\xi = 0$, see figure 9.16. The mean of $1.053 \cdot 10^{-3}$ in SD events in the MC without a vertex requirement shows that a peak in data with a mean at exactly $\xi = 0$ cannot be expected in data. The mean in the MC with a vertex requirement is closer to data, and when efficiencies of the ATLAS ID are taken into account it will most likely diminish the difference. However, one should keep in mind that also the elastic events are reconstructed with mean $\xi > 0$, see table 10.1, thus the difference is probably not only due to the limitations of the toy model MC.

The flat plateau at $0.05 < \xi < 0.15$ is not present in the MC for SD processes and is in disagreement with Regge theory, section 4.4.3. The vertex requirement raises the predicted cross section in this region, but there is still a decrease with increasing $\xi$. The contribution from ND is not large enough to explain the plateau. The cross section for ND is about 4 times larger than for SD, but only a small fraction of ND events produces protons with kinematics inside the ALFA acceptance.

The steep fall off in data at $\xi \approx 0.17$ is probably due to the ALFA acceptance shown in figure 8.1d in chapter 8. A high transverse momentum is necessary if protons with $\xi > 0.17$ should be within the ALFA acceptance, but the differential cross section fall rapidly with $p_T$ for SD events, section 4.4.3. This $p_T$ behaviour does not apply for ND events, hence this fall off is not seen for ND events and the predicted ND distribution actually overestimates the high-$\xi$ tail compared to data.

Reconstructed horizontal momentum

The reconstructed horizontal momentum distribution is shown in figure 10.5. About $0.04\%$ of the reconstructed events lie outside the ALFA acceptance region for $p_x$ inferred from figure 8.1f, and the majority of these events have a bad probability of the reconstruction fit.

The distribution is too broad compared with the SD MC samples shown in figure 10.6. The requirement of a vertex seems to slightly broaden the $p_x$ distribution. Both data and MC have a negative $p_x$ mean, which may be due to the asymmetry of the $p_x$ acceptance region shown in figure 8.1e and 8.1f.

In MC there is a clear bump at $p_x \approx -1.5$ which is not due to the acceptance. It can simply be ascribed to the double minimum structure in the reconstruction discussed in section 9.3 resulting in a worse resolution for negative $p_x$ values than for positive in figure 9.17. Events with a true horizontal momentum of $\sim -1$ GeV may therefore be reconstructed to give the observed bump. This effect is not as visible in data, possibly because of background events.

The ND events are able to explain only the outermost parts of the tails.
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[Graph showing distribution of reconstructed SD proton momenta]

**Figure 10.5:** Distribution of the reconstructed $p_x$ of SD protons in data.

**Figure 10.6:** Distributions of the reconstructed $p_x$ of protons in data and MC. The MC distributions are normalised to data. Only protons where $1 - \text{Prob}(\chi^2, \text{ndf}) < 68.27\%$ is plotted.

**Reconstructed vertical momentum**

The plots for $p_y$ are shown in figure 10.7 and 10.8. There are almost no events with $|p_y| < 0.1$ GeV due to the acceptance of the ALFA detector. Neither the data distribution nor the MC distributions are symmetric around $p_y = 0$, which is a consequence of the beam spot not being a $y = 0$ mm. While this effect does not play a role in the elastic events where we have the parallel-to-point feature. This is not the case in diffractive events where the energy might be low and the vertex effects come into play, see figure 9.7b. The positive $y$ coordinate of the beam spot therefore favours events with positive $p_y$ because the $\Delta C_y$’s are negative. Again, the distribution in data is to broad compared to the SD MC distributions. The contribution from ND events is somewhat ambiguous since it is larger than data at negative $p_y$ while reasonable at high $p_y$.

**Conclusion on the kinematic distributions**

The kinematic distributions in data are not in agreement with a simple toy model MC. The requirement of a reconstructed vertex in the MC gives a smaller discrepancy to the peak in data at $\xi \approx 0$ than without the vertex requirement, but neither of the two are able to explain the observed plateau at $0.05 < \xi < 0.15$. The two MC simulations give too narrow distributions of horizontal and vertical momentum.

The MC simulation of ND events does not improve the understanding of the kinematic distributions in data. It covers some parts of the tails in the horizontal and vertical momentum distributions while clearly over- and underestimates other parts. ND gives too large a contribution at $\xi > 0.25$ and does not significantly diminish the discrepancy at the plateau. A full simulation in the future may of course alter the behaviour of the ND sample.

Of course the discrepancy between data an the toy model MC can be ascribed
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Figure 10.7: Distribution of the reconstructed $p_y$ of SD protons in data.

Figure 10.8: Distributions of the reconstructed $p_y$ of protons in data and MC. The MC distributions are normalised to data. Only protons where $1 - \text{Prob}(\chi^2, \text{ndf}) < 68.27\%$ is plotted.

to a wrong theoretical understanding of the single diffractive processes. However, a careful study of background sources and the full simulated MC are needed before such a conclusion may be considered.

10.6 ATLAS ID and proton momentum correlation

In a perfect detector with full phase space covering and 100% efficiency for all particles, there should be a correlation factor of -1 between the momentum of the SD proton and the sum of momentum in ATLAS due to momentum conservation. This can of course not be expected in data since the ATLAS ID does not track neutral particles, it has a limited acceptance region, and only particles with $p_T > 100$ MeV have a reasonable tracking efficiency.

Figure 10.9 shows that the momentum correlation in data is small. The linear fit to the mean values fitted in $[-2, 2]$ GeV for $p_x$ and $[-0.4, 0.4]$ GeV for $p_y$ shows that on average there is a negative correlation for both $p_x$ and $p_y$ as expected, but it is not strong. The difference in the slopes between $p_x$ and $p_y$ are within the errors of the fit, hence the better resolution in reconstructed $p_y$ does not have an influence.

Figure 10.9 can be made for different intervals of $\xi$, and it turns out that the correlation decreases with increasing $\xi$ of the proton. This is somewhat surprising since higher $\xi$ means lower rapidity gap between the dissociated system and the proton, equation (4.24). Therefore, the fraction of particles of the system measured in ATLAS rises with $\xi$, and the momentum sum of the vertex should be less dominated by statistical fluctuations. The decreasing correlation is not due to background events, since the toy MC shows similar behaviour. Here, the correlation at $\xi \sim 0$ is stronger than in data, but at higher $\xi$ there seems to be no correlation at all. A full simulated MC may shed light on this issue.

On an event-by-event basis the correlation is non-existing. The RMS is of the order...
2 GeV in both the vertical and horizontal momentum and can not be used to improve the kinematic reconstruction algorithm. The ATLAS ID momentum measurement could have been used as input to guide the reconstruction fit if the correlation had been stronger on an event-by-event basis.

The ID also provides a measure of the invariant mass of the particles which puts an upper limit on the energy of the proton, see equation 4.16. From equation (4.16) we find

\[ E_3 \approx \frac{s - M^2}{2\sqrt{s}} \Rightarrow E_3 \lesssim \frac{s - M^2_{\text{measured}}}{2\sqrt{s}} . \]  

(10.4)

Unfortunately, the invariant mass in the SD events does not exceed 300 GeV which only gives an upper limit of the proton energy of \( \approx 3994 \) GeV. This is way to small an effect to be used to guide the kinematic reconstruction.

The inclusion of calorimeter data is likely to be a useful tool in the study of energy and momentum correlations between the proton tagged in ALFA and the dissociated system in ATLAS. First of all, the calorimeters provide an energy measurement in a much larger range of pseudorapidity than the ID which will lower the upper limit on the proton energy in equation (10.4). Hopefully, this will be useful in the reconstruction, at least for some events with large activity in the calorimeters. Secondly, the calorimeters can give an idea of the amount and direction of transverse momentum not measured in the ID.

Figure 10.9: Correlation between momentum measured in ATLAS ID and reconstructed momentum of proton in ALFA for events where \( 1 - \text{Prob}(\chi^2, \text{ndf}) < 68.27\% \). (a) is the horizontal momentum correlation and (b) is the vertical momentum correlation.
11  |  Conclusion

The work of this thesis has resulted in an algorithm used to reconstruct the kinematics of diffractively scattered protons at the LHC. The algorithm has been developed for a $\sqrt{s} = 8$ TeV, $\beta^* = 90$ m optics, but should in principle be easy to modify in order to be useful in other optics.

Knowledge about the magnetic lattice of the LHC between the interaction point (IP) in the ATLAS detector and the ALFA detector, where the protons are tagged, has been used to parametrize the proton impact in the ALFA detectors in the transverse plane to the beam at $z = 237$ m and $z = 241$ m. The $x$ and $y$ coordinates are almost decoupled since only dipoles and quadrupoles are located between ATLAS and ALFA. The $x$ coordinate depends on the energy, the horizontal momentum and the horizontal component of the IP and vice versa for the $y$ coordinate. Only the energy of the proton couples the two directions.

The $\chi^2$ function for a single diffractive proton includes the $(x, y)$ measurement of a reconstructed vertex in the ATLAS inner detector, which will be regarded as the IP, and the $(x, y)$ coordinates of the proton impact in the two ALFA stations at $z = 237$ m and $z = 241$ m. Minuit is used to minimize the $\chi^2$ function and thereby reconstruct the kinematics but also the IP. Much effort has been put into the minimization procedure since the multi-dimensionality of the fit parameter space as well as strong correlations between the fit parameters requires special treatment. Five fits with different starting values for energy and horizontal momentum are performed for each event to find the correct minimum. Simulating the detector resolutions of ATLAS and ALFA reveals that a local minimum in some of the events gives a kinematics closer to the true than the global minimum. The level of such events is about 1 %, but to determine a better estimate of these event in single diffractive data, a full Monte Carlo simulation is needed. It is found that taking the minimum of the five fits with the lowest energy and not the lowest $\chi^2$ gives the best result in total.

A simulation including the detector resolutions shows that the reconstruction has a relative resolution of the energy of $\sim 1\%$, a vertical momentum resolution of $\sim 1$ MeV and a horizontal momentum resolution of $\sim 100$ MeV. Uncertainties on the positions of the ALFA detectors of $\approx 5\ \mu$m are expected which gives rise to offsets in the reconstructed kinematics of about a factor 10 smaller than the resolution due to detector resolutions and is therefore not important. An uncertainty on the ATLAS position of $200\ \mu$m wrt. the LHC will give offsets of the same order as the resolution but may be eliminated using elastic data. An effective optics at $\sqrt{s} = 7$ TeV, $\beta^* = 90$ m has been found by the elastic analysis group and is used to estimate the effect of the uncertainty on the optics on the kinematic reconstruction. The effect on the energy reconstruction is about a factor 10 smaller than the resolution, whereas the optics effect and the resolution are comparable for the momentum reconstruction.

Cuts have been applied to data in order to obtain as pure a single diffractive
sample as possible. The kinematics of the protons in ALFA are reconstructed and the distributions of relative energy loss, vertical and horizontal momentum are in rough agreement with a simple toy model Monte Carlo simulation. A peak in the relative energy loss distribution at slightly above zero is observed which is expected from Regge theory. However, the distribution has a flat plateau which is not seen in the simulation and an explanation is still lacking. The transverse momentum distributions are broader than in the simulation. A simulated contamination of the single diffractive Monte Carlo sample with non-diffractive events does not seem to better explain the data.

Data shows a correlation factor of around -0.14 between the reconstructed momentum of the proton and the momentum sum of tracks of the vertex measured in ATLAS in both the horizontal and vertical direction. On an event-by-event basis, the correlation is too small to be useful to guide the kinematic reconstruction of the fit.

11.1 Outlook

The work with the kinematic reconstruction algorithm is almost finished at the $\sqrt{s} = 8$ TeV, $\beta^* = 90$ m optics. Elastic data will provide an effective optics in the future which can be used to find the true effect on the kinematic reconstruction from the optics uncertainty. The elastic data at $\sqrt{s} = 7$ TeV, $\beta^* = 90$ m optics shows no sign of an ATLAS offset, but this can be investigated further at $\sqrt{s} = 8$ TeV where more data is available.

ALFA is planned to be running in the beginning after the first long shut down of the LHC where the luminosity will be low. Design optics at $\sqrt{s} = 14$ TeV, which are already available, can be used to optimize the reconstruction algorithm to work at this new energy regime.

The data analysis is still on a very preliminary level. The effective optics and the final alignment of the ALFA detectors will give a more correct kinematic reconstruction. Calorimeter data can be used to select single diffractive events and make the study of rapidity gaps possible. A full Monte Carlo simulation will give a clear sign of the discrepancies (if any) between expectations and data.

The study of central diffractive processes is in progress. It is of course interesting in its own rights, but may also provide insight in the amount of background in the single diffractive sample.
Appendices
A | Rapidity calculations

The transformation of rapidity under a Lorentz boost along the $z$-axes:

$$
y = \frac{1}{2} \ln \frac{E + p_z c}{E - p_z c}
$$

$$
= \frac{1}{2} \ln \gamma (E + \beta p_z) + \gamma (p_z + \beta E)
- \frac{1}{2} \ln \gamma (E - p_z) - \gamma (p_z - \beta E)
$$

$$
= \frac{1}{2} \ln \gamma [(E + p_z)(1 + \beta)]
- \frac{1}{2} \ln \gamma [(E - p_z)(1 - \beta)]
$$

$$
= y + \frac{1}{2} \ln \left(1 + \frac{\beta}{1 - \beta}\right)
$$

$$
= y + \frac{1}{2} \ln \left(\gamma \sqrt{1 + \beta}\right), \quad (A.1)
$$

In the massless limit, the rapidity can be approximated by the pseudorapidity:

$$
y = \frac{1}{2} \ln \left[\frac{E + p_z c}{E - p_z c}\right]
\approx \frac{1}{2} \ln \left[\frac{|p| c + |p| c \cos \theta}{|p| c - |p| c \cos \theta}\right]
$$

$$
= \frac{1}{2} \ln \left[\frac{1 + \cos \theta}{1 - \cos \theta}\right]
$$

$$
= \ln \left[\sqrt{\frac{(1 + \cos \theta)(1 + \cos \theta)}{(1 - \cos \theta)(1 + \cos \theta)}\right]
$$

$$
= -\ln \left[\frac{1 - \cos \theta}{\sin \theta}\right]
$$

$$
= -\ln \left[i \frac{2 - e^{i \theta} + e^{-i \theta}}{e^{i \theta} - e^{-i \theta}}\right]
$$

$$
= -\ln \left[i \frac{(e^{i \theta/2} - e^{-i \theta/2})^2}{(e^{i \theta/2} - e^{-i \theta/2})(e^{i \theta/2} - e^{-i \theta/2})}\right]
$$

$$
= -\ln \left[\tan \frac{\theta}{2}\right]
= \eta. \quad (A.2)
$$
User Manual for the kinematic reconstruction algorithm

The kinematic reconstruction algorithm is made as a class with a constructor and some functions.

B.1 Constructor

The constructor takes four inputs. The first input is a file where the relevant information about the optics is given. Choosing an input file gives the user the opportunity to use the reconstruction code for different optics. At this point, only files for the $\beta^* = 90$ m optics at $\sqrt{s} = 7$ and 8 TeV are available since these optics are the only two where the reconstruction has been fully tested. However, the code is made in a way such that the generalisation to other optics should be easy, but the exact minimization procedure, which takes care of the double minimum structure, may change.

The main content of the input file are coordinates in ALFA at the two stations together with the corresponding values of $E$, $p_x$ and $p_y$. There are four sets of coordinates for each energy since two combinations of $p_x, p_y$ and two combinations of $v_x, v_y$ are needed to find the parametrization of the proton trajectory at a given energy. The other information stored in the input file are the fit ranges and step sizes of $E, p_x, p_y$ as well as the beam energy and the crossing angle between the colliding beams. The transverse momentum components are not required to be within the fit ranges, but the ranges are used to find start values. The crossing angle is zero in the optics analysed in this thesis, but this is not always the case and must be taken into account by the input file.

The second input to the constructor determines whether interpolators in energy or fourth degree polynomials in inverse total momentum of the proton are used for the parametrisation. Interpolators are chosen as default as this is the most precise, but the polynomials are slightly faster.

The last two inputs to the constructor are the $(x, y)$ coordinates of the average beam spot for elastic events over the entire run. These are used to make the coordinate transformation of the ALFA coordinates described in section 9.1.3. The user has to find the beam spot himself.
B.2 Functions

The class has several functions available to the user:

**ProtonRec** is the most important one as it is the function performing the minimization. The first two inputs are vectors of doubles. *ProtonRec* saves the final parameter estimates in the first vector and the estimated errors in the second one. The next four inputs are the \((x, y)\)-coordinates of the IP and the errors either taken from the reconstructed vertex in the ATLAS ID or the beam spot. The next six arguments are \((x_{237}, y_{237}, z_{237})\) and \((x_{241}, y_{241}, z_{241})\). The \(z\) coordinates are only used to determine whether it is A-side or C-side. Only one \(z\) coordinate is needed, but both must be given to *ProtonRec* in order to avoid confusion for the user. The last six arguments are optional and are the coordinates of the second proton if the user wants a two-proton fit.

**SetALFAResolution** gives the user the opportunity to change the default values of the ALFA resolution, which is 30 microns for the inner stations and 40 microns for the outer. This may be useful if at some point one want to assign a resolution depending on for instance the number of layer hits in ALFA.

**SetPrintLevel** determines how much information from the minimization should be printed. This is useful in debugging if *ProtonRec* returns strange parameters. The function is the same as for ROOT::Math::Minimizer [55].

**SetMaxIterations** determines the number of allowed iteration. By default, this number is so high that will never have any impact. It is the same as for ROOT::Math::Minimizer.

**SetMaxFunctionCalls** determines how many times the \(\chi^2\) function (9.10) can be called for each iteration. It is the same as for ROOT::Math::Minimizer.

**FunctionValue** returns the \(\chi^2\) at the minimum. The goodness of the fit as a probability can be calculated from the \(\chi^2\) and the number of degrees of freedom (NDF). The NDF is one for the one-proton fit and two for the two-proton fit since the protons are forced to have the same interaction point.

**FunctionDerivative** returns the derivative of the \(\chi^2\) wrt. the parameters. These should of course be close to one at the minimum.
C | Plots for the kinematic reconstruction
C.1 Pull distributions

Figure C.1: Pull distributions for the five parameters, where $\sigma$ are the errors returned by Minuit. The vertex position and ALFA coordinates have been smeared with a gaussian number with a width of 0.3 mm and 30 $\mu$m, respectively. The values for $\chi^2$, $\mu$ and $\sigma$ are for a gaussian fit in the range $[-2; 2]$.
C.2 Plots for systematic uncertainties

ALFA offset

Figure C.2: The difference between the true and the reconstructed horizontal momentum from simulation for a horizontal offset of -5 \( \mu m \) in the inner ALFA detector and 5 \( \mu m \) in the outer at (a) \( E_{proton} = 3500 \) GeV and (b) \( E_{proton} = 4000 \) GeV. Error bars indicate the RMS of the distributions, and each bin contains 5000 events.

Figure C.3: The difference between the true and the reconstructed vertical momentum from simulation for a vertical offset of -5 \( \mu m \) in the inner ALFA detector and 5 \( \mu m \) in the outer at (a) \( E_{proton} = 3500 \) GeV and (b) \( E_{proton} = 4000 \) GeV. Error bars indicate the RMS of the distributions, and each bin contains 5000 events. The large error bars, which are present for some bins, are due to few outliers.
**APPENDIX C. PLOTS FOR THE KINEMATIC RECONSTRUCTION**

**ATLAS offset**

![Graph](image)

**Figure C.4:** The difference between the true and the reconstructed horizontal momentum from simulation for an offset of 200 µm in x and y between ATLAS and LHC CS at (a) $E_{\text{proton}} = 3500$ GeV and (b) $E_{\text{proton}} = 4000$ GeV. Error bars indicate the RMS of the distributions, and each bin contains 5000 events.

![Graph](image)

**Figure C.5:** The difference between the true and the reconstructed vertical momentum from simulation for an offset of 200 µm in x and y between ATLAS and LHC CS at (a) $E_{\text{proton}} = 3500$ GeV and (b) $E_{\text{proton}} = 4000$ GeV. Error bars indicate the RMS of the distributions, and each bin contains 5000 events.
APPENDIX C. PLOTS FOR THE KINEMATIC RECONSTRUCTION

C.3 Plots for comparison of the 7 TeV optics

Figure C.6: Reconstructed horizontal momentum for elastic protons in run 191373 with design and effective 7 TeV, $\beta^* = 90$ m optics.

Figure C.7: Reconstructed vertical momentum for elastic protons in run 191373 with design and effective 7 TeV, $\beta^* = 90$ m optics.
Figure C.8: Reconstructed horizontal vertex coordinate for elastic protons in run 191373 with design and effective 7 TeV, $\beta^* = 90$ m optics.

Figure C.9: Reconstructed vertical vertex coordinate for elastic protons in run 191373 with design and effective 7 TeV, $\beta^* = 90$ m optics.
Bibliography


BIBLIOGRAPHY


[37] The ALFA group. Measurement of the total cross section in pp collisions 1 at $\sqrt{s} = 7$ TeV from elastic scattering with the ATLAS detector. *Work in progress*.


[45] Private communication with Karlheinz Hiller.


